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Quantum gravity effects on compact star cores

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ABSTRACT

Using the Tolman–Oppenheimer–Volkoff equation and the equation of state of zero temperature ultra-relativistic Fermi gas based on generalized uncertainty principle (GUP), the quantum gravitational effects on the cores of compact stars are discussed. Our results show that $2m(r)/r$ varies with r. Quantum gravity plays an important role in the region $r \sim 10^3 r_0$, where $r_0 \sim \beta_0 l_p$, l_p is the Planck length and β_0 is a dimensionless parameter accounting for quantum gravity effects. Furthermore, near the center of compact stars, we find that the metric components are $g_{tt} \sim r^4$ and $g_{rr} = [1 - r^2/(6r_0^2)]^{-1}$. All these effects are different from those obtained from classical gravity. These results can be applied to neutron stars or denser ones like quark stars. The observed masses of neutron stars ($\leqslant 2M_\odot$) indicate that β_0 can not exceed 10^{37}, not as good as the upper bound $\beta_0 < 10^{34}$ from simple electroweak consideration. This means that incorporating either quantum gravity effects or nuclear interactions, one obtains almost the same mass limits of neutron stars.

The configuration of a spherically symmetric static star, composed of perfect fluids, is determined by the Tolman–Oppenheimer–Volkoff (TOV) equation (in c.g.s. units) [1,2]

$$\frac{dP}{dr} = -\left(\rho + P/c^2\right)\frac{Gm(r) + 4\pi G r^3 P/c^2}{r[r - 2Gm(r)/c^2]}, \tag{1}$$

with

$$\frac{dm(r)}{dr} = 4\pi r^2 \rho(r), \tag{2}$$

where c is the velocity of light. G is the gravitational constant. P and ρ are respectively the pressure and the macroscopic energy density measured in proper coordinates. Supplied with an equation of state and appropriate boundary conditions, Eq. (1) and Eq. (2) determine $P(r)$, $m(r)$ and $\rho(r)$. If the pressure and gravitational potential is everywhere small, i.e., $P(r) \ll \rho c^2$, $2Gm(r)/c^2 r \ll 1$, the TOV equation reduces to the fundamental equation of Newtonian astrophysics

$$\frac{dP}{dr} = -\rho(r)\frac{Gm(r)}{r^2}. \tag{3}$$

Most of the low density compact stars like white dwarfs are well described by Newtonian gravity. For compact stars like neutron stars and other exotic compact stars, general relativity plays an important role [3]. The ideal neutron star is the simplest model in which nuclear interactions are ignored and the pressure of cold degenerate neutrons contends against the gravitational collapse [2]. There are basically two ways to improve the model of compact stars. The first one is to discuss more realistic structures of neutron stars and other Fermi stars in theoretical and observational perspectives [4–11,18,19]. In these works, various types of equation of state (EOS) are introduced to represent strongly interacting components and nuclear interactions. Nuclear interactions significantly lift the maximum mass of neutron stars from the Oppenheimer limit $0.7M_\odot$ to $2M_\odot$. A more detailed discussion and references therein refer to [11]. Another direction is to introduce $f(R)$ theory or quantum gravity effects into the models [12–17]. This way is of interest when addressing high density and high pressure cold Fermi stars. This is the purpose of this Letter. As a first step in this direction, we adopt the ideal model without nuclear interactions and the TOV equation.

In the absence of a full theory of quantum gravity, effective models are useful tools to gain some features from quantum theory of gravity. One of the most important models is the generalized uncertainty principle (GUP), derived from the modified fundamental commutation relation [20–26]

$$[x, p] = i\hbar\left(1 + \beta p^2\right), \tag{4}$$

where $\beta = \beta_0 l_p^2/\hbar^2 = \beta_0/c^2 M_p^2$, $l_p^2 = G\hbar/c^3$, $M_p^2 = \hbar c/G$. $\hbar = h/2\pi$ is the Planck constant and β_0 is a dimensionless parameter. With this modified commutator, one can easily derive the generalized uncertainty principle (GUP)

$$\Delta x \Delta p \geqslant \frac{\hbar}{2}\left[1 + \beta(\Delta p)^2\right], \tag{5}$$

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which in turn gives the absolutely smallest uncertainty in positions, i.e., the minimum measurable length

$$\Delta x \geqslant \Delta_{\min} = \hbar\sqrt{\beta} = \sqrt{\beta_0} l_p. \tag{6}$$

Note that the model in (4) considers only the minimal uncertainty in position. In this case, the quantum mechanics structure underlying the GUP has been studied in full detail [23]. The statistics of ideal gases based on GUP has been discussed by many authors [29–33]. In our recent work, we have studied a system composed of zero temperature ultra-relativistic Fermi gas based on GUP [33]. The Newtonian equation with uniform pressure was employed to discuss stellar structures. The proper particle number, energy density and pressure for an ultra-relativistic system were given in [33]

$$\frac{N}{V} = \frac{8\pi}{(hc)^3} E_H^3 f(\kappa), \tag{7}$$

$$\rho = \frac{8\pi}{c^2 (hc)^3} E_H^4 h(\kappa), \tag{8}$$

$$P = \frac{8\pi}{(hc)^3} E_H^4 g(\kappa), \tag{9}$$

where $E_H = c/\sqrt{\beta} = M_p c^2/\sqrt{\beta_0}$ denotes the Hagedorn energy, introduced in [33] and $\kappa = \varepsilon_F \sqrt{\frac{\beta}{c^2}} = \varepsilon_F / E_H$. Moreover

$$h(\kappa) \equiv \frac{1}{4} \frac{\kappa^4}{(1+\kappa^2)^2}, \tag{10}$$

$$f(\kappa) \equiv \frac{1}{8}\left[\frac{\kappa(\kappa^2-1)}{(1+\kappa^2)^2} + \tan^{-1}(\kappa)\right], \tag{11}$$

$$g(\kappa) \equiv \kappa f(\kappa) - h(\kappa). \tag{12}$$

It is worth noting that when κ increases, the proper pressure blows up, while the proper energy density and the proper number density are both bounded. This is a manifestation of the minimal length.

The size of β_0 signals when quantum gravity effects enter the story. In [27], based on the precision measurement of Lamb shift, an upper bound of β_0 is given by $\beta_0 < 10^{36}$. A relatively rough but stronger restriction is estimated in [28]. However, a better bound is gained from simple electroweak consideration $\beta_0 < 10^{34}$. For $\beta_0 = 10^{34}$, we rewrite Eqs. (8) and (9) as

$$\rho = 5.24 \times 10^{95} \frac{1}{\beta_0^2} h(\kappa) \sim 10^{27} h(\kappa) \left(\mathrm{kg\,m^{-3}}\right), \tag{13}$$

$$P = 4.73 \times 10^{112} \frac{1}{\beta_0^2} g(\kappa) \sim 10^{44} g(\kappa) \text{ (Pascals)}. \tag{14}$$

Comparing these with the normal nuclear density $\rho_n = 2.7 \times 10^{17}\ \mathrm{kg\,m^{-3}}$ and the pressure $P_n \sim 10^{34}$ Pascals, the highest pressure recorded under laboratory controlled conditions, we can find that in the vicinity of nuclear matter equilibrium density, quantum gravitational effects are not important. However, for density higher than the normal nuclear one, it is of interest to investigate the cores of compact stars like neutron stars and other exotic compact stars where quantum gravity may play a leading role. As first approximation, we consider only the degeneracy pressure regardless of the interaction correction. On the other hand, to date, several accurate masses determinations of neutron stars are available from radio binary pulsars, as we will find that this may be used to constrain the magnitude of β_0.

Two configurations of compact stars have been addressed in [33], by applying the Newtonian limit equation (3) with uniform density. One is that the star is almost composed of ultra-relativistic particles. The other is that the major contribution to the mass is from non-relativistic cold nuclei. However, to discuss the core of ultra-compact stars like neutron stars, one should use TOV equations. Setting $r = r_0\tilde{r}$, $m = m_0\tilde{m}$, $P = P_0\tilde{P}$ and

$$\rho = \frac{m_0}{4\pi r_0^3}\tilde{\rho} \equiv \rho_0\tilde{\rho}, \qquad P_0 = \rho_0 c^2, \qquad \frac{G m_0}{c^2 r_0} \equiv 1, \tag{15}$$

the TOV equations (1) and (2) are reduced to the following dimensionless ones

$$\frac{d\tilde{P}}{d\tilde{r}} = -(\tilde{\rho} + \tilde{P}) \frac{\tilde{m} + \tilde{r}^3 \tilde{P}}{\tilde{r}(\tilde{r} - 2\tilde{m})}, \tag{16}$$

$$\frac{d\tilde{m}}{d\tilde{r}} = \tilde{r}^2 \tilde{\rho}. \tag{17}$$

When there is no introduction of quantum gravity, for a system almost composed of ultra-relativistic fermions, the equation of state is $\tilde{P} = \tilde{\rho}/3$. An exact solution is given in [34]

$$\frac{2\tilde{m}(\tilde{r})}{\tilde{r}} = \frac{3}{7}, \qquad \tilde{P}(\tilde{r}) = \frac{1}{14}\tilde{r}^{-2}. \tag{18}$$

The pressure is not zero on the surface of the star. This does not meet the physical boundary conditions. However, the point is that it is an analytic solution describing the central region of compact stars with divergent pressure in the center [2]. Note that the length scale r_0 in Eq. (15) is uncertain. Thus r, m, ρ and P can be of any size.

From Eq. (18), the pressure is divergent in the center. Therefore, influences from quantum gravity should be included in the discussion. Obviously, near the surface, particles are non-relativistic while in the region around the center, particles are ultra-relativistic [2]. This determines the equations of state and boundary conditions.

In the vicinity of $r = 0$, the equation of state is given by Eq. (7), Eq. (8) and Eq. (9). Under the limit $\kappa \to 0$, it is straightforward to recover $P = \rho/3c^2$. Defining $r = r_0\tilde{r}$, $m = m_0\tilde{m}$ with

$$r_0^{-2} \equiv \frac{4\pi G}{c^4} \frac{8\pi}{(hc)^3} E_H^4, \tag{19}$$

$$m_0 \equiv 4\pi r_0^3 \frac{8\pi}{c^2 (hc)^3} E_H^4 = 1.93 \times 10^{-8} \beta_0 \text{ (kg)}, \tag{20}$$

$$P_0 = \rho_0 c^2, \quad \rho_0 = \frac{8\pi}{c^2 (hc)^3} E_H^4, \tag{21}$$

where r_0 is the minimum radius in [33]

$$r_0 = \sqrt{\frac{\pi}{4}} \beta_0 l_p = \sqrt{\frac{\pi}{4}} \sqrt{\beta_0} \Delta_{\min} = 1.43 \times 10^{-35} \beta_0 \text{ (m)}. \tag{22}$$

Since r_0 in Eq. (19) comes from Eqs. (1), (2), (8) and (9), r_0 represents the proper length. The expressions (21) and (22) show that the system cannot be arbitrary scale, determined entirely by β_0. This indicates that our discussion is focused on the central region of compact stars. Substituting the above expressions for P and ρ (Eq. (8) and Eq. (9)) into Eq. (16) and Eq. (17), one gets

$$\frac{d\tilde{m}(\tilde{r})}{d\tilde{r}} = \tilde{r}^2 h(\kappa), \tag{23}$$

$$\frac{d\kappa(\tilde{r})}{d\tilde{r}} = \frac{-\kappa(\tilde{r})[\tilde{m}(\tilde{r}) + \tilde{r}^3 g(\kappa)]}{\tilde{r}[\tilde{r} - 2\tilde{m}(\tilde{r})]}. \tag{24}$$

Since the density is regular in the center, one has $m(0) = 0$ as a boundary condition. After setting $\kappa_0 \equiv \kappa(0)$ as another boundary condition, Eq. (23) and Eq. (24) are integrated numerically in Tables 1–5.

In Tables 1–3, we perform the integration with different $\kappa(\tilde{r})$. Four conclusions can be drawn from these tables:

Table 1
Integration from κ_0 to $\kappa(\tilde{r}) = 0.1$. The value of $2\tilde{m}(\tilde{r})/\tilde{r}$ is insensitive to initial condition κ_0. $2\tilde{m}(\tilde{r})/\tilde{r}$ has relatively large deviation from 0.429 since at $\tilde{r} = 97.7$ quantum gravity has evident effects.

κ_0	$\kappa(\tilde{r})$	$m(\tilde{r})$	$\tilde{r}$	$2\tilde{m}(\tilde{r})/\tilde{r}$
1000.0	0.1	21.88	97.70	0.448
100.0		21.87	97.70	0.448
50.0		21.85	97.70	0.447
20.0		21.79	97.60	0.446
10.0		21.71	97.60	0.445
8.0		21.65	97.50	0.444
5.0		21.53	97.50	0.442
3.0		21.29	97.40	0.437
1.0		19.72	95.20	0.414
0.5		18.36	87.90	0.418

Table 2
Integration from κ_0 to $\kappa(\tilde{r}) = 0.01$. The different initial value κ_0 has almost no effect on $2\tilde{m}(\tilde{r})/\tilde{r}$. $\tilde{r}$ is large enough to overwhelm quantum gravity influences.

κ_0	$\kappa(\tilde{r})$	$\tilde{m}(\tilde{r})$	$\tilde{r}$	$2\tilde{m}(\tilde{r})/\tilde{r}$
1000.0	0.01	1985.47	9250.70	0.429
100.0		1985.48	9250.80	0.429
50.0		1985.51	9251.00	0.429
20.0		1985.61	9251.60	0.429
10.0		1985.76	9252.50	0.429
8.0		1985.86	9253.00	0.429
5.0		1986.14	9254.40	0.429
3.0		1986.72	9257.20	0.429
1.0		1988.16	9269.70	0.429
0.5		1981.62	9265.40	0.428
0.1		2016.18	9405.60	0.429

Table 3
Integration from κ_0 to $\kappa(\tilde{r}) = 0.001$. The numerical results completely match the asymptotic solution of Eq. (25).

κ_0	$\kappa(\tilde{r})$	$\tilde{m}(\tilde{r})$	$\tilde{r}$	$2\tilde{m}(\tilde{r})/\tilde{r}$
1000.0	0.001	198372.71	925778.50	0.429
100.0		198372.88	925778.70	0.429
50.0		198373.07	925778.90	0.429
20.0		198373.65	925779.50	0.429
10.0		198374.58	925780.30	0.429
5.0		198376.38	925781.40	0.429
1.0		198393.76	925802.80	0.429
0.1		198516.29	925868.20	0.429
0.01		201594.58	940268.10	0.429

- Different from the results obtained in classical gravity, $2\tilde{m}(\tilde{r})/\tilde{r}$ varies with $\tilde{r}$ but not a constant 3/7. For example, with $\kappa(\tilde{r}) = 0.1$ in Table 1, the deviation of $2\tilde{m}(\tilde{r})/\tilde{r}$ is about 4%.
- $2\tilde{m}(\tilde{r})/\tilde{r}$ is not sensitive to different initial value κ_0.
- For large $\kappa(\tilde{r})$ or small $\tilde{r}$, quantum gravity contribution is important to the value of $2\tilde{m}(\tilde{r})/\tilde{r}$. As $\kappa(\tilde{r})$ decreases, or $\tilde{r}$ increases, the configuration approaches the classical one obtained in [34], with a constant $2\tilde{m}(\tilde{r})/\tilde{r} = 3/7$.
- Quantum gravity plays an important role in the region $r \sim 10^3 r_0$.

Some analytic solutions can be obtained in extreme cases as follows.

- Under $\kappa \to 0$, it is easy to see that $h(\kappa) \sim \kappa^4/4$, $g(\kappa) \sim \kappa^4/12$. Then from Eq. (23) and Eq. (24), we obtain

$$\frac{2\tilde{m}(\tilde{r})}{\tilde{r}} = \frac{3}{7}, \qquad \kappa(\tilde{r}) = \left(\frac{6}{7}\right)^{1/4} \tilde{r}^{-1/2},$$
$$\tilde{P}(\tilde{r}) = \frac{1}{12}\kappa^4 = \frac{1}{14}\tilde{r}^{-2}, \quad \text{for large } \tilde{r}. \tag{25}$$

This solution is nothing but the classical one without quantum gravity.

Table 4
Integration from κ_0 to $\kappa(\tilde{r}) = 20$. The large value of κ corresponds to $r \to 0$. $2\tilde{m}(\tilde{r})/\tilde{r}$ depends sensitively on $\tilde{r}$.

κ_0	$\kappa(\tilde{r})$	$\tilde{m}(\tilde{r})$	$\tilde{r}$	$2\tilde{m}(\tilde{r})/\tilde{r}$
1000.0	20	2.85×10^{-2}	0.700	8.15×10^{-2}
500.0		2.77×10^{-2}	0.693	7.99×10^{-2}
200.0		2.52×10^{-2}	0.672	7.51×10^{-2}
100.0		2.14×10^{-2}	0.636	6.72×10^{-2}
50.0		1.41×10^{-2}	0.554	5.10×10^{-2}
30.0		0.60×10^{-2}	0.417	2.89×10^{-2}

- Under $r \to 0$ and $\kappa \to \infty$, Eq. (23) and Eq. (24) can be replaced by asymptotic expressions

$$\frac{d\tilde{m}(\tilde{r})}{d\tilde{r}} = \frac{1}{4}\tilde{r}^2, \tag{26}$$
$$\frac{d\kappa(\tilde{r})}{d\tilde{r}} = \frac{-\kappa(\tilde{r})[\tilde{m}(\tilde{r}) + \tilde{r}^3 \frac{\pi}{16}\kappa(\tilde{r})]}{\tilde{r}[\tilde{r} - 2\tilde{m}(\tilde{r})]}. \tag{27}$$

The solution of these equations is

$$\tilde{m}(\tilde{r}) = \frac{\tilde{r}^3}{12}, \qquad \kappa(\tilde{r}) = \frac{32}{\pi}\frac{1}{\tilde{r}^2},$$
$$P(\tilde{r}) = \frac{2}{\tilde{r}^2}, \quad \text{for } \tilde{r} \to 0. \tag{28}$$

The solution (28) represents the situation where quantum gravity dominates. This happens near the center of ultra-compact stars. One can see that it is quite different from the solution of classical gravity. Table 4 is the numerical result integrated for large $\kappa(\tilde{r})$, well consistent with the asymptotic solution (28).

For a spherically symmetric static compact star, the metric is given by [3]

$$g_{rr} \equiv A(r) = \left(1 - \frac{2Gm(r)}{rc^2}\right)^{-1} = \left(1 - \frac{2\tilde{m}(\tilde{r})}{\tilde{r}}\right)^{-1}. \tag{29}$$
$$g_{tt} \equiv -B(r), \qquad \frac{1}{B}\frac{dB}{dr} = \frac{2G}{c^2 r^2}\left[m(r) + \frac{4\pi r^3 P}{c^2}\right]\left[1 - \frac{2Gm}{c^2 r}\right]^{-1}. \tag{30}$$

Then for $r \to 0$, from (28), we have

$$A(r) = \frac{1}{1 - \tilde{r}^2/6}, \qquad B(r) \sim \tilde{r}^4. \tag{31}$$

One may compare (31) with the classical results

$$A(r) = \frac{7}{4}, \qquad B(r) \sim \tilde{r}^{1/2}. \tag{32}$$

In Table 5, Eq. (23) and Eq. (24) are integrated with a large initial κ_0. It is interesting that $2\tilde{m}/\tilde{r}$ reaches a maximum value 0.734 in the vicinity of $r = 3.00 r_0$. Our calculation shows that near the center, $2\tilde{m}/\tilde{r} = \tilde{r}^2/6$ which indicates that $2\tilde{m}/\tilde{r}$ increases with $\tilde{r}$. On the other hand, as $\tilde{r} \to \infty$, $2\tilde{m}/\tilde{r} \to 3/7$. Therefore, the maximum of $2\tilde{m}/\tilde{r}$ at $r = 3.00 r_0$ is a turning point, where quantum gravity effect starts to dwindle. From Table 5, one also finds that g_{rr} has a small range of fluctuation. A minimum $(1 - 0.279)^{-1} = 1.39$ is achieved at $r \simeq 12.5 r_0$. This minimum is about one-third of the maximum $(1 - 0.734)^{-1} = 3.76$ at $r \simeq 3.00 r_0$. We do not have good explanation for this fluctuation. It may be caused by the effectiveness of our model. Finally, g_{rr} tends to the constant 7/4 at large $\tilde{r}$ as expected. The profile of $2\tilde{m}/\tilde{r}$ versus $\tilde{r}$ is plotted in Fig. 1. One can see that the upper limit, 8/9 on the surface of a spherically symmetric static star, is well satisfied.

Table 6 shows the integrations from $\kappa_0 = 10$ to the nuclear density $\rho_n \simeq 10^{17}$ kg/m^3 for different β_0. The fifth line represents the

Table 5
Integration with a fixed $\kappa_0 = 1000$. $2\tilde{m}/\tilde{r}$ reaches its maximum value 0.734 in the vicinity of $r = 3.00r_0$. $2\tilde{m}/\tilde{r}$ has a small range of fluctuation and achieves a minimum value 0.279 at $r \simeq 12.5r_0$. Eventually, $2\tilde{m}/\tilde{r}$ tends to the constant 7/4 at large $\tilde{r}$.

κ_0	$\kappa(\tilde{r})$	$\tilde{m}(\tilde{r})$	$\tilde{r}$	$2\tilde{m}(\tilde{r})/\tilde{r}$
1000.0	100.0	2.32×10^{-3}	0.303	1.53×10^{-2}
	50.0	7.05×10^{-3}	0.439	3.21×10^{-2}
	20.0	2.85×10^{-2}	0.700	8.14×10^{-2}
	10.0	7.87×10^{-2}	0.984	0.160
	5.0	0.205	1.365	0.301
	1.0	0.983	2.706	0.727
	0.8	1.100	3.001	**0.734**
	0.7	1.169	3.220	0.726
	0.5	1.344	4.052	0.663
	0.3	1.825	8.091	0.451
	0.25	2.359	12.465	**0.279**
	0.2	4.062	22.801	0.356
	0.1	21.880	97.689	0.448
	0.01	1985.47	9250.70	**0.429**

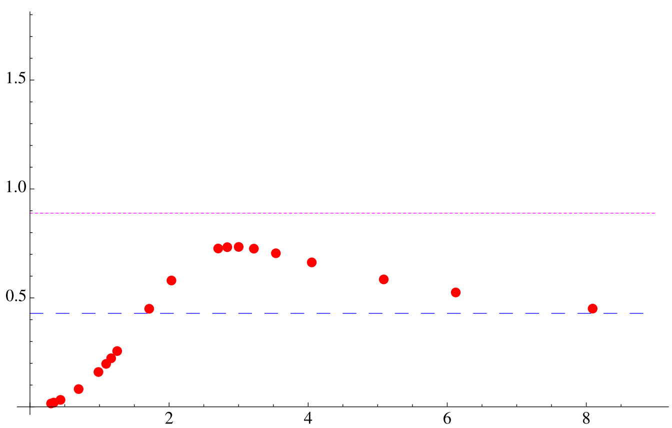

Fig. 1. For a fixed $\kappa_0 = 1000$, $2m/r$ versus the radius $r = r_0\tilde{r}$. As $\tilde{r} \to 0$, $2m/r \sim r^2$. $2m/r$ has a maximum around $\tilde{r} = 3$. $2m/r$ acquires the asymptotic value 0.429 at large r. The dashed line represents $2m/r = 0.429$ while the dotted line represents $2m/r = 8/9$, the upper limit of $2m/r$ on the surface of a spherically symmetric static star.

values κ corresponding to the nuclear density. The last two lines give the masses (in solar mass units) and radii, when the stellar surface density is taken as the nuclear density. From the second conclusion drawn from Table 1 to Table 3, the results are insensitive to κ_0 provided $\kappa_0 \geqslant 5$. Therefore, the currently observed masses of neutron stars ($\leqslant 2M_\odot$) indicates β_0 can not be greater than 10^{37}. This conclusion compatible with that from precision measurements of Lamb shift. In another words, when incorporating the influence of quantum gravity, one obtains the same mass limit of neutron stars as that from considering nuclear interactions.

Table 7 shows the integrations from $\kappa_0 = 10$ to $\kappa = 0.01$ for different β_0. In this region, quantum gravity plays an important role. The last line lists $\rho(\kappa = 0.01)$ for different β_0, with reference to Eq. (10) and Eq. (13). In the region where the density is less than $\rho(0.01)$, quantum gravity effects almost have no effect. For $\beta_0 = 1$, the volume in which quantum gravitational effects are important is in fact minuscule. Therefore, the observation of quantum gravity effects depends only on the size of β_0. The precise determination of the neutron star masses determines only the upper limit of β_0.

In summery, we discussed the structure of ultra-compact star cores by a simple effective quantum gravity model. The model, GUP, introduces a new equation of state, determined by Eqs. (7), (8) and (9). By plugging the equation of state into TOV equations, we found some different features from previous works in literature.

Since quantum gravitational effects play an important role only in high density, we considered configurations in which a star is almost composed of ultra relativistic particles. The asymptotic solutions near the center are given by (28) and (31). The complete picture is given by numerical calculation. Quantum gravitational effects play a leading role only in a relatively small range $\sim 10^3 r_0 = 10^3\sqrt{\beta_0}\Delta_{\min}$. Outside this region, the solutions are determined by Eqs. (18) and (32). Our discussion can be applied to neutron stars, for example. An upper bound of $\beta_0 < 10^{37}$ was also achieved in Table 6. However, this bound is larger than $\beta_0 < 10^{36}$, obtained from the precision measurement of Lamb shift. On the other hand, simple electroweak estimation gives a better bound $\beta_0 < 10^{34}$ than both of them. There are two ways to model compact stars. One is including the nuclear interactions and another is to incorporate quantum gravity effects. It is of interest that our results show that the two ways give the same mass limit of neutron stars. It would be of importance in the future work to combine both methods together in modelling compact stars. We hope the refined models can further narrow the range of β_0.

Table 6
Integration from $\kappa_0 = 10$ to the nuclear density $\rho_n \simeq 10^{17}$ kg/m^3 for different β_0. The fifth line shows the values κ corresponding to the nuclear density. The last two lines give the masses (in solar mass units) and radii, when the stellar surface density is taken as the nuclear density. The precise mass determinations of neutron stars that have masses not larger than $2M_\odot$ indicates β_0 can not be greater than 10^{37}.

β_0	10^{37}	10^{35}	10^{33}
$\rho = 5.24 \times 10^{95} \frac{1}{\beta_0^2} h(\kappa)$	$5.24 \times 10^{21} h(\kappa)$	$5.24 \times 10^{25} h(\kappa)$	$5.24 \times 10^{29} h(\kappa)$
$m_0 = 1.93 \times 10^{-8} \beta_0$	1.93×10^{29} kg	1.93×10^{27} kg	1.93×10^{25} kg
$r_0 = 1.43 \times 10^{-35} \beta_0$	1.43×10^{2} m	1.43×10^{0} m	1.43×10^{-2} m
$\rho_n \simeq 10^{17}$ kg/m^3	0.1	0.01	0.001
$M/M_\odot$	2.11	1.93	1.93
R	1.40×10^{4} m	1.32×10^{4} m	1.32×10^{4} m

Table 7
Integration from $\kappa_0 = 10$ to $\kappa = 0.01$ for different β_0. In this region, quantum gravity plays an important role. The last line lists $\rho(\kappa = 0.01)$ for different β_0, with reference to Eq. (10) and Eq. (13). In the region where the density is less than $\rho(0.01)$, quantum gravity effects is negligible.

β_0	10^{37}	10^{36}	10^{35}	10^{34}
m_0	1.93×10^{29} kg	1.93×10^{28} kg	1.93×10^{27} kg	1.93×10^{26} kg
r_0	1.43×10^{2} m	1.43×10^{1} m	1.43×10^{0} m	1.43×10^{-1} m
$M = 1985.76 m_0$	3.83×10^{32} kg	3.83×10^{31} kg	3.83×10^{30} kg	3.83×10^{29} kg
$R = 9252.50 r_0$	1.32×10^{6} m	1.32×10^{5} m	1.32×10^{4} m	1.32×10^{3} m
$\rho(0.01)$	1.31×10^{12} kg/m^3	1.31×10^{14} kg/m^3	1.31×10^{16} kg/m^3	1.31×10^{18} kg/m^3

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