STOCHASTIC MODELLING AND OPTIMIZATION OF WATER RESOURCES SYSTEMS

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Communicated by Richard Bellman

Abstract—This work involves the simultaneous optimization of the initial design and operating policy over the life of multipurpose multiservoir water resources systems receiving stochastic inflows. The approach is based on the division of the reservoir into two imaginary water storage pools, namely, the conservation and flood pools. Based on this treatment, the optimization problem is stated using the concepts of Lagrange multipliers and parameter optimization. Two nonlinear programming techniques, namely, the generalized reduced gradient technique and the gradient projection technique, combined independently with Markovian decision are proposed to solve such a problem. To illustrate the use of the proposed techniques, the Walnut River Basin in southeastern Kansas, is employed in this work.

INTRODUCTION

Much attention and effort have been placed upon the planning, design, and operation of water resources systems, because of their importance to national welfare. The multiple uses of a water resources system can be approximately classified into three major types: withdrawal consumptive use, nonwithdrawal nonconsumptive use, and withholding (or retardation) use. Irrigation and urban water supply belong to the first type, hydroelectric power generation and recreation to the second type, and flood control to the third type. The multipurpose water resources system may be investigated under the assumptions of steady-state operating conditions, dynamic or unsteady state operating conditions, dynamic or unsteady state operating conditions, and stochastic conditions. All three types of problems are complicated for large water resources systems. However, the degree of complications and the nature of difficulties are different. Of the above three types of problems, the stochastic type has so far received only some, but not much, attention. However, in real life, water resources systems are stochastic in nature. The purpose of this work is twofold: one is to build a realistic but computationally amenable stochastic model which includes the benefits or losses of the complementary and/or competing water uses; the other is to develop efficient computational procedures to optimize such a model.

Several papers have appeared on the modelling and optimization of water reservoirs. Cocks [7], Loucks [17, 19], Revelle et al. [23], Yeh et al. [32, 33], Houck and Cohon [34, 35], and Nelson et al. [36] applied linear programming to solve reservoir optimization problems. Kirshen [37], Nieh [38], and Windsor [39] used mixed integer programming
optimization models to reservoir design. Jonch-Clausen [40], and Koris and Nagy [41], employed quadratic programming for the optimal regulation of reservoirs. Turgeon [42], and Chu and Yeh [43], determined optimal reservoir operations through nonlinear programming. Bather [2], Buras [3,4], Butcher [5], Fukao and Nureki [11,12], Hall et al. [13], Loucks [18], Meier and Beightler [20], Mobasher et al. [21], Strupczewski [25], Tzvetanov [27], Young [28], Tauxe et al. [44,45], Beard and Chang [46], Jensen et al. [47], Sniedovich [48], Arunkumar [49], Bogardi et al. [50], Klemes [51], and Glanville [52] resorted to dynamic programming to obtain reservoir operating policies. Prekopa et al. [53] applied stochastic programming to design serially linked reservoirs.

Eichert and Davis [54], Maidment and Chow [55], Keifer et al. [56], Riley and Scherer [57], Deb [58], Stedinger and Bell-Graf [59], Kaczmarek et al. [60], Hall [61], Bogardi et al. [62], Thompstone et al. [63], Croley II and Rao [64], and Franke and Leipold [65] dealt with systems analysis in the management of water reservoirs. Chow [6], Hufschmidt and Fiering [14], Law [16], Young et al. [29], Major and Lenten [66], White and Christophodoulou [67], Eichert [68], Singh [69], Sniedovich [70], and Wright [71] discussed water resources simulation procedures. Sung [72], Olenik [73], Mades and Tauxe [74], Muselman and Talavage [75], Goicoechea et al. [76], Ambrosino et al. [77], Loucks [78,81], Sakawa [79], Neuman and Krzysztofowicz [80], and Passy [82] considered multiobjective analysis in water resources planning. Fiering [10], Langbein [15], and Thomas and Watermeyer [26] used queueing theory in the design of water resources systems. Morel-Seytoux [83], and Lauffer and Morel-Seytoux [84] employed decision theory for reservoir optimization. Ikada and Yoshikawa [85] presented a game theory approach to water resources planning. Gal [9] formulated a parameters iteration method for optimal reservoir management. Arunkumar and Chou [86], and Akleswaran et al. [87], devised heuristic approaches for the optimal control of reservoirs. All the above publications treated a multipurpose reservoir to be a single lumped pool of water for modelling and optimization purposes. Also, they did not consider the simultaneous treatment of the various water benefits, along with the losses when the demands are not met.

This paper finds the best initial design and the optimal operating policy over the life of multipurpose multireservoir systems, without the unrealistic assumption of a single lumped pool of water. The objective is to optimize the flood and various water usage benefits/losses simultaneously along with the capital and operating costs.

A multipurpose reservoir generally serves two basically different usages, namely the prevention of flood damages during excessively high inflow periods and the conservation of water for various purposes, or uses during drought periods. Accordingly, it is proposed to consider a multipurpose reservoir to be consisting of two water storage pools, namely, the flood pool and the conservation pool. Thus, the flood pool is for the storage of accumulated water during flooding situations; and the conservation pool is for the storage of water for various purposes such as water supply, water quality control, recreation, etc. Apart from the above two pools, there is another pool known as the sedimentation pool for sedimentation purposes. It may be noted that the sedimentation pool is the bottom pool, the conservation pool is the middle pool and the flood pool is the top pool. The sedimentation pool volume is determined by the soil condition and the probable sedimentation rate. Thus, the sedimentation pool capacity is a given constant and need not be considered for modelling and optimization purposes. Then the best initial design involves finding the optimal conservation and flood pool capacities.

In general, the operating policy is designed to meet the following requirements: (i) to maintain a nearly constant conservation pool level to meet the various water demands; (ii) to maintain a nearly zero flood pool level, except during flooding situations, to store future flood waters; and (iii) to maintain a minimum release so that sufficient water is maintained in the channel below the reservoir for the conservation of fish and wildlife.
Then the optimal operating policy is to determine the actual optimal values of the decision and state variables.

The major difficulty in water resources modelling and optimization lies in the stochastic nature of the inflows to the reservoirs. Because of this, stochastic modelling and optimization should be used. The stochastic inflows may be "independent" or "serially correlated." In this paper "first order" or "lag one" serially correlated inflows are considered. This means that the inflow of each month is dependent only on the inflow of the previous month, forming a Markov chain. The stochastic optimization procedures advocated in this work are the two versions of the nonlinear programming approach, namely, the generalized reduced gradient (GRG) technique \[11\] and the gradient projection (GP) technique \[24\], each separately combined with Markovian decision.

The basic idea of combining the GRG/GP technique with Markovian decision is to convert the probabilistic nature of the Markovian decision problem into an equivalent deterministic model and then solving it by the GRG/GP technique. The tool used here for smoothing out the probabilistic nature of the problem is the expected value criterion, which is based primarily on the law of large numbers.

In finding the optimum of the problem, the whole life of the reservoir must be considered. In general, the useful life of a reservoir is assumed to be 100 years. Furthermore, in order to consider the flood benefits or losses, the duration of a stage must be in the order of the duration of the high intensity rainfalls. This duration is generally in the order of hours or days. Thus, the number of stages to be optimized is extremely large. This large problem is impractical to solve in terms of computer requirements. Moreover, data collection on a daily or monthly basis would be extremely time-consuming and impractical.

Two approximations are used to overcome this difficulty. The first approximation is to use a typical one year duration to represent the 100 years duration. In other words, it is assumed that this one year represents the average of the 100 years. This obviously is only an approximation. The use of stochastic inflows compensates for some of the approximation.

The second approximation concerns the inflow or runoff rate. It may be noted that although only one year duration is considered, the number of stages can still be very large if hourly or daily inflows are used. In this work, monthly inflows or runoffs are used. Thus, only twelve stages or months are needed to solve the problem. However, the monthly inflows cannot consider flood benefits or losses. In order to consider the flood effects, certain assumptions are made.

The computation for all the benefits and costs, except for the flood benefits, is carried out on a monthly basis. But, the flooding which is generally caused by a relatively short duration high intensity rainfall is treated differently. To devise a feasible approach to handle flood, it should be noted that flood damage depends primarily on the maximum overflow. This flood damage seldom occurs twice or more times in a month. Even if flooding occurs twice within a relatively short time span, the added flood damage for the second occurrence is small if the amount of rainfall or overflow in the second flood is not larger than that in the first one. Based on these observations, an approximate scheme is devised to handle the flooding situation by using the monthly runoff data.

It is assumed that flooding can occur only once a month. Also, based on rough estimates for the Walnut river basin, it is assumed that this flooding is caused by a high intensity rainfall of two days duration. In other words, the excess water must be released within this short duration to avoid damage. Furthermore, it is assumed that if flooding occurs in a month, all the runoff for that month comes from the high intensity rainfall of the two days duration. No rainfall occurs in the other days of the month.

It is obvious that the above assumptions are not completely realistic. However, the
degree of accuracy for a given problem can be improved, if more accurate data like daily or hourly rainfall records are available.

To illustrate the application of the above concepts and methods, the Walnut River Basin in southeastern Kansas is employed in this work. The data used for the above system are furnished by the Corps of Engineers, Tulsa, Oklahoma District [8] and the Kansas Water Resources Board. An IBM 360/50 computer with a Fortran compiler is used in the computations.

**WALNUT RIVER BASIN**

*Description of the system*

Walnut River is a tributary of the Arkansas River (see Fig. 1). There are four major tributaries of Walnut River, namely, West Branch, Whitewater River, Little Walnut Creek, and Timber Creek. The basin contains an area of 1955 square miles. Three reservoirs were suggested for construction for the purposes of flood control, water supply, water quality control, recreation, and fish and wildlife conservation. The three reservoirs are El Dorado on Walnut River, Towanda on Whitewater River, and Douglas on Little Walnut Creek.

It appears that groundwater supply in this area is limited. Droughts and extended periods of low flow have caused serious water shortages in the basin. On the other hand, fifty-six storms with precipitation averaging 3.5 inches or more have occurred during the period January, 1922, through December, 1961. An average of one flood occurs on the main stream and major tributaries each year. Thus, flood control, water supply, and quality are suitable purposes for the reservoirs.

The problem is to find the conservation pool and the flood pool capacities of each of the three reservoirs so as to maximize the net benefit over the expected life. The various costs and benefits data are furnished by the Corps of Engineers, Tulsa, Oklahoma District. These data are correlated into benefit and cost equations by the use of polynomial regression [22].

**MODELLING**

*Optimization problem*

The problem is to find the expected values of the conservation pool capacity, the flood pool capacity, and the operating policy during the life of each of the reservoirs so as to maximize the various benefits minus costs.

*Material balance equation*

Let a water resources system consisting of a single reservoir be represented by the following performance equation:

\[ x_c(n) + x_f(n) = x_c(n - 1) + x_f(n - 1) + F(n) - R(n) \]  

with the initial condition

\[ x_c(0) = x_c^i \]

\[ x_f(0) = 0, \]
Fig. 1. Walnut River Basin.
where

\[ n = \text{specific month, } 1 \ldots 12; \]
\[ x_c(n) = \text{state variable representing the conservation pool level in } 10^3 \text{ acre-feet at the end of month } n; \]
\[ x_f(n) = \text{state variable representing the flood pool level in } 10^3 \text{ acre-feet at the end of month } n; \]
\[ R(n) = \text{control or decision variable representing the amount of water release in } 10^3 \text{ acre-feet during month } n; \]
\[ F(n) = \text{inflow in } 10^3 \text{ acre-feet into the reservoir due to rainfall and other discharges during month } n; \]
\[ x_c^d = \text{desired conservation pool volume in } 10^3 \text{ acre-feet.} \]

Now introducing the Markov process in Eq. (1), we obtain

\[ x_c(j, n) + x_f(j, n) = E[x_c(n - 1)] + E[x_f(n - 1)] + F(j, n) - R(j, n), \quad j = 1 \ldots 4; n = 1 \ldots 12 \quad (2) \]

with the initial condition

\[ E[x_c(0)] = x_c^d \]
\[ E[x_f(0)] = 0, \]

where

\[ j = \text{specific inflow class, } 1 \ldots 4; \]
\[ E = \text{expectation;} \]
\[ x_c(j, n) = \text{new state variable representing the conservation pool level in } 10^3 \text{ acre-feet, for inflow class } j \text{ at the end of month } n; \]
\[ x_f(j, n) = \text{flood pool level in } 10^3 \text{ acre-feet, for the inflow class } j \text{ at the end of month } n; \]
\[ R(j, n) = \text{new control or decision variable representing the amount of water release in } 10^3 \text{ acre-feet, for inflow class } j \text{ during the month } n; \]
\[ F(j, n) = \text{lag-one (or) first-order (or) one-step serially correlated random variable representing the mean in } 10^3 \text{ acre-feet of inflow class } j \text{ into the reservoir due to rainfall and other discharges during month } n. \]

In Eq. (2), for any \( n \), \( E[x_c(n - 1)] \) and \( E[x_f(n - 1)] \) are known. The inflow rate into the reservoir is stochastically derived from the past rainfall record. The other three variables in Eq. (2), namely, \( x_c(j, n) \), \( x_f(j, n) \), and \( R(j, n) \) are unknown. Of these three unknowns, one can be obtained by optimization and another one can be obtained by solving Eq. (2). In order to determine the third one, the conservation and flood pools are considered separately using weighting factors (Lagrange multipliers).

\textit{Constraints}

For any given reservoir, because of hydrological reasons, the total reservoir capacity \( z \) (constant sedimentation pool volume plus conservation pool volume plus flood pool volume) should have an upper limit. Thus

\[ z(j, n) \leq z_{\text{max}}, \quad j = 1 \ldots 4; n = 1 \ldots 12. \quad (3) \]
Stochastic modelling and optimization of water resources systems

The lower limits on the conservation and flood pool volumes are given as

\[ x_c(j, n) \geq x_{c, \text{min}}, \quad j = 1 \ldots 4; \quad n = 1 \ldots 12 \]
\[ x_f(j, n) \geq 0. \]  

(4)

Because of spillway and outlet works design and channel capacity, there exists a maximum release rate. In order to conserve water quality and fish and wildlife in the channel, a minimum release rate must also be imposed. Thus,

\[ R_{\text{min}} \leq R(j, n) \leq R_{\text{max}}^*, \quad j = 1 \ldots 4; \quad n = 1 \ldots 12, \]  

(5)

where \( R_{\text{max}}^* \) represents the maximum amount that can be released in two days.

Objective function

It is required to maximize the following expected value of the objective function:

\[ \psi_i = E(G\{1, E[x_c(0)], E[x_f(0)], F(i, 0)\}), \quad i = 1 \ldots 4 \]
\[ = \sum_{n=1}^{12} \sum_{j=1}^{4} H[x_c(j, n), x_f(j, n), F(j, n), R(j, n)]p(i, j, n), \quad i = 1 \ldots 4 \]
\[ = \sum_{n=1}^{12} \sum_{j=1}^{4} [U_S(j, n) + U_q(j, n) + U_R(j, n)]p(i, j, n), \quad i = 1 \ldots 4 \]
\[ = \sum_{n=1}^{12} \sum_{j=1}^{4} [J_S(j)\rho_S(n) + J_q(j)\rho_q(n) + J_R(j)\rho_R(n)]p(i, j, n), \quad i = 1 \ldots 4, \]  

(6)

where

\( G\{1, E[x_c(0)], E[x_f(0)], F(i, 0)\} \) = value of the objective function for a reservoir starting with month 1 (Jan), initial expected conservation pool level \( E[x_c(0)] \), initial expected flood pool level \( E[x_f(0)] \) and initial inflow \( F(i, 0) \), \( i = 1 \ldots 4 \);
\( p(i, j, n) \) = the first-order transition probability of moving from the \( i \)th class inflow of the \( (n-1) \)th month, denoted by \( F(i, n-1) \), to the \( j \)th class inflow of the \( n \)th month, denoted by \( F(j, n) \);
\( H \) = value of the objective function for inflow class \( j \) and month \( n \);
\( U_S(j, n) \) = WSB for inflow class \( j \) and month \( n \);
\( U_q(j, n) \) = WQB for inflow class \( j \) and month \( n \);
\( U_R(j, n) \) = RB for inflow class \( j \) and month \( n \);
\( \rho_S(n) \) = WSB coefficient for month \( n \);
\( \rho_q(n) \) = WQB coefficient for month \( n \);
\( \rho_R(n) \) = RB coefficient for month \( n \);
\( J_S(j) \) = annual WSB for inflow class \( j \);
\( J_q(j) \) = annual WQB for inflow class \( j \);
\( J_R(j) \) = annual RB for inflow class \( j \);
\( \text{WSB} \) = water supply benefits;
\( \text{WQB} \) = water quality benefits;
\( \text{RB} \) = recreation benefits.
Net objective function

After introducing the flood benefits and the costs in the objective function, the expected value of the net objective function is given by

$$\zeta_i = \psi_i + E(J_F) - RP_{KN}E(C_C) - E(0_C), \quad i = 1 \ldots 4,$$  \hspace{1cm} (7)

where

$$E(J_F) = E(\text{flood benefits}) = f(\max \{E[x_i(n)]\});$$

$$E(C_C) = E(\text{capital cost}) = f(\max \{E[x_c(n)] + E[x_f(n)]\});$$

$$E(0_C) = E(\text{OMR cost}) = f(\max \{E[x_c(n)] + E[x_f(n)]\});$$

$$f(\cdot) = \text{function};$$

$$RP_{KN} = \text{capital recovery factor} = \frac{k(1+k)^N}{(1+k)^N - 1};$$

where

$k = \text{interest rate};$

$N = \text{expected economic life of the reservoir (assumed to be 100 years for all the reservoirs)}.$

Modified objective function

The expected value of the modified objective function is given as

$$\phi_i = \zeta_i - \lambda \sum_{n=1}^{12} E[x_c(n) - x_c^d]^2 + \mu \sum_{n=1}^{12} E[x_f(n) - 0]^2, \quad i = 1 \ldots 4$$

$$= \zeta_i - \lambda \sum_{n=1}^{12} \sum_{j=1}^{4} [x_c(j, n) - x_c^d]^2p(i, j, n)$$

$$- \mu \sum_{n=1}^{12} \sum_{j=1}^{4} [x_f(j, n) - 0]^2p(i, j, n), \quad i = 1 \ldots 4.$$  \hspace{1cm} (8)

where

$\lambda = \text{a weighting factor corresponding to the Lagrange multiplier in nonlinear programming;}$

$\mu = \text{a weighting factor (similar to } \lambda) \text{ corresponding to the Lagrange multiplier in nonlinear programming.}$

The parameters $\lambda$, $\mu$, and $x_c^d$ are used to maintain the optimal conservation pool level $x_c$ as close to $x_c^d$ as possible and to maintain the optimal flood pool level at nearly zero except during flooding periods. Thus, these parameters also separate the conservation and flood pools functionally. By using very high values of $\lambda$ and $\mu$, the optimal conservation pool volume would always be nearly the same as the desired volume $x_c^d$ and the optimal flood pool volume would be nearly zero except during flooding periods.

Operating policy

Let

$$S(j, n) = E[x_c(n - 1)] + E[x_f(n - 1)] + F(j, n) - R(j, n).$$  \hspace{1cm} (9)
Then the operating policy may be written as

(a) If $S(j, n) \leq x^d_c$

$$x_c(j, n) = S(j, n)$$

$$x_t(j, n) = 0.$$  

(b) If $S(j, n) > x^d_c$

$$x_c(j, n) = x^d_c$$

$$x_t(j, n) = S(j, n) - x^d_c.$$  

OPTIMIZATION

This section deals with the data input, preliminary calculations, and the detailed plan and discussion for the independent optimization of each reservoir by the two proposed methods.

Runoff data

The inflow rate into the reservoir depends on the amount of rainfall, drainage area, and hydrology. The inflow rate $F(j, n)$ in the material balance equation is stochastically derived from the past rainfall record. The sample record of monthly and annual flows at the El Dorado reservoir site for the forty-year period October 1921 through September 1961 is shown in Table 1.

Inflow classes

For any reservoir, the number of inflow classes or intervals is to be so chosen as to cover a wide range of runoff rates and at the same time to make a compromise between (i) too large a computer memory as well as time due to too many classes and (ii) too obscure stochastic properties due to too few inflow classes. Here four inflow classes are used for each reservoir of the Walnut River Basin. Each of the class intervals has the same percentage of the ranked observations of the inflow data for each reservoir. The inflow intervals and their expected values for each reservoir of the Walnut River Basin are depicted in Table 2.

Transition probability

The first-order transition probability of moving from the $i$th class inflow of the $(n - 1)$th month, denoted by $F(i, n - 1)$, to the $j$th class inflow of the $n$th month, denoted by $F(j, n)$, is $p(i, j, n)$. This probability can be calculated from the actual operating inflow data (Tables 1 and 2) as follows:

$$p(i, j, n) = \frac{\text{Numerator}}{\text{Denominator}},$$

where

Numerator = number of inflows in class $j$ of the $n$th month corresponding to the inflows in class $i$ of the $(n - 1)$th month

Denominator = total number of inflows in class $i$ of the $(n - 1)$th month.

In this case, there are $i = j = 4$ inflow classes. Also, $F(i, n - 1) = F(j, n)$, $i = j = 1 \ldots 4$, 

Table 1. Estimated monthly and annual flows in acre-feet at El Dorado dam site.

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</table>

Mean: 2610 1980 5050 10,320 11,900 10,270 8200 4180 4340 4580 3730 2400 69,560
Table 2. Information on the inflow classes of the Walnut River Basin.

<table>
<thead>
<tr>
<th>Inflow Class</th>
<th>El Dorado</th>
<th>Towanda</th>
<th>Douglas</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Class Interval (acre-ft)</td>
<td>Expected Value (acre-ft)</td>
<td>Class Interval (acre-ft)</td>
</tr>
<tr>
<td>1</td>
<td>0.000-0.432</td>
<td>0.216</td>
<td>0.000-0.781</td>
</tr>
<tr>
<td>2</td>
<td>0.432-1.530</td>
<td>0.981</td>
<td>0.781-2.710</td>
</tr>
<tr>
<td>3</td>
<td>1.530-5.840</td>
<td>3.685</td>
<td>2.710-10.500</td>
</tr>
<tr>
<td>4</td>
<td>5.840-80.100</td>
<td>42.970</td>
<td>10.500-137.500</td>
</tr>
</tbody>
</table>

where $F(i, n - 1)$ and $F(j, n)$ represent the means of the respective inflow classes and stages (months). There are twelve transition probability matrices representing the transition from December to January, January to February... November to December.

A typical transition probability matrix to represent the transition from the $(n - 1)$th stage to the $n$th stage can be represented by Table 3.

A sample transition probability matrix representing the transition from January to February for a typical reservoir is shown in Table 4.

Parameter optimization

It is to be noted that $x^d_c$ is essentially an unknown parameter before the optimum is obtained. It is true that the desired level $x^d_c$ can be approximately obtained by engineering and hydrological studies. However, the true optimum value for $x^d_c$ cannot be obtained except by optimization calculations. This parameter optimization problem can be solved by two different approaches.

The first approach can be called the trial-and-error approach. A series of values for $x^d_c$ are assumed. An optimization problem is solved for each assumed value of $x^d_c$. That value of $x^d_c$ corresponding to the maximum expected value of the modified objective function is the optimal desired value of $x^d_c$. The second approach is to consider both the releases and the $x^d_c$ as control variables and solve this optimization problem directly. In this work, the former approach is employed. Three different values are employed for $x^d_c$ of each reservoir. These values are listed in Table 5. The values in set 2 are the proposed design capacities for the conservation pools of the three reservoirs, by the Corps of Engineers, Tulsa, Oklahoma.

Global optimum

The proposed approaches do not guarantee global optimum if each reservoir has several local optima. It should be pointed out that there exists no optimization technique

<table>
<thead>
<tr>
<th>n-th Stage</th>
<th>i</th>
<th>j</th>
<th>F(1, n)</th>
<th>F(2, n)</th>
<th>F(3, n)</th>
<th>F(4, n)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>F(1, n - 1)</td>
<td>F(2, n - 1)</td>
<td>F(3, n - 1)</td>
<td>p(i, j, n)</td>
<td>F(4, n - 1)</td>
<td></td>
</tr>
</tbody>
</table>
Table 4. Sample transition probability matrix for the El Dorado Reservoir.

<table>
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<tr>
<th>January-February</th>
<th>0.86</th>
<th>0.14</th>
<th>0.00</th>
<th>0.00</th>
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</thead>
<tbody>
<tr>
<td>0.15</td>
<td>0.54</td>
<td>0.31</td>
<td>0.00</td>
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<tr>
<td>0.00</td>
<td>0.10</td>
<td>0.90</td>
<td>0.00</td>
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<tr>
<td>0.00</td>
<td>0.00</td>
<td>0.67</td>
<td>0.33</td>
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</table>

Table 5. Desired conservation pool volume, $x_i^j$.

<table>
<thead>
<tr>
<th>$x_i^j$, 10^3 acre-ft</th>
<th>El Dorado</th>
<th>Towanda</th>
<th>Douglas</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>65.0</td>
<td>34.0</td>
<td>67.0</td>
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<tr>
<td>2</td>
<td>74.9</td>
<td>46.5</td>
<td>77.3</td>
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<tr>
<td>3</td>
<td>85.0</td>
<td>55.0</td>
<td>87.0</td>
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</table>

which can guarantee the optimum of a general nonlinear problem with several local optima. To overcome this difficulty partially, several different sets of starting points can be used. If the results, by using these different sets, converge to the same optimum, this optimum is most probably the global optimum.

**Problem details**

The optimization problem has 144 constraints for each reservoir; 48 of these constraints are material balance equations which are equality constraints [Eq. (2)] and the remaining 96 are bounds [Eq. (5)]. There are 48 control variables, namely, $R(j, n_i)$, $j = 1 \ldots 4$; $n_i = 1 \ldots 12$.

**Markovian decision combined with the GP/GRG technique**

Either of these methods solves only one problem at a time. Thus, in both the methods, there is provision for the calculation of only one value of the objective function in a single computer run, corresponding to the specified initial inflow class. Hence the value of the objective function, for each combination of the specified initial inflow class, the $x_i^j$ value and the initial approximation for the control variables, has to be calculated in each separate computer run. Therefore the total number of trials for each reservoir by MDGP (Markovian decision combined with the GP technique), or MDGRG (Markovian decision combined with the GRG technique) is $(\text{the number of classes}) \times (\text{the number of } x_i^j \text{ values}) \times (\text{the number of initial approximations}) = 4 \times 3 \times 2 = 24$.

**COMPUTATIONAL EXPERIENCE AND DISCUSSION OF RESULTS**

Table 6 shows the sample results for the flooding situation of the El Dorado Reservoir using MDGRG. The same results using MDGP are given in Table 7. In both the above cases, the starting conservation pool level is taken to be the desired conservation pool volume of the reservoir and the starting flood pool level is taken to be zero.

It may be noted that for the initial inflow class 1 of the El Dorado Reservoir using MDGP, 53 iterations and 259 functional evaluations are needed with the initial approximation for releases being 10,000 acre-feet, while only 8 iterations and 9 functional
evaluations are needed with the initial releases being 5000 acre-feet. Similar results are obtained for the same reservoir with the initial inflow classes 2, 3, and 4. Also, the same experience is acquired for all four inflow classes of the Douglas reservoir using MDGP. This appears to be caused by the irregularity of the objective function with the initial releases being 10,000 acre-feet. This irregularity may appear as sharp ridges or very flat regions [30, 31]. Actual computational experience tells us that due to this irregularity, the maximum step-size is continuously reduced by the problem until the step-size is so small that no significant improvement in the objective function can be made. Thus, the true optimum is never obtained with high accuracy by this approach, when the initial approximation for releases is 10,000 acre-feet.

It is interesting to note that this difficulty is not encountered when this approach is used for the Towanda Reservoir. Also, MDGRG never experienced such a problem for any of the reservoirs.

It may be noted from the results that the optimal desired conservation pool volume is the set 1, \( x_{k}^{A} \) value for the El Dorado Reservoir. Similarly, the optimal desired conservation pool volumes for the Towanda and Douglas reservoirs have been found to be set 3 and set 1 values, respectively.

Typical convergence rates for the optimal net objective function, the norm of the projected gradient and the norm of the reduced gradient are shown in Figs. 2 through 5. Although the initial approximations are far removed from the optimal values for illustrative purposes, only a reasonable number of iterations are required by both the MDGP and MDGRG methods. The optimal operating variables for a typical case are listed in Table 8.

<table>
<thead>
<tr>
<th>Initial Approx. for ( R(j, n) ) ( 10^3 ) acre-ft</th>
<th>Expected Optimal Flood Control Pool Capacity ( 10^3 ) acre-ft</th>
<th>Expected Optimal Net Objective Function ( 10^3 ) Dollars</th>
<th>No. of Iterations</th>
<th>No. of Functional Evaluations</th>
<th>Computer Time (hours)</th>
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<tbody>
<tr>
<td>( x_{k}^{d} ), set</td>
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<tr>
<td>1</td>
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<td>292.51</td>
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<tr>
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<td>22.72</td>
<td>293.52</td>
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<td>9</td>
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<td>22.52</td>
<td>287.38</td>
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<td>10</td>
<td>22.52</td>
<td>280.21</td>
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<td>5</td>
<td>22.72</td>
<td>281.17</td>
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</table>

Table 6. Results for the flooding situation of El Dorado Reservoir by the Markovian decision approach combined with the GRG technique. Starting conservation pool level = desired conservation pool volume. Starting flood pool level = 0.

Table 7. Results for the flooding situation of El Dorado Reservoir by the Markovian decision approach combined with the GP technique. Starting conservation pool level = desired conservation pool volume. Starting flood pool level = 0.
Fig. 2. Convergence rate of net optimal benefits for Towanda Reservoir with inflow class 1 and $x_f$ set 1 by the Markovian decision approach combined with the GRG technique.

Fig. 3. Convergence rate of the norm of the reduced gradient for Towanda Reservoir with inflow class 1 and $x_f$ set 1.
Fig. 4. Convergence rate of net optimal benefits for Towanda Reservoir with inflow class 1 and \( x^f \) set 1 by the Markovian decision approach combined with the GP technique.

\[ -R(j,n)_{\text{initial}} = 10^4 \]
\[ -R(j,n)_{\text{initial}} = 15 \times 10^3 \]

Fig. 5. Convergence rate of the norm of the projected gradient for Towanda Reservoir with inflow class 1 and \( x^f \) set 1.
Table 8. Optimal operating variables for the flooding situation with inflow class 1 and $x^*_T$ set 1. Starting conservation pool level = desired conservation pool volume. Starting flood pool level = 0.

<table>
<thead>
<tr>
<th>Month, n</th>
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<th></th>
<th>Towanda</th>
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<th>Douglas</th>
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<tr>
<td></td>
<td>$E[R(n)]$</td>
<td>$E[x_c(n)]$</td>
<td>$E[x_f(n)]$</td>
<td>$E[R(n)]$</td>
<td>$E[x_c(n)]$</td>
<td>$E[x_f(n)]$</td>
<td>$E[R(n)]$</td>
<td>$E[x_c(n)]$</td>
<td>$E[x_f(n)]$</td>
<td>$E[R(n)]$</td>
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The MDGP method requires $132, 10^3$ bytes of memory while MDGRG requires $298, 10^3$ bytes of memory in an IBM 360/50 computer for each combination of the $x^d_i$ value, the inflow interval and the initial approximation for releases.

CONCLUSION

Since a multipurpose reservoir generally serves two basically different usages—namely, the prevention of flood damages during excessively high inflow periods and the conservation of water for various purposes like water supply, water quality, recreation, etc.—this work was based on the imaginary division of the reservoir into flood and conservation pools.

This paper illustrated the use of the two nonlinear programming techniques—namely, the generalized reduced gradient technique and the gradient projection technique, each independently combined with Markovian decision (MDGRG and MDGP)—in the planning, design, and operation of the three reservoirs of the Walnut river basin receiving stochastic inflows. Both MDGRG and MDGP presented no convergence difficulties. The results of both approaches are in good agreement. Although for each method, two different initial approximations were used, both of them converged to the same optimal solution.

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