Available online at www.sciencedirect.com

ScienceDirect

Procedia Engineering 127 (2015) 485 – 492

Procedia Engineering

www.elsevier.com/locate/procedia

International Conference on Computational Heat and Mass Transfer-2015

Flow and Heat Transfer of Quiescent Non-Newtonian Power-Law Fluid Driven by a Moving Plate: An Integral Approach

BH.Lakshmipathi Raju[a]*,V.Dharma Rao[b], B.Bala Krishna[c]

[a]*Mechanical Department,GITAM University,Visakhapatnam-45, Inida*
[b]*Mechanical Department,GVPCE,Visakhapatnam-48,Inida ,*[c]*Mechanical Department,JNTUK,Kakinada-03,Inida*

Abstract

The problem of boundary layer flow and heat transfer from a plate moving in a quiescent non-Newtonian power law fluid medium is solved numerically to obtain results showing the effects of power-law index and Reynolds number on friction and heat transfer coefficients. A decrease in friction coefficients and an increase in heat transfer coefficients is observed for power-law liquids in comparison with water. Further in the case of a power-law liquid, an increase in the velocity of the plate also produces a reduction in drag and an increase in heat transfer. Compared to a Newtonian liquid, the friction coefficient decreases by 57% for a non-Newtonian liquid of power-law index, n = 0.568 at Re=2000. By increasing the Reynolds number to 10000, the friction coefficient decreases to 66% in comparison with a Newtonian liquid.

© 2015 The Authors. Published by Elsevier Ltd. This is an open access article under the CC BY-NC-ND license (http://creativecommons.org/licenses/by-nc-nd/4.0/).
Peer-review under responsibility of the organizing committee of ICCHMT – 2015

Keywords: laminar; sakiadis flow; power law fluid; integral method; heat transfer.

1. Introduction

When it is desired to reduce drag in ships and under-water vehicles, water-soluble polymer in concentrated form is ejected through the bow part of the vehicle, which dissolves in water, forms a boundary layer close to the body of the vehicle, and helps in reducing the drag [1]. The fluid in boundary layer contains dilute polymer

* Corresponding author. Tel.: +91990878204;
*E-mail address:*bhlpr19@rediffmail.com

1877-7058 © 2015 The Authors. Published by Elsevier Ltd. This is an open access article under the CC BY-NC-ND license (http://creativecommons.org/licenses/by-nc-nd/4.0/).
Peer-review under responsibility of the organizing committee of ICCHMT – 2015
doi:10.1016/j.proeng.2015.11.402

solution, and thus is a non-Newtonian fluid. The body of the vehicle can be considered a moving plate immersed in quiescent water medium. The applications of this phenomenon can also be found in many industrial applications such as polymer extrusion, drawing of copper wires, continuous stretching of plastic films and artificial fibres, hot rolling, wire drawing, glass-fibre, metal extrusion, and metal spinning [2]. This problem has been tackled in the case of Newtonian fluids using several analytical and experimental techniques. Sakiadis [3] analysed such flows in Newtonian fluids based on the boundary layer concept using similarity solutions and thereafter the flow is named as Sakiadis flow. This work is extended to heat transfer applications by Tsou et al. [4] who conducted experiments to analyse the boundary layer flow over a moving wall subjected to a constant wall temperature. The case of exponentially stretching sheet in viscous flow is studied by Crane [5]. Erickson et al. [6] investigated numerically the effect of blowing or suction at stretching sheet under constant speed on heat and mass transfer characteristics within the boundary layer flow. Similar work with exponentially stretching sheet is studied by Elbashbeshy [7] and results are obtained using similarity solution method. The effect of temperature dependent viscosity on steady flow and heat transfer in the boundary layer flow is studied by Hossain and Munir [8]. Similar flow configuration with different thermal boundary conditions is studied by various researchers (Fang[9], Fang and Lee [10]). All the works cited above are concerned with the boundary layer flow of Newtonian fluids. However the industrial applications, such as drag reduction use non-Newtonian liquids. Schowalter [11] extended the stretching sheet problem to non-Newtonian pseudo-plastic fluid and obtained similarity solutions. Acrivos et al. [12] also studied the same problem for the generalized power-law non-Newtonian fluids. Aziz [13] obtained flow and heat transfer solutions by direct method for permeable stretching sheet under convective surface boundary condition.

No theoretical studies are found on the hydrodynamics and heat transfer related to the moving plate in a quiescent non-Newtonian power-law fluid medium. Hence the same is theoretically tackled using von Karman integral method to obtain the momentum and heat transfer characteristics. The velocity and temperature profiles are presented for different values of power-law indices of non-Newtonian power-law fluid flow over a moving plate.

2. Physical model and Mathematical formulation

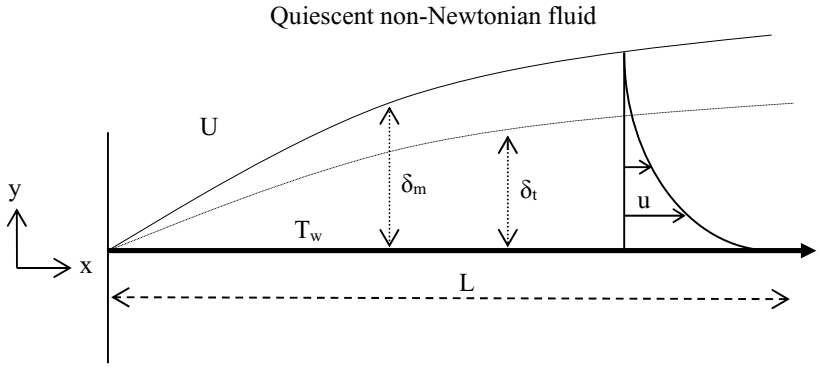

Fig.1 Schematic diagram of boundary layer flow developed by moving plate

Fig.1. shows the boundary layer flow of quiescent non-Newtonian fluid over a moving plate. The plate is moving with a constant velocity (U) is maintained at a constant temperature (T_w). The temperature of quiescent non-Newtonian fluid is at a temperature T_∞. The induced flow of the fluid is laminar. δ and δ_t are thicknesses of momentum and thermal boundary layers respectively. It can be found from the velocity profile shown in Fig.1 that the velocity of the fluid at the plate is equal to that of the plate, and gradually decreases to zero at the end of the boundary layer. Viscous heat dissipation effect is neglected. The problem is governed by the continuity, momentum and energy balance equations as shown below.

Continuity equation

$$\frac{\partial u}{\partial x}+\frac{\partial v}{\partial y}=0 \tag{1}$$

Momentum balance equation

$$u\frac{\partial u}{\partial x}+v\frac{\partial v}{\partial y}=\frac{1}{\rho}\left(\frac{\partial \tau}{\partial y}\right) \tag{2}$$

The shear stress (τ) of the fluid is defined by power-law (Ostwald-de-Wäle) model for non-Newtonian fluid flow

$$\tau = K\left(\frac{\partial u}{\partial y}\right)^n \tag{3}$$

Energy balance equation

$$u\frac{\partial T}{\partial x}+v\frac{\partial T}{\partial y}=\alpha\frac{\partial}{\partial y}\left(\frac{\partial T}{\partial y}\right) \tag{4}$$

Boundary conditions:

u=U at y=0, and

u=0 and (du/dy)=0 at y= δ (5)

$T=T_w$ at y=0 and

$T=T_\infty$, (dT/dy)=0 and $(d^2T/dy^2)=0$ at $y=\delta_t$ (6)

2. 1. Integral momentum balance equation

The momentum equation is transformed into integral form using von-Karman momentum integral method.

$$\frac{d}{dx}\int_0^\delta \frac{u}{U}\left(1-\frac{u}{U}\right)dy = -\frac{K}{\rho U^2}\left(-\frac{\partial u}{\partial y}\right)^n_{y=0} \tag{7}$$

A second order polynomial is assumed for the velocity profile with the aid of the boundary conditions given in Eq.(5).

$$\frac{u}{U}=\left(1-\frac{y}{\delta}\right)^2=(1-\eta)^2 \tag{8}$$

where $\eta=\frac{y}{\delta}$

2. 2. Hydrodynamic boundary layer

Substituting Eq. (8) in Eq. (7) and rearranging

$$\frac{d\delta}{dx}=\frac{K}{\rho U^2}\left(\frac{2U}{\delta}\right)^n\frac{15}{2}=\frac{2^{(n-1)}15}{\mathrm{Re}_\delta} \tag{9}$$

where $\mathrm{Re}_\delta=\frac{\rho U^{(2-n)}\delta^n}{K}$ (10)

Integrating Eq. (9) to obtain δ

$$\frac{\delta}{x}=\left(\frac{15(n+1)2^{(n-1)}}{\mathrm{Re}_x}\right)^{\frac{1}{1+n}}=\left(\frac{C}{\mathrm{Re}_x}\right)^{\frac{1}{1+n}} \tag{11}$$

here $\mathrm{Re}_x=\frac{\rho U^{(2-n)}x^n}{K}$ and $C=15(n+1)2^{(n-1)}$ (12)

2. 3. Local friction coefficient

The friction coefficient can be computed from the shear stress by using the following relation

$$\frac{f}{2}=\frac{\tau}{\rho U^2}=\frac{K}{\rho U^2}\left(\frac{2U}{\delta}\right)^n \tag{13}$$

The friction coefficient can be obtained by using the equations (11) and (13)

$$\frac{f}{2}=\frac{2^{\left(\frac{2n}{1+n}\right)}}{15^{\left(\frac{n}{1+n}\right)}(1+n)^{\left(\frac{n}{1+n}\right)}}\left(\frac{1}{\mathrm{Re}_x^{\left(\frac{1}{1+n}\right)}}\right)=\left(\frac{C'}{\mathrm{Re}_x}\right)^{\frac{1}{1+n}} \tag{14}$$

where $C'=\dfrac{2^{2n}}{15^n(1+n)^n}$

3. Integral Energy Balance Equation

The energy balance equation, viz., Eq. (4) is integrated partially with respect to y to get

$$\frac{d}{dx}\int_0^{\delta_t}u(T_\infty-T)dy=\alpha\left(\frac{dT}{dy}\right)_{y=0} \tag{15}$$

Temperature profile is assumed up to 3rd order polynomial and obtained temperature profile after imposing thermal boundary conditions Eq. (6) as

$$\frac{T-T_w}{T_\infty-T_w}=1-\left(1-\frac{y}{\delta_t}\right)^3 \tag{16}$$

The following equation is obtained by solving the Eqs. (8), (16) and (15)

$$(15-12\xi+3\xi^2)\frac{d\delta_t}{dx}=\frac{180\alpha}{\delta_t U}+(2\xi^3-\xi^2)\frac{2^{(n-1)}15}{\mathrm{Re}_\delta} \tag{17}$$

where, $\dfrac{\delta_t}{\delta}=\xi$ (18)

3.1. Local Prandtl number

The thermal boundary layer thickness is obtained by solving the equation (17) using Euler's method.

$$\delta_t=\left[\left(\frac{24}{15}\right)\left(\frac{\alpha}{2^{n-1}}\right)\frac{\rho U^{(1-n)}}{(n+1)K}\right]^{0.5}\delta^{\frac{n+1}{2}} \tag{19}$$

The simplified form of ξ

$$\xi=\left(\frac{24}{15(n+1)}\right)^{1/2}\mathrm{Pr}^{-1/2} \tag{20}$$

where $\mathrm{Pr}=\dfrac{C_p}{k}K\left(\dfrac{2U}{\delta}\right)^{n-1}$ (21)

3.2 Local Nusselt number

The local convection heat transfer coefficient is defined by the equation

$$\left(\frac{\partial T}{\partial y}\right)_{y=0}=h\left(T_w-T_\infty\right) \tag{22}$$

The Nusselt number equation can be obtained by using the heat fluxes of conduction and convection at the moving plate and given as

$$Nu = \frac{hx}{k} = \frac{3x}{\delta_t} \tag{23}$$

4. Results and Discussion

The mathematical expressions developed in the previous section have been solved for the study of hydrodynamic and heat transfer in a quiescent power-law fluid driven by a moving plate for different combinations of power-law index, Reynolds number and Prandtl number. Results are obtained for different combinations of above-said parameters.

4.1. Frictional characteristics

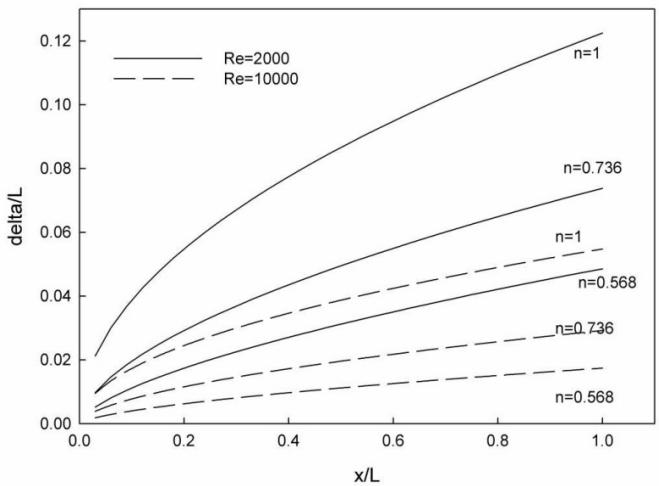

Fig.2variation of dimensionless boundary layer thicknessalong the plate for different Reynolds numbers

To analyze the variation of momentum boundary layer thickness, equation (11) has been solved for different n and Re. Fig. 2 represents the variation of boundary layer thickness along the plate for different values of n and Reynolds numbers. The thickness of the hydrodynamic boundary layer decreases with an increase in power law index n. Also δ decreases with an increase in Reynolds number for all n. The friction coefficient calculated from the equation (14).

The effect of Reynolds number and power-law index on local friction coefficient over the plate is shown in Fig3. The friction coefficient at any x increases with an increase in n or a decrease in Re. The theoretical results of Papanastasiou et al. [14] for a Newtonian fluid driven by a moving plate are also shown in Fig. 3 for validation of the present theory. It can be found that the results, obtained from the present theory for n = 1, are in good agreement with those of Papanastasiou et al. [14].The average friction coefficient is an integrated average of the local friction coefficient over the prescribed length of the plate.

The average friction coefficient is an integrated average of the local friction coefficient over the prescribed length of the plate.

The average friction coefficient is plotted versus Reynolds number in Fig. 4 for different values of n. For any Re, the average friction coefficient increases with an increase in n.

The results presented in Figs. 3 and 4 show a remarkable decrease in friction coefficients or the amount of drag in the case of power-law fluids in comparison with Newtonian fluids.

The average friction coefficient reduces to 57% of that of Newtonian liquid for n = 0.568 at Re = 2000. The drag further reduces to 66% at Re = 10000, viz., when the velocity of the plate is increased by five times.

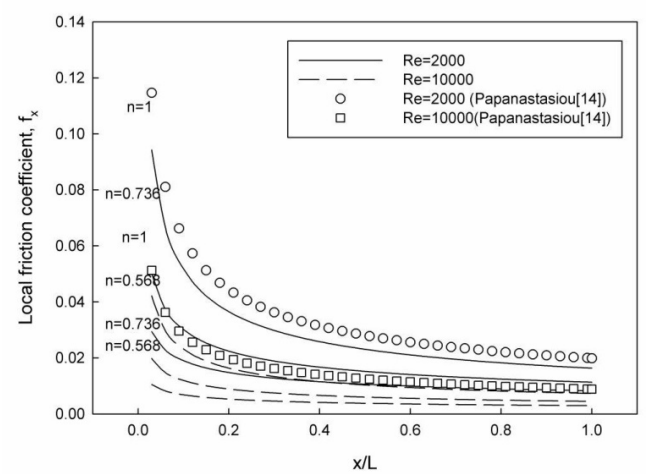

Fig.3Effect of power law index on the variation of local friction coefficient along the length of the plate for different Reynolds numbers

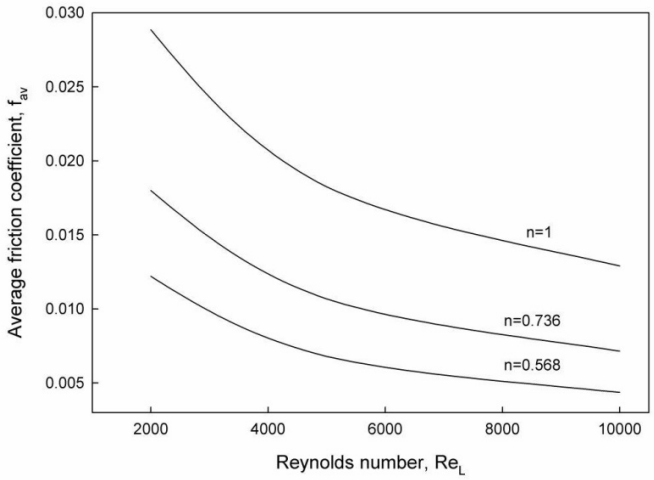

Fig.4variation of average friction coefficient for different Reynolds numbers at n=0.568, 0.736, and 1.0

4.2 Heat transfer characteristics

The ratio of the thermal and momentum boundary layer thickness ξ is calculated from Eq. (17) and are shown plotted in Fig. 5. The effect of n on the variation of zeta (ξ) along the length of the plate is presented in Fig. 5, which indicates that ξ decreases as n decreases.

The local Nusselt numbers are obtained from Eq. (23). Figure 6 shows the variation of local Nusselt number along the length of the plate for different power law indices and Reynolds. Fig 6 shows that the local Nusselt number increases with an increase in n.

Fig. 7 illustrates the effect of Reynolds number on the average Nusselt number. The Nusselt number is found to be larger for non-Newtonian power-law fluids compared to Newtonian fluids. A comparison of the fluids of n=0.736 and n=0.568 shows that the Nusselt number increases by 90%, 93.8% at Re=2000, and by 88.6%, 92.5% at Re=10000. Hence the heat transfer coefficients increase by increase the speed of the moving plate.

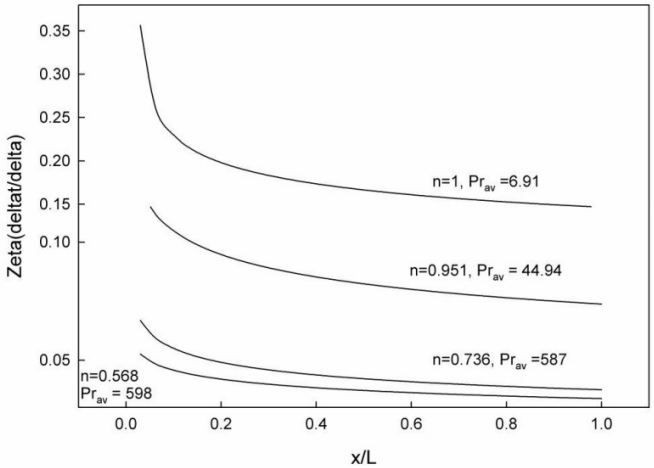

Fig.5 variation of Zeta along the length of the plate for different power-law indices

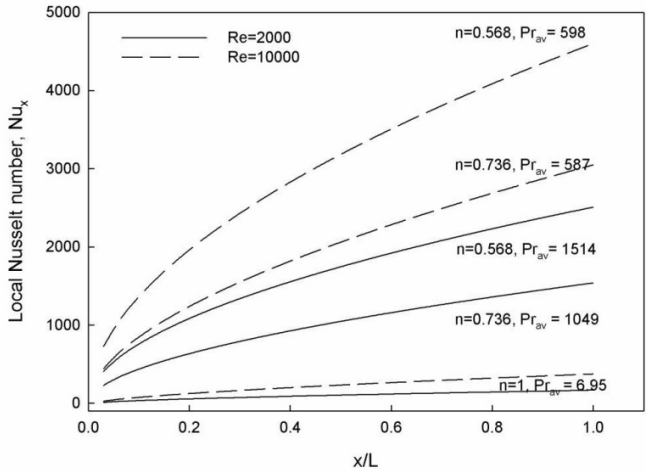

Fig.6 Effect of power-law index on variation of local Nusselt number along the length of plate for different Reynolds numbers

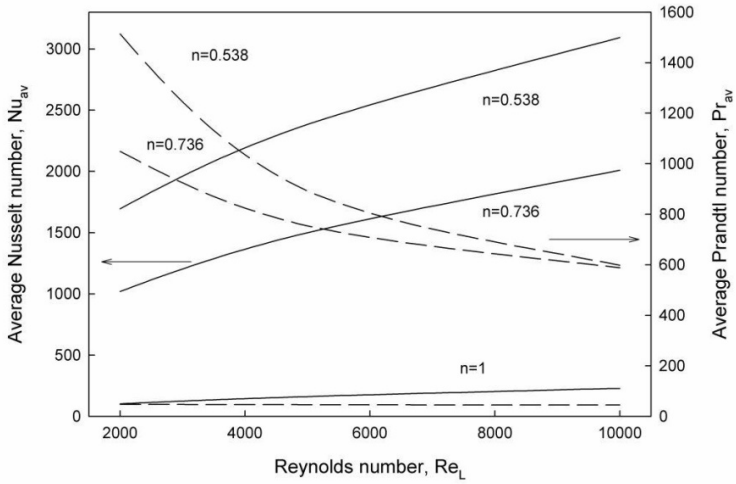

Fig.7 variation of average Nusselt and Prandtl numbers with Reynolds numbers for different power-law indices

The theoretical results for the local friction coefficients and Nusselt numbers obtained from the present analysis for different values of Re_x and n are subjected to non-linear regression analysis to yield the following equation.

$$f = \frac{0.931n^{1.297}}{\mathrm{Re}_x^{0.5313}} \tag{24}$$

Standard deviation = 2.9%

$$Nu_x = 0.5487n^{-1.099}\,\mathrm{Re}_x^{0.6288}\,\mathrm{Pr}_x^{0.3888} \tag{25}$$

Standard deviation = 6.6%

$$\overline{Nu} = 0.9421n^{-1.686}\,\mathrm{Re}_L^{0.5316}\,\overline{\mathrm{Pr}}^{0.3628} \tag{26}$$

Standard deviation = 1.78%

5. Conclusion

The present work provides analytical solutions for boundary layer flow and heat transfer coefficients of moving plate through a quiescent power law fluid. The effect of power-law index and Reynolds number on friction and heat transfer has been analyzed.

It is found from the numerical results that there is a decrease in friction coefficients and an increase in heat transfer coefficients with a decrease in the power-law index n or with an increase in Reynolds number.

A moving plate experiences lesser drag in non-Newtonian power-law fluids than that in a Newtonian liquid. If its velocity is increased there is a further decrease in drag on the plate.

However the convection heat transfer coefficients are more for the moving plate in the power-law liquids than those in Newtonian liquids

References

[1] Fabula, Andrew G., James H. Green, and William F. Madison. Torpedo drag reduction employing polymer ejection. U.S. Patent No. 4,186,679. 5 Feb. (1980).

[2] Xu, Hang, and Shi-Jun Liao. "Laminar flow and heat transfer in the boundary-layer of non-Newtonian fluids over a stretching flat sheet." *Computers & Mathematics with Applications* 57.9 (2009): 1425-1431.

[3] Sakiadis, B. C. Boundary-layer behavior on continuous solid surfaces: II. The boundary layer on a continuous flat surface. AIChE Journal 7.2, 221-225 (1961).

[4] Tsou, F. K., E. M. Sparrow, and R. Jh Goldstein. Flow and heat transfer in the boundary layer on a continuous moving surface. International Journal of Heat and Mass Transfer 10.2, 219-235 (1967).

[5] Crane, Lawrence J. Flow past a stretching plate. Zeitschrift für angewandte Mathematik und Physik ZAMP 21.4, 645-647 (1970).

[6] Erickson, L. E., L. T. Fan, and V. G. Fox. Heat and mass transfer on moving continuous flat plate with suction or injection. Industrial & Engineering Chemistry Fundamentals 5.1, 19-25 (1966).

[7] Elbashbeshy, E. M. A. Heat transfer over an exponentially stretching continuous surface with suction. Archives of Mechanics 53.6, 643-651 (2001).

[8] Hossain, Md Anwar, Md Sazzad Munir, and David Andrew S. Rees. Flow of viscous incompressible fluid with temperature dependent viscosity and thermal conductivity past a permeable wedge with uniform surface heat flux. International journal of thermal sciences 39.6, 635-644 (2000).

[9] Fang, T. Similarity solutions for a moving-flat plate thermal boundary layer. Acta mechanica 163.3-4, 161-172 (2003).

[10] Fang, Tiegang, and F. Lee Chia-fon. A moving-wall boundary layer flow of a slightly rarefied gas free stream over a moving flat plate. Applied mathematics letters 18.5, 487-495 (2005)

[11] Schowalter, W. R. The application of boundary-layer theory to power-law pseudoplastic fluids: Similar solutions. AIChE Journal 6.1, 24-28 (1960).

[12] Acrivos, Andreas, M. J. Shah, and E. E. Petersen. Momentum and heat transfer in laminar boundary-layer flows of non-Newtonian fluids past external surfaces. AIChE Journal 6.2, 312-317 (1960).

[13] Aziz, Abdul. A similarity solution for laminar thermal boundary layer over a flat plate with a convective surface boundary condition. Communications in Nonlinear Science and Numerical Simulation 14.4, 1064-1068 (2009).

[14] Papanastasiou, T., Georgiou, G., & Alexandrou, A. N. (1999). Viscous fluid flow. CRC Press.