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Decay constants of the pion and B mesons with the Bethe–Salpeter equation

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Abstract

In this Letter, we investigate the under-structures of the π and B mesons in the framework of the Bethe–Salpeter equation with the confining effective potential (infrared modified flat bottom potential). In bare quark–gluon vertex approximation, we obtain the algebraic expressions for the solutions of the coupled rainbow Schwinger–Dyson equation and ladder Bethe–Salpeter equation. Firstly, we neglect the rainbow Schwinger–Dyson equation, take the bare quark propagator and solve the Bethe–Salpeter equation numerically alone. Although the bare quark propagator cannot embody dynamical chiral symmetry breaking and has a mass pole in the time-like region, it can give reasonable results for the values of decay constants f_π and f_B compared with the values of experimental data and other theoretical calculations, such as lattice simulations and QCD sum rules. Secondly, we explore those mesons within the framework of the coupled rainbow Schwinger–Dyson equation and ladder Bethe–Salpeter equation. The Schwinger–Dyson functions for the u and d quarks are greatly renormalized at small momentum region and the curves are steep at about $q^2 = 1\ \mathrm{GeV}^2$ which indicates an explicitly dynamical symmetry breaking. The Euclidean time Fourier-transformed quark propagator has no mass poles in the time-like region which naturally implements confinement. As for the b quark, the current mass is very large, the renormalization is more tender, however, mass pole in the time-like region is also absent. The Bethe–Salpeter wavefunctions for both the π and B mesons have the same type (Gaussian type) momentum dependence as the corresponding wavefunctions with the bare quark propagator, however, the quantitative values are changed and the values for the decay constants f_π and f_B are changed correspondingly.

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1. Introduction

Quantum chromodynamics (QCD) is the appropriate theory for describing the strong interaction at high energy region, however, the strong gauge cou-

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pling at low energy destroys the perturbative expansion method. The physicists propose many nonperturbative approaches to deal with the long distance properties of QCD, such as chiral perturbation theory [1], heavy quark effective theory [2], QCD sum rule [3], lattice QCD [4], perturbative QCD [5], coupled Schwinger–Dyson equation (SDE) and Bethe–Salpeter equation (BSE) method [6], etc. All of those approaches have both outstanding advantages and obvious shortcomings. For example, lattice simulations are rigorous in view of QCD, they suffer from lattice artifacts and uncertainties, such as Gribov copies, boundary conditions and so on, furthermore, current technique cannot give reliable result below 1 GeV, where the most interesting and novel behavior is expected to lie. The coupled SDE and BSE have given a lot of successful descriptions of the long distance properties of strong interactions and the QCD vacuum, for a recent review one can see Ref. [7]. The SDE can provide a natural way to embody the dynamical symmetry breaking and confinement which are two crucial features of QCD, although they correspond to two very different energy scales [8,9]. On the other hand, the BSE is a conventional approach in dealing with the two body relativistic bound state problems [10]. From the solutions of the BSE, we can obtain useful information about the under-structure of the hadrons and thus obtain powerful tests for the quark theory of the mesons. However, the main drawback can be traced back to the fact that when we solving the SDE and BSE, model dependent kernels for the gluon two-point Green's function have to be used, furthermore, the coupled SDE and BSE are a divergent series of equations, we have to make truncations in one or the other ways. Numerical calculations indicate that the coupled rainbow SDE and ladder BSE with phenomenological potential models can give satisfactory results. The usually used effective potentials are confining Dirac δ function potential, Gaussian distribution potential and flat bottom potential (FBP) [11–13]. The FBP is a sum of Yukawa potentials, which not only satisfies gauge invariance, chiral invariance and fully relativistic covariance, but also suppresses the singular point which the Yukawa potential has. It works well in understanding the dynamical chiral symmetry braking, confinement and the QCD vacuum as well as the meson structures, such as electromagnetic form factor, radius, decay constant [14,15].

The decay constant of the B meson f_B plays an important role in modern physics with the assumption of current-meson duality. The precise knowledge of the value of the f_B will provide great improvement in our understanding of various processes convolving the B meson decays. At present, it is a great challenge to extract the value of the B meson decay constant f_B from experimental data. So it is interesting to combine the those successful potentials within the framework of coupled SDE and BSE to calculate the decay constants of both the π and B mesons. In this Letter, we use an infrared modified flat-bottom potential (IMFBP) which takes the advantages of both the Gaussian distribution potential and the FBP to calculate both the π and B mesons decay constants. Certainly, our potential model can be used to investigate the properties of other pseudoscalar mesons, such as $K, D, D_s, \ldots$. For example, we can obtain the decay constants $f_\pi = 127$ MeV, $f_K = 156$ MeV, $f_D = 238$ MeV, and $f_B = 192$ MeV with the same parameters, while a detailed studies of those mesons $K, D, D_s, \ldots$ may be our next work, they are not our main concern in this Letter.

The Letter is arranged as follows: we introduce the infrared modified flat bottom potential in Section 2; in Sections 3 and 4, we solve the Schwinger–Dyson equation and the Bethe–Salpeter equation and obtain the decay constants for both the π and B mesons; Section 5 is reserved for conclusion and discussion.

2. Infrared modified flat bottom potential

The infrared structure of the gluon propagator has important implication for the quark confinement. One might expect that the behavior of the quark interaction in the region of small space-like p^2 determines the long range properties of the $q\bar{q}$ potential and hence implements confinement, however, the present techniques in QCD manipulation cannot give satisfactory small r behavior for the gluon propagator, on the other hand, the phenomenological confining potential models give a lot of successes in dealing with the low energy hadron physics, such as dynamical chiral symmetry braking, pseudoscalar mesons electromagnetic form factors, mass formulations, π–π scattering parameters, etc [7,11,16]. In this Letter, we use a Gaussian distribution function to represent the infrared behavior of the gluon propagator,

$$4\pi G(k^2) = 3\pi^2 \frac{\varpi^2}{\Delta^2} \mathrm{e}^{-k^2/\Delta}, \tag{1}$$

which determines the quark–quark interaction through a strength parameter ϖ and a ranger parameter Δ.[2] This form is inspired by the δ function potential (in other words the infrared dominated potential) used in Ref. [11], which it approaches in the limit $\Delta \to 0$. For the intermediate momentum, we take the FBP as the best approximation and neglect the large momentum contributions from the perturbative QCD calculations as the coupling constant at high energy is very small. The FBP is a sum of Yukawa potentials which is an analogous to the exchange of a series of particles and ghosts with different masses (Euclidean form),

$$G(k^2) = \sum_{j=0}^{n} \frac{a_j}{k^2 + (N + j\rho)^2}, \tag{2}$$

where N stands for the minimum value of the masses, ρ is their mass difference, and a_j is their relative coupling constant.

The definition of momentum regions between infrared and intermediate momentum is about $\Lambda_{\mathrm{QCD}} = 200$ MeV, which is naturally set up by the minimum value of the masses $N = 1\Lambda_{\mathrm{QCD}}$, where the Gaussian function $\mathrm{e}^{-k^2/\Delta}$ decays to about 0.3 of its original values. Certainly, there are some overlaps between those regions, in this way, we can guarantee the continuity of the momentum. The asymptotic freedom tell us that at high energy the gauge coupling is very small and can be neglected safely, on the other hand, our phenomenological potential at energy about $N + j\rho,\ j > 3$ is already extend to the perturbative region and catches some perturbative physical effects. Thus, our phenomenological infrared modified FBP is supposed to embody a great deal of physical information about all the momentum regions.

Due to the particular condition we take for the FBP, there is no divergence in solving the SDE. In its three-dimensional form, the FBP takes the following form:

$$V(r) = -\sum_{j=0}^{n} a_j \frac{e^{-(N+j\rho)r}}{r}. \tag{3}$$

[2] Here we correct a writing error in the first version.

In order to suppress the singular point at $r = 0$, we take the following conditions:

$$V(0) = \text{const},$$
$$\frac{dV(0)}{dr} = \frac{d^2V(0)}{dr^2} = \cdots = \frac{d^nV(0)}{dr^n} = 0. \tag{4}$$

So we can determine a_j by solve the following equations, inferred from the flat bottom condition Eq. (4),

$$\sum_{j=0}^{n} a_j = 0,$$
$$\sum_{j=0}^{n} a_j (N + j\rho) = V(0),$$
$$\sum_{j=0}^{n} a_j (N + j\rho)^2 = 0,$$
$$\vdots$$
$$\sum_{j=0}^{n} a_j (N + j\rho)^n = 0. \tag{5}$$

As in previous literature [13–15], n is set to be 9.

3. Schwinger–Dyson equation

The Schwinger–Dyson equation, in effect the functional Euler–Lagrange equation of the quantum field theory, provides a natural framework for investigating the nonperturbative properties of the quark and gluon Green's functions. By studying the evolution behavior and analytic structure of the dressed quark propagator, one can obtain valuable information about the dynamical symmetry breaking phenomenon and confinement. The SDE for the quark takes the following form:

$$S^{-1}(p) = i\gamma \cdot p + m + \frac{16\pi i}{3} \int \frac{d^4k}{(2\pi)^4} \Gamma_\mu S(k) \gamma_\nu G_{\mu\nu}(k - p), \tag{6}$$

where

$$S^{-1}(p) = iA(p^2)\gamma \cdot p + B(p^2) \equiv A(p^2)\big[i\gamma \cdot p + m(p^2)\big], \tag{7}$$

$$G_{\mu\nu}(k) = \left(\delta_{\mu\nu} - \frac{k_\mu k_\nu}{k^2}\right) G(k^2), \tag{8}$$

and m stands for an explicit quark mass-breaking term. In the rainbow approximation, we take $\Gamma_\mu = \gamma_\mu$. With the explicit small mass term for the u and d quarks, we can preclude the zero solution for the $B(p)$ and in fact there indeed exists a small bare current quark mass. In this Letter, we take Landau gauge. This dressing comprises the notation of constituent quark by providing a mass $m(p^2) = B(p^2)/A(p^2)$, which is corresponding to the dynamical symmetry breaking. Because the form of the gluon propagator $G(p)$ in the infrared region cannot be exactly inferred from the $SU(3)$ color gauge theory, one often uses model dependent forms as input parameters in the previous studies of the rainbow SDE [6,7,13–16], in this Letter we use the infrared modified FBP to substitute for the gluon propagator.

In this Letter, we assume that a Wick rotation to Euclidean variables is allowed, and perform a rotation analytically continuing p and k into the Euclidean region where them can be denoted by $\bar{p}$ and $\bar{k}$, respectively. Alternatively, one can derive the SDE from the Euclidean path-integral formulation of the theory, thus avoiding possible difficulties in performing the Wick rotation [17]. As far as only numerical results are concerned, the two procedures are equal. In fact, the analytic structure of quark propagator has interesting information about confinement, we will go to this topic again in the third subsection of Section 4.

The Euclidean rainbow SDE can be projected into two coupled integral equations for $A(\bar{p}^2)$ and $B(\bar{p}^2)$, the explicit expressions for those equations can be found in Ref. [14,15]. For simplicity, we ignore the bar on p and k in the following notations.

4. Bethe–Salpeter equation

The BSE is a conventional approach in dealing with the two body relativistic bound state problems [10]. The quark theory of the mesons indicate that the mesons are quark and antiquark bound states. The precise knowledge about the quark structures of the mesons will result in better understanding of their properties. In the following, we write down the BSE for the pseudoscalar mesons,

$$\begin{aligned} &S_+^{-1}(q+\xi P)\chi(q,P)S_-^{-1}\big(q-(1-\xi)P\big) \\ &\quad = \frac{16\pi i}{3}\int \frac{d^4k}{(2\pi)^4}\Gamma_\mu \chi(k,P)\Gamma_\nu G_{\mu\nu}(q-k), \end{aligned} \tag{9}$$

where $S(q)$ is the quark propagator function, $G_{\mu\nu}(k)$ is the gluon propagator, P_μ is the four-momentum of the center of mass of the pseudoscalar mesons, q_μ is the relative four-momentum between the quark and antiquark in the pseudoscalar mesons, Γ_μ is the full vertex of quark–gluon, ξ is the center of mass parameter which can be chosen to between 0 and 1, and $\chi(q,P)$ is the Bethe–Salpeter wavefunction (BSW) of the bound state. In the limit $\Gamma_\mu = \gamma_\mu$, we obtain the ladder BSE.

After we perform the Wick rotation analytically and continue q and k into the Euclidean region, the Euclidean pseudoscalar BSW $\chi(q,P)$ can be expanded in Lorentz-invariant functions:

$$\begin{aligned} \chi(q,P) = \gamma_5\big[&iF_1(q,q\cdot P) + \gamma\cdot P F_2(q,q\cdot P) \\ &+ \gamma\cdot q F_3(q,q\cdot P) \\ &+ i[\gamma\cdot q,\gamma\cdot P]F_4(q,q\cdot P)\big]. \end{aligned} \tag{10}$$

The BSW F_i can be expressed in terms of the $SO(4)$ eigenfunctions, the Tchebychev polynomials $T_n^{1/2}(\cos\theta)$,

$$F_i(q,q\cdot P) = \sum_0^\infty F_i^n(q,P) q^n P^n T_n^{1/2}(\cos\theta), \tag{11}$$

where $n=$ even if $i=1,2,4$; $n=$ odd if $n=3$, $T_n^{1/2}(\cos\theta) = \cos(n\cos\theta)$ and θ is the included angle between q and P. In solving the coupled BSEs for F_i^n, it is impossible to solve an infinite series of coupled equations, we have to make truncations in one or the other ways in practical manipulations. Numerical calculations indicate that taking only $n=0,1$ terms can give satisfactory results:

$$\begin{aligned} \chi(q,P) = \gamma_5\big[&iF_1^0(q,P) + \gamma\cdot P F_2^0(q,P) \\ &+ \gamma\cdot q q\cdot P F_3^1(q,P) \\ &+ i[\gamma\cdot q,\gamma\cdot P]F_4^0(q,P)\big]. \end{aligned} \tag{12}$$

For a thorough investigation of the solutions of the above BSWs, we must take full quark propagator and full quark–gluon vertex, again we are led to solve a divergent series of coupled SDEs and BSEs, truncations in one or the other ways for the quark propagator and quark–gluon vertex are also necessary. In this Letter, we take the bare vertex for both the SDE and BSE.

In solving the BSEs, it is important to translate the wavefunctions F_i^n into the same dimension,

$$\begin{aligned}
&F_1^n \to \Lambda^{2n} F_1^n, \qquad F_2^n \to \Lambda^{2n+1} F_2^n, \\
&F_3^n \to \Lambda^{2n+1} F_3^n, \qquad F_4^n \to \Lambda^{2n+2} F_4^n, \\
&q \to q/\Lambda, \qquad P \to P/\Lambda,
\end{aligned} \tag{13}$$

where Λ is some quantity of the dimension of mass.

Here we take a short digression to discussing the spectrum of the BSEs. In ideal conditions, a precise solution of the BSE for the bound states of definite quantum numbers will reproduce the full spectrum with the fundamental parameters of QCD, such as $SU(3)$ gauge invariance, quark masses, etc. For example, the solutions of the BSE for 0^{-+} mesons will result in a full pseudoscalar spectrum for both the fundamental states and excited states such as $\pi^0, \pi(1300), \ldots$. However, the present conditions are far from the case, the truncated BSEs always result in a spectrum with more bound states (artifact) [18]. Moreover, the spectrum is not the major subject which the present Letter concern. So in the Letter, we take the masses of the pseudoscalar mesons as input parameters and make an investigation of the π and B mesons BSWs for both ladder approximation and bare quark propagator approximation.

The ladder BSE can be projected into four coupled integral equations in the following form:

$$\begin{aligned}
&H(1,1)F_1^0(q,P) + H(1,2)F_2^0(q,P) \\
&\quad + H(1,3)F_3^1(q,P) + H(1,4)F_4^0(q,P) \\
&\qquad = \int_0^\infty k^3\,dk \int_0^\pi \sin^2\theta\, K(1,1), \\
&H(2,1)F_1^0(q,P) + H(2,2)F_2^0(q,P) \\
&\quad + H(2,3)F_3^1(q,P) + H(2,4)F_4^0(q,P) \\
&\qquad = \int_0^\infty k^3\,dk \int_0^\pi \sin^2\theta\, \big(K(2,2) + K(2,3)\big), \\
&H(3,1)F_1^0(q,P) + H(3,2)F_2^0(q,P) \\
&\quad + H(3,3)F_3^1(q,P) + H(3,4)F_4^0(q,P) \\
&\qquad = \int_0^\infty k^3\,dk \int_0^\pi \sin^2\theta\, \big(K(3,2) + K(3,3)\big), \\
&H(4,1)F_1^0(q,P) + H(4,2)F_2^0(q,P) \\
&\quad + H(4,3)F_3^1(q,P) + H(4,4)F_4^0(q,P) \\
&\qquad = \int_0^\infty k^3\,dk \int_0^\pi \sin^2\theta\, K(4,4),
\end{aligned} \tag{14}$$

the expressions of the $H(i,j)$ and $K(i,j)$ are cumbersome and neglected here, the interested readers can get the word-version from the author.

Here we give some explanations about the expressions of $H(i,j)$. The $H(i,j)$'s are functions of the quark's Schwinger–Dyson functions (SDF)

$$\begin{aligned}
&A\big(q^2 + \xi^2 P^2 + \xi q\cdot P\big), \quad B\big(q^2 + \xi^2 P^2 + \xi q\cdot P\big), \\
&A\big(q^2 + (1-\xi)^2 P^2 - (1-\xi) q\cdot P\big), \\
&B\big(q^2 + (1-\xi)^2 P^2 - (1-\xi) q\cdot P\big).
\end{aligned}$$

The relative four-momentum q is a quantity in Euclidean space–time while the center of mass four-momentum P is a quantity in Minkowski space–time. The present theoretical techniques cannot solve the SDE in Minkowski space–time, we have to expand A and B in terms of Taylor series of $q\cdot P$,

$$\begin{aligned}
&A\big(q^2 + \xi^2 P^2 + \xi q\cdot P\big) \\
&\quad = A\big(q^2 + \xi^2 P^2\big) + A\big(q^2 + \xi^2 P^2\big)' \xi q\cdot P + \cdots, \\
&\vdots \\
&B\big(q^2 + \xi^2 P^2 + \xi q\cdot P\big) \\
&\quad = B\big(q^2 + \xi^2 P^2\big) + B\big(q^2 + \xi^2 P^2\big)' \xi q\cdot P + \cdots.
\end{aligned} \tag{15}$$

The other problem is that we cannot solve the SDE in the time-like region as the two-point gluon Green's function cannot be exactly inferred from the $SU(3)$ color gauge theory even in the low energy space-like region. In practical manipulations, we can extract the values of A and B from the space-like region smoothly to the time-like region with the polynomial functions. To avoid possible violation with confinement in sense of the appearance of pole masses $q^2 = -m(q^2)$, we must be care in the choice of polynomial functions [11].

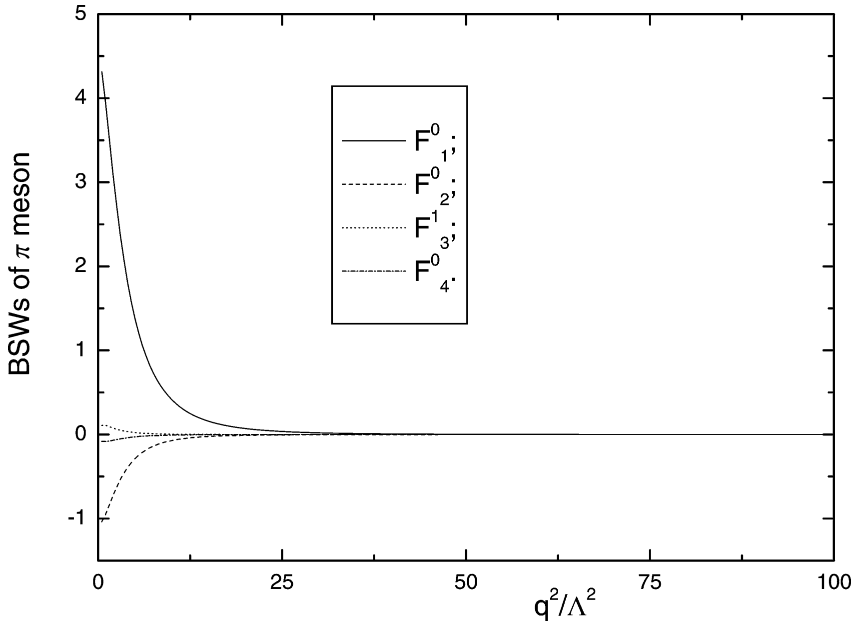

Fig. 1. BSWs of π meson.

Finally, we write down the normalization condition for the BSW,

$$\int \frac{d^4q}{(2\pi)^4} \times \left\{ \bar{\chi} \frac{\partial S^{-1}(q+\xi P)}{\partial P_\mu} \chi(q,P) S^{-1}\big(q-(1-\xi)P\big) + \bar{\chi} S^{-1}(q+\xi P)\chi(q,P) \times \frac{\partial S^{-1}(q-(1-\xi)P)}{\partial P_\mu} \right\} = 2P_\mu, \tag{16}$$

where $\bar{\chi} = \gamma_4 \chi^+ \gamma_4$. We can substitute the expressions of the BSWs and SDFs into the above equation and obtain the precise result, however, the expressions are cumbersome and neglected here.

4.1. Decay constants of pseudoscalar mesons

The decay constants of the pseudoscalar mesons are defined by the following current-meson duality:

$$i f_\pi P_\mu = \langle 0|\bar{q}\gamma_\mu\gamma_5 q|\pi(P)\rangle = \sqrt{N_c}\int \mathrm{Tr}\left[\gamma_\mu\gamma_5\chi(k,P)\frac{d^4k}{(2\pi)^4}\right], \tag{17}$$

here we use π to represent the pseudoscalar mesons.

4.2. Bethe–Salpeter equation with bare quark–gluon vertex and bare quark propagator

In this subsection, we investigate the BSWs of the π and B mesons with the quark–gluon vertex and quark propagator are both taken to be bare,

$$\Gamma_\mu = \gamma_\mu, \qquad S^{-1}(p) = i\gamma\cdot p + M, \tag{18}$$

where the effective mass M is taken to be the constituent quark mass for the u, d and b quarks. In this Letter, we take the effective mass M as an input parameter. Such a two-point quark Green's function cannot embody dynamical chiral symmetry breaking and has a mass pole in the time-like region. However, as a first step, we can study the BSWs for those pseudoscalar mesons with the crude approximation. The algebraic expressions for the BSWs can be obtained easily with a simple substitution of $A(p) \to 1$ and $B(p) \to M$ in Eq. (14). After solving the BSEs numerically by iterations, we plot the BSWs $F_1^0, F_2^0, F_3^1, F_4^0$ as functions of the relative four-momentum q for the π meson and B meson, respectively. In this Letter, we take the π meson BSWs explicitly shown in Fig. 1 as an example and neglect others for simplicity. As the values of the wavefunctions F_3^1, F_4^0 are tiny, we plot them perspicuously in another figure. From those figures, we can see that the first two wavefunctions are dominating, $F_1^0, F_2^0 \gg F_3^1, F_4^0$, and all of the four wavefunctions are Gaussian-type and center around very small momentum, i.e., near zero momentum which indicates that the bound states must exist at the small momentum region or in other words confinement occurs at the infrared region. Based on the numerical values of the BSWs of the π and B mesons, we can obtain the corresponding decay constants.

$$f_\pi = 134\ \text{MeV}, \qquad f_B = 164\ \text{MeV}, \tag{19}$$

which are compatible with the experimental, lattice and QCD sum rule results, $f_\pi = 130$ MeV (Exp) and $f_B \approx 150$–210 MeV (Latt, sumrule) [20–22]. In calculation, the input parameters are $N = 1.0\Lambda$, $V(0) = -10.0\Lambda$, $\rho = 5.0\Lambda$, $M_u = M_d = 530$ MeV, $M_b = 5200$ MeV, $\Lambda = 200$ MeV, $\varpi = 1.3$ GeV and $\Delta = 0.03\ \text{GeV}^2$.

4.3. Coupled rainbow Schwinger–Dyson equation and ladder Bethe–Salpeter equation

In this subsection, we explore the coupled equations of the rainbow SDE and ladder BSE with the bare quark–gluon vertex for both the π and B mesons. The algebraic expressions for those solutions are obtained already in Section 3 and forepart of Section 4, here we will not repeat the tedious routine. In solving those equations numerically, the simultaneous iterations converge quickly to an unique value independent of the choice of initial wavefunctions. The final results for the SDFs and BSWs are plotted as functions of the square momentum q^2.

The quark–gluon vertex can be dressed through the solutions of the Ward–Takahashi identity or Slavnov–Taylor identity and taken to be the Ball–Chiu vertex and Curtis–Pennington vertex [23,24]. Although it is possible to solve the SDE with the dressed vertex, our analytical results indicate that the expressions for the BSEs with the dressed vertex are cumbersome and not suitable for numerical iterations.[3]

In order to demonstrate the confinement of quark, we have to study the SDF of the quark and prove that there no poles on the real timelike p^2 axial. So it is necessary to perform an analytic continuation of the dressed quark propagator from Euclidean space into Minkowski space $p_4 \to ip_0$. However, we have no knowledge of the singularity structure of quark propagator in the whole complex plane. One can take an alternative safe procedure and stay completely in Euclidean space avoiding analytic continuations of the dressed propagators [25]. It is sufficient to take the Fourier transform with respect to the Euclidean time T for the scalar part S_s,

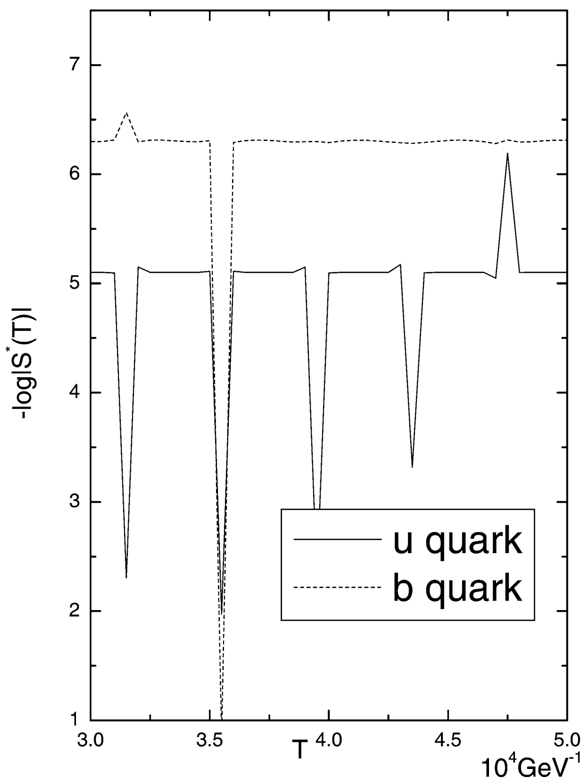

Fig. 2. $-\log|S^*(T)|$.

$$S_s^*(T) = \int_{-\infty}^{+\infty} \frac{dq_4}{2\pi} e^{iq_4T} S_s = \int_{-\infty}^{+\infty} \frac{dq_4}{2\pi} e^{iq_4T} \frac{B(q^2)}{q^2A^2(q^2) + B^2(p^2)}. \tag{20}$$

If $S(p)$ had a pole at $p^2 = -m^2$, the Fourier transformed $S_s^*(T)$ would fall off as e^{-mT} for large T or $\log S_s^* = -mT$.

[3] This observation is based on the authors' work in USTC.

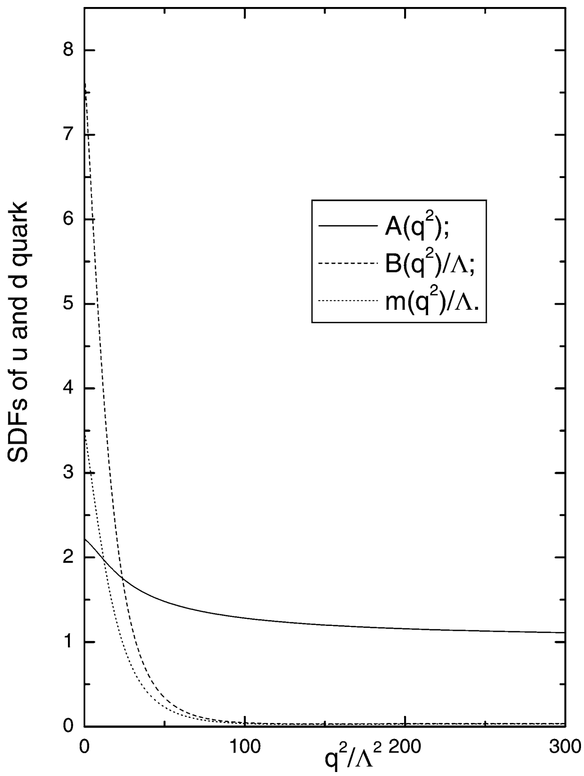

Fig. 3. SDFs of u and d quark.

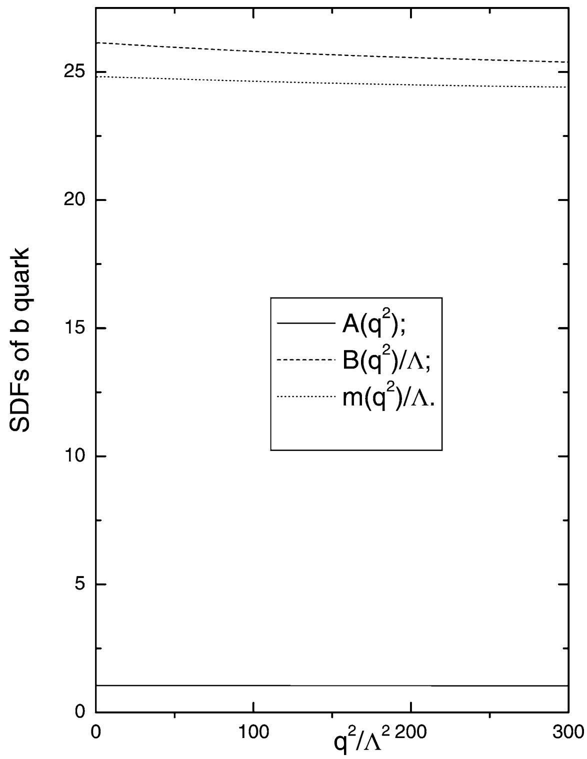

Fig. 4. SDFs of b quark.

In our calculation, for large T, the values of S_s^* is negative, except occasionally a very small fraction positive values. We can express S_s^* as $|S_s^*|e^{in\pi}$, n is an odd integer. $\log S_s^* = \log|S_s^*| + in\pi$. If we neglect the imaginary part, we find that when the Euclidean time T is large, there indeed exists a crudely approximated (almost flat) linear function with about zero slope for all the u, d (the curve for the d quark has the same behavior as the u quark in the limit of Isospin symmetry is exact) and b quarks with respect to T, which is shown in Fig. 2. Here the word 'crudely' should be understand in the linearly fitted sense, to be exact, there is no linear function. However, such fitted linear functions are hard to acquire physical explanation and the negative values for S_s^* indicate an explicit violation of the axiom of reflection positivity [26], in other words, the quarks are not physical observable, i.e., confinement.

From Fig. 3, we can see that for the u and d quarks, the SDFs are greatly renormalized at small momentum region and the curves are steep at about $q^2 = 1\ \mathrm{GeV}^2$ which indicates an explicit dynamical chiral symmetry breaking, while at large q^2, they take asymptotic behavior. As for the b quark, shown in Fig. 4, the current mass is very large, the renormalization is more tender, however, mass pole in the time-like region is also absent, which can be seen from Fig. 2. At zero momentum, $m_u(0) = 688$ MeV, $m_d(0) = 688$ MeV and $m_b(0) = 4960$ MeV, which are compatible with the constituent quark masses. In fact, the connection of $m(p)$ to constituent masses is somewhat less direct for the light quarks and is precise only for the heavy quarks. For heavy quarks, $m_{\text{constituent}}(p) = m\ (p = 2m_{\text{constituent}}(p))$, for light quarks, it only makes a crude estimation [19]. From the plotted BSWs (neglected here for simplicity), we can see that the BSWs for both the π and B mesons have the same type momentum dependence as the corresponding wavefunctions with the bare quark propagators, however, the quantitative values are changed. The Gaussian type BS wavefunctions which center around small momentum indicate that the bound states exist only in the infrared region, in other words confinement. Finally we obtain the values for the decay constants of the π and B mesons,

$$f_\pi = 127\ \text{MeV}, \qquad f_B = 192\ \text{MeV}, \tag{21}$$

which are compatible with the experimental, lattice and QCD sum rule results, $f_\pi = 130$ MeV (Exp) and $f_B \approx 150$–210 MeV (Latt, sumrule) [20–22]. In calculation, the input parameters are $N = 1.0\Lambda$, $V(0) = -11.0\Lambda$, $\rho = 5.0\Lambda$, $m_u = m_d = 6$ MeV, $m_b = 4700$ MeV, $\Lambda = 200$ MeV, $\varpi = 1.6$ GeV and $\Delta = 0.04\ \text{GeV}^2$.

From the variations of the values for the decay constants of both the π and B mesons, we can estimate that the full vertex approximation will not change those values greatly.

5. Conclusion and discussion

In this Letter, we investigate the under-structures of the π and B mesons in the framework of the Bethe–Salpeter equation with the confining effective potential (infrared modified flat bottom potential). In bare quark–gluon vertex approximation, we obtain the algebraic expressions for the solutions of the coupled rainbow SDE and ladder BSE for those mesons. At the first step, we neglect the rainbow SDE, take the bare quark propagator and solve the BSE numerically alone. Although the bare quark propagator cannot embody dynamical chiral symmetry breaking and has a mass pole in the time-like region, it can give reasonable results for the values of the decay constants f_π, f_B compared with the values of the experimental data and other theoretical calculations, such as lattice simulations and QCD sum rules. In calculation, we obtain the BSWs for the π and B mesons, which center in the small momentum region, are compatible with confinement. Secondly, we explore the coupled equations of the rainbow SDE and ladder BSE with the bare quark–gluon vertex for those mesons. The quark–gluon vertex can be dressed through the solutions of the Ward–Takahashi identity or Slavnov–Taylor identity and taken to be the Ball–Chiu vertex and Curtis–Pennington vertex, however, a consistently numerical manipulation is unpractical. After we solve the coupled rainbow SDE and ladder BSE numerically, we obtain both the SDFs and BSWs for both the π and B mesons. The SDFs for the u and d quarks are greatly renormalized at small momentum region and the curves are steep at about $q^2 = 1\ \text{GeV}^2$ which indicates an explicitly dynamical chiral symmetry breaking. After we take Euclidean time Fourier transformation about the quark propagator, we can find that there is no mass pole in the time-like region and obtain satisfactory result about confinement. As for the b quark, the current mass is very large, the renormalization is more tender, however, mass pole in the time-like region is also absent. The BSWs for both the π and B mesons have the same type momentum dependence as the corresponding wavefunctions with the bare quark propagators, however, the quantitative values are changed and the corresponding values for the decay constants f_π and f_B are changed, but not greatly. We can estimate that the full vertex approximation will not change those values greatly. Once the SDFs and BSWs for both the π and B mesons are known, we can use them to investigate a lot of important quantities in the B meson decays, such as B–π, B–K, B–ρ former factors, Isgur–Wise functions, strong coupling constants, etc.

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