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Taming the higher power corrections in semileptonic B decays

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ABSTRACT

We study the effect of dimension 7 and 8 operators on inclusive semileptonic B decays and the extraction of $|V_{cb}|$. Using moments of semileptonic B decay spectra and information based on the Lowest-Lying State saturation Approximation (LLSA) we perform a global fit of the non-perturbative parameters of the heavy quark expansion including for the first time the $\mathcal{O}(1/m_b^{4,5})$ contributions. Higher power corrections appear to have a very small effect on the extraction of $|V_{cb}|$, independently of the weight we attribute to the LLSA.

1. Introduction

The results of the B Factories and LHC place stringent constraints on new physics in the flavor sector. Only small deviations from the SM are allowed, and their detection represents an experimental and theoretical challenge. In the next few years a wealth of new experimental results will come from Belle-II and from the high-luminosity phase of LHC. In this context, the precise determination of the parameters of the Cabibbo–Kobayashi–Maskawa (CKM) matrix remains a high priority, as it is instrumental to constraining new physics models and to setting bounds on the scale of new effective interactions. However, the determination of the CKM element V_{cb}, which plays a special role in tests of the CKM unitarity and in FCNC transitions, is plagued by a long-standing $\sim 3\sigma$ tension between the analyses based on inclusive and exclusive decays. This is unlikely to signal new physics [1] and calls for a thorough investigation of all possible sources of theoretical uncertainty.

The determination of $|V_{cb}|$ from inclusive semileptonic B decays is based on an Operator Product Expansion (OPE) [2–5] which allows us to parameterize all of the non-perturbative physics in terms of the expectation values of local operators in the B-meson to be extracted from experimental data. Since the contribution of higher dimensional operators is suppressed by powers of the heavy quark mass, only the operators of low dimension are expected to be relevant. Current fits of inclusive semileptonic B decays [6] use experimental data on the moments of kinematic distributions to constrain the power corrections up to $1/m_b^3$ terms, corresponding to dimension ≤ 6 operators, and neglect higher power corrections altogether.

While present data appear to be well described by these fits, investigations of higher power corrections are mandatory to test the convergence of the heavy quark mass expansion. Moreover, the OPE does not lead to an expansion of inclusive observables in inverse powers of m_b but also contains terms of $\mathcal{O}(1/m_b^n\ 1/m_c^k)$, with odd $n \geq 3$ and even $k \geq 2$, sometimes dubbed *intrinsic charm* (IC) contributions [7–9], which alter the actual power counting since numerically $m_c^2 \sim \Lambda_{QCD} m_b$ and thus $\mathcal{O}(1/m_b^3 m_c^2) \simeq \mathcal{O}(1/m_b^4)$. Higher power corrections have been studied in [10,11], where nine new operators of dimension 7 and eighteen new operators of dimension 8 have been identified and their Wilson coefficients computed at the tree-level. A rough estimate of the matrix elements of these 27 new operators is given by the Lowest-Lying State Approximation (LLSA) [11,12], which assumes that the lowest lying heavy meson states saturate a sum-rule for the insertion of a heavy meson state sum. The LLSA relates higher-order matrix elements to lower dimensional ones and to the excitation energy ϵ and is expected to be valid within 50–100% [12].

In this Letter, after briefly reviewing the structure of the $1/m_b^{4,5}$ corrections computed in [11], we study their inclusion in the fit of Ref. [6] and discuss how the results depend on the uncertainty associated to the LLSA.

2. Power corrections and matrix elements

Our analysis is based on the calculation of higher power corrections of [11], which is performed at leading order in α_s. The

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inclusive observables considered below (width, moments of kinematic distributions) can be calculated by an appropriate (weighted) phase-space integral of the differential decay width

$$\mathrm{d}\Gamma = 16\pi G_F^2 |V_{cb}|^2 W_{\mu\nu} L^{\mu\nu} \mathrm{d}\phi\,, \tag{1}$$

where all the soft hadronic information is contained in the hadronic tensor $W_{\mu\nu} = -\frac{1}{\pi}\,\mathrm{Im}\,T_{\mu\nu}$. The hadronic tensor is the imaginary part of the forward matrix element of a time-ordered product of weak currents. The charm quark in this forward matrix element propagates in a background field. We expand the background field propagator S_{BGF}, with momentum $Q^\mu = m_b v^\mu + k^\mu - q^\mu$, in powers of k^μ/m_b, where $k^\mu \to iD^\mu$ is the residual momentum of the b-quark inside the B-meson

$$\begin{aligned} T_{\mu\nu} &= \langle B(p)|\bar{b}_v \Gamma_\mu i S_{\mathrm{BGF}} \Gamma_\nu^\dagger b_v|B(p)\rangle \\ &= \sum_i \mathrm{Tr}\left\{\Gamma_\mu \frac{1}{\not{Q} - m_c + i\epsilon}\Gamma_\nu^\dagger\,\hat{\Gamma}^{(i)}\right\} A^{(i,0)} \\ &+ \sum_i \mathrm{Tr}\left\{\Gamma_\mu \frac{1}{\not{Q} - m_c + i\epsilon}\gamma^{\mu_1}\frac{1}{\not{Q} - m_c + i\epsilon}\Gamma_\nu^\dagger\,\hat{\Gamma}^{(i)}\right\} A^{(i,1)}_{\mu_1} \\ &+ \cdots \end{aligned} \tag{2}$$

The coefficients $A^{(i,m)}_{\mu_1\mu_2\ldots\mu_m}$ containing the non-perturbative parameters are known analytically at $\mathcal{O}(1/m_b^2)$ [4,5] (corresponding to $m = 2$), at $\mathcal{O}(1/m_b^3)$ [13], and at order $1/m_b^{4,5}$ [11]. At the lowest non-trivial order, corresponding to dimension 5 operators, the non-perturbative parameters are given by

$$\begin{aligned} 2M_B\,\mu_\pi^2 &= -\langle \bar{B}|\bar{b}_v\, iD_\rho iD_\sigma\, b_v|\bar{B}\rangle\ \Pi^{\rho\sigma}\,, \\ 2M_B\,\mu_G^2 &= \frac{1}{2}\langle \bar{B}|\bar{b}_v\left[iD_\rho, iD_\sigma\right]\left(-i\sigma_{\alpha\beta}\right) b_v|\bar{B}\rangle\ \Pi^{\alpha\rho}\Pi^{\beta\sigma}\,, \end{aligned} \tag{3}$$

where $\Pi^{\mu\nu} = g^{\mu\nu} - v^\mu v^\nu$, and v^μ is the heavy quark velocity. At each higher order in $1/m_b$ we have one more derivative in $A^{(i,m)}_{\mu_1\mu_2\ldots\mu_m}$. Thus the number of parameters proliferates. We have only 2 parameters, ρ_D^3 and ρ_{LS}^3, at $O(1/m_b^3)$, but there are nine additional ones at $O(1/m_b^4)$ and eighteen at $O(1/m_b^5)$. As mentioned in the Introduction, upon integration over the phase space the Wilson coefficient of some of the dimension 8 operators are sensitive to the (infrared) charm mass scale and represent the IC terms of $O(1/m_b^3 m_c^2)$, which numerically dominate the $O(1/m_b^5)$ contributions.

In the following we will include the $O(1/m_b^{4,5})$ corrections in the fit to the semileptonic moments on which the inclusive determination of $|V_{cb}|$ is based. We will use the LLSA ansatz, proposed in [11] and made more systematic in [12], to constrain the 27 new parameters.

The goal of LLSA is to estimate expectation values of local operators of the form $\bar{b}_v iD_{\mu_1} iD_{\mu_2}\ldots iD_{\mu_n}\Gamma b_v$, where Γ is a Dirac matrix. Splitting the chain of covariant derivatives into two shorter ones labeled by A_1^k and C_k^n and inserting a full set of intermediate states between them one finds in the heavy quark limit [11,12]

$$\begin{aligned} &\langle \bar{B}|\bar{b}_v\, A_1^k\, C_k^n\, \Gamma\, b_v|\bar{B}\rangle = \\ &\qquad \frac{1}{2M_B}\sum_n \langle \bar{B}|\bar{b}_v\, A_1^k\, b_v(0)|H_n\rangle\langle H_n|\bar{b}_v(0)\, C_k^n\, \Gamma\, b_v|\bar{B}\rangle\,, \end{aligned} \tag{4}$$

where $|H_n\rangle$ are hadronic states with the appropriate quantum numbers. The LLSA assumes that the sum of intermediate states is saturated by the lowest-lying state that can contribute, i.e. either the ground-state multiplet B, B^* or the first excited states with $\ell = 1$. Indeed, the matrix elements involving time derivatives like $\langle B|\bar{b}iD_j iD_0^k iD_l b|B\rangle$ are saturated by P-wave intermediate states, with parity opposite to that of the ground state. Including these states in the sum leads to extra powers of the P-wave excitation energy, $\epsilon = M_P - M_B$. While there exist separate contributions coming from the spin $\frac{1}{2}, \frac{3}{2}$ light degrees of freedom, we assume $\epsilon_{1/2} = \epsilon_{3/2} = \epsilon \simeq 0.4$ GeV.

Table 1
LLSA expressions for the higher-order non-perturbative parameters.

$\overline{m}_1$	$\frac{5(\mu_\pi^2)^2}{9}$	r_6	$\epsilon^2\rho_D^3$
$\overline{m}_2$	$-\epsilon\rho_D^3$	r_7	0
$\overline{m}_3$	$-\frac{(\mu_G^2)^2}{6}$	r_8	$\epsilon^2\rho_{LS}^3$
$\overline{m}_4$	$\frac{(\mu_G^2)^2}{8} + \frac{(\mu_\pi^2)^2}{6}$	r_9	$-\mu_\pi^2\rho_{LS}^3$
$\overline{m}_5$	$-\epsilon\rho_{LS}^3$	r_{10}	$\mu_G^2\rho_D^3$
$\overline{m}_6$	$\frac{(\mu_G^2)^2}{6}$	r_{11}	$\frac{\mu_G^2\rho_D^3}{3} - \frac{\mu_G^2\rho_{LS}^3}{6} + \frac{\mu_\pi^2\rho_{LS}^3}{3}$
$\overline{m}_7$	$-\frac{\mu_G^2\mu_\pi^2}{3}$	r_{12}	$-\frac{\mu_G^2\rho_D^3}{3} - \frac{\mu_G^2\rho_{LS}^3}{6} - \frac{\mu_\pi^2\rho_{LS}^3}{3}$
$\overline{m}_8$	$-\mu_G^2\mu_\pi^2$	r_{13}	$-\frac{\mu_G^2\rho_D^3}{3} + \frac{\mu_G^2\rho_{LS}^3}{6} + \frac{\mu_\pi^2\rho_{LS}^3}{3}$
$\overline{m}_9$	$\frac{(\mu_G^2)^2}{8} - \frac{5\mu_G^2\mu_\pi^2}{12}$	r_{14}	$\rho_{LS}^3\left(\epsilon^2 + \frac{\mu_G^2}{6} - \frac{\mu_\pi^2}{3}\right) + \frac{\mu_G^2\rho_D^3}{3}$
r_1	$\epsilon^2\rho_D^3$	r_{15}	0
r_2	$-\mu_\pi^2\rho_D^3$	r_{16}	0
r_3	$-\frac{\mu_G^2\rho_{LS}^3}{6} - \frac{\mu_\pi^2\rho_D^3}{3}$	r_{17}	$\epsilon^2\rho_{LS}^3$
r_4	$\epsilon^2\rho_D^3 + \frac{\mu_G^2\rho_{LS}^3}{6} - \frac{\mu_\pi^2\rho_D^3}{3}$	r_{18}	0
r_5	0		

In the following we use the notation of [11], according to which the nine matrix elements that occur at $O(1/m^4)$ are denoted by m_i, and the eighteen at $O(1/m^5)$ by r_i. The operators involved coincide with those identified in [12], even though different notations are adopted. It is useful to redefine the $1/m_b^4$ parameters to account for combinatorial factors. In practice, we expand the (anti-)commutators and count the number of terms after expunging those which are of higher order in $1/m_b$ due to the equations of motion. We then expect the parameters to have a *natural* scale of $O(\Lambda^n_{QCD})$, with n the dimension of the corresponding operator, as is also the case for the parameters in Eq. (3). The rescaled parameters are

$$\begin{aligned} &\overline{m}_1 = m_1 && \overline{m}_2 = m_2 && \overline{m}_3 = m_3/4 \\ &\overline{m}_4 = m_4/8 && \overline{m}_5 = m_5 && \overline{m}_6 = m_6/4 \\ &\overline{m}_7 = m_7/8 && \overline{m}_8 = m_8/8 && \overline{m}_9 = m_9/8\,. \end{aligned} \tag{5}$$

No such redefinition is necessary for the $1/m_b^5$ parameters, as they were already defined in this way. The LLSA expressions for the $\overline{m}_i, r_i$ are reported in Table 1.

3. Inclusive observables

The OPE allows us to express sufficiently inclusive observables as a double series in α_s and Λ_{QCD}/m_b. In fact, the non-perturbative corrections to the semileptonic differential rate start at $\mathcal{O}(1/m_b^2)$. Perturbative corrections are known up to NNLO [14–17] and the mixed $\mathcal{O}(\alpha_s\mu^2_{\pi,G}/m_b^2)$ corrections [18–20] have also been calculated. The expansion requires knowledge of the expectation values of local operators in the B-meson. These non-perturbative parameters can be determined from measurements of the normalized moments of the lepton energy and invariant hadronic mass distributions in inclusive $B \to X_c\ell\nu$ decays,

$$\begin{aligned} \langle E_\ell^n\rangle &= \frac{1}{\Gamma_{E_\ell > E_{\mathrm{cut}}}}\int_{E_\ell > E_{\mathrm{cut}}} E_\ell^n\,\frac{d\Gamma}{dE_\ell}\,dE_\ell\,, \\ \langle M_X^{2n}\rangle &= \frac{1}{\Gamma_{E_\ell > E_{\mathrm{cut}}}}\int_{E_\ell > E_{\mathrm{cut}}} M_X^{2n}\,\frac{d\Gamma}{dM_X^2}\,dM_X^2\,, \end{aligned} \tag{6}$$

where E_ℓ is the lepton energy, m_X^2 the invariant hadronic mass squared and E_{cut} an experimental lower cut on the lepton energy applied by the experiments. The cut dependence of the moments provides additional information on the OPE parameters we are fitting. For moments with $n > 1$, it is convenient to employ *central* moments, computed relative to $\langle E_\ell \rangle \equiv \ell_1$ and $\langle m_X^2 \rangle \equiv h_1$,

$$\ell_n(E_{cut}) = \langle (E_\ell - \langle E_\ell \rangle)^n \rangle_{E_\ell > E_{cut}},$$
$$h_n(E_{cut}) = \langle (M_X^2 - \langle M_X^2 \rangle)^n \rangle_{E_\ell > E_{cut}}. \tag{7}$$

We also have information on the lepton energy cut dependence of the inclusive width, which can be studied introducing $R^* = \Gamma_{E_\ell > E_{\rm cut}} / \Gamma_{\rm tot}$. The information on the non-perturbative parameters obtained from a fit to these observables enables us to then extract $|V_{cb}|$ from the total semileptonic width [6,21–24].

All analyses have so far considered only the minimal set of four matrix elements which appear at $\mathcal{O}(1/m_b^{2,3})$. The $\mathcal{O}(1/m_b^{4,5})$ contributions have never been included, although a rough estimate of their importance has been given in [11]. From the results of that paper we have computed all the $\mathcal{O}(1/m_b^{4,5})$ corrections to the first three hadronic and leptonic moments and to R^*; we will now employ these expressions in the global fit to determine $|V_{cb}|$. The result for the width is given in the Appendix. Notice that normalized moments are ratios of two heavy quark expansions; re-expanding these ratios in inverse powers of m_b one finds that the $\mathcal{O}(1/m_b^{4,5})$ corrections also include products of $\mathcal{O}(1/m_b^2)$ with $\mathcal{O}(1/m_b^{2,3})$ terms.

4. The fit

We upgrade the fit strategy introduced in [24] in the kinetic scheme, and use as a baseline the default parameters and settings most recently employed in [6]. In particular, we use the same experimental data; the full list of available measurements [25–31] and the leptonic energy cuts employed in the fit is given in Table 1 of Ref. [24]. We also employ the $\overline{\rm MS}$ scheme for the charm mass and use the constraints $\overline{m}_c(3\text{ GeV}) = 0.986(13)$ GeV [32], $\mu_G^2(m_b) = 0.35(7)$ GeV^2, $\rho_{LS}^3 = -0.15(10)$ GeV^3.

The inclusion of higher power corrections allows us to slightly decrease the theoretical errors, which are estimated using the method of Ref. [24], i.e. varying the HQE parameters by fixed amounts in the calculation of an observable. Here we use the same settings as in [6], except for the variation in $\rho_{D,LS}^3$, which we decrease from 30% to 22%, to take into account the inclusion of $\mathcal{O}(1/m^4, 5)$ power corrections. For what concerns the correlations among theoretical errors we choose scenario **D** of Ref. [24], where different central moments are uncorrelated and the correlation between measurements of the same moment with E_{cut} differing by 100 MeV is given by a factor which becomes smaller for increasing E_{cut}.

The results of the default fit performed in [6] read

$$\begin{aligned} m_b^{kin} &= 4.553(20), & \overline{m}_c(3\text{ GeV}) &= 0.987(13), \\ \mu_\pi^2 &= 0.465(68), & \mu_G^2 &= 0.332(62), \\ \rho_D &= 0.170(38), & \rho_{LS}^3 &= -0.150(96), \end{aligned} \tag{8}$$

where all parameters except for m_c are in the kinetic scheme with cutoff $\mu_{kin} = 1$ GeV. Using $\tau_B = 1.579$ ps, Ref. [6] gets $|V_{cb}| = 42.21(78)\ 10^{-3}$.

As a first step in the analysis, we repeat exactly the same fit to the $\mathcal{O}(1/m_b^{2,3})$ parameters but include the $\mathcal{O}(1/m_b^{4,5})$ corrections in the theoretical predictions. We fix their values using the LLSA expressions for the matrix elements $\overline{m}_i, r_i$, computed using the central values in (8) and $\epsilon = 0.4$ GeV. The products of $1/m_b^2$ and $1/m_b^3$ effects are also computed using (8) and cannot vary in the fit. The results are similar to those in (8), except that μ_π^2 and ρ_D^3 get a significant shift up, $\mu_\pi^2 = 0.506(74)$ GeV^2, $\rho_D^3 = 0.257(42)$ GeV^3, and that the central value of $|V_{cb}|$ is 42.47 10^{-3}. This total 0.7% increase in $|V_{cb}|$ occurs despite the $\mathcal{O}(1/m_b^{4,5})$ contributions increase the semileptonic width by more than 1%, leading to a *direct* reduction of $|V_{cb}|$. A similar pattern (larger μ_π^2, ρ_D^3, and $|V_{cb}|$) is observed if we fix only the matrix elements $\overline{m}_i, r_i$ to their LLSA values, and let the products of $1/m_b^2$ and $1/m_b^3$ effects to vary.

While the LLSA can set the scale of the higher power effects, it is certainly subject to large corrections. We therefore assign an error to the LLSA predictions and assume gaussian priors for all the $\overline{m}_i, r_i$, which are then fit along with the other parameters. The accuracy of the LLSA is hard to quantify. At $\mathcal{O}(1/m_b^3)$ the values of ρ_D^3 and ρ_{LS}^3 in (8) match well the LLSA expressions $\rho_D^3 = \epsilon\mu_\pi^2$ and $\rho_{LS}^3 = -\epsilon\mu_G^2$. Ref. [12] estimates a $\sim$50% uncertainty, which obviously does not hold when the LLSA leads to zero matrix elements. Ref. [33] in Sec. 6.5 found indications for large non-factorisable corrections, which could reach 100% in some expectation values not affected by cancellations. Dimensionally, we know that the non-perturbative parameters of the OPE are quantities of $\mathcal{O}(\Lambda_{QCD}^n)$. There are in fact two scales involved in their determination: $M_B - m_b$ and the mass splitting $\epsilon \simeq 0.4$ GeV between the B meson and the lowest P-wave excitation. Accordingly, we prescribe the error to be the maximum of either 60% of the parameter's value or $\Lambda_{LL}^n/2$ ($n = 4, 5$), where we use a scale $\Lambda_{LL} = 0.55$ GeV which roughly corresponds to the average of the two relevant scales. The fit is performed starting with LLSA central values based on Eq. (8) and $\epsilon = 0.4$ GeV. The LLSA central values are then updated to the results of the new fit, iterating the procedure until the results stabilize.

5. Results

We report the results of the default fit in Table 2. In Fig. 1 we compare the $\mu_{\pi,G}^2, \rho_{D,LS}^3$ results of the 2014 fit in (8) with those of the new default fit. We also compare the LLSA predictions for $\overline{m}_i, r_i$ based on (8) with the results of the default fit. The LLSA uncertainty is computed as explained in the previous paragraph. We can see that most of the new parameters do not change much from their LLSA value, reflecting the low sensitivity of the fit to higher power parameters. However, there are exceptions, especially among the $\overline{m}_i$: the largest shift occurs for $\overline{m}_2$ and corresponds to $1.2\sigma_{LLSA}$. Indeed, the hadronic moments at higher cuts are specif-

Table 2
Default fit results: the second and third columns give the central values and standard deviations.

m_b^{kin}	4.546	0.021	r_1	0.032	0.024
$\overline{m}_c$(3 GeV)	0.987	0.013	r_2	−0.063	0.037
μ_π^2	0.432	0.068	r_3	−0.017	0.025
μ_G^2	0.355	0.060	r_4	−0.002	0.025
ρ_D^3	0.145	0.061	r_5	0.001	0.025
ρ_{LS}^3	−0.169	0.097	r_6	0.016	0.025
$\overline{m}_1$	0.084	0.059	r_7	0.002	0.025
$\overline{m}_2$	−0.019	0.036	r_8	−0.026	0.025
$\overline{m}_3$	−0.011	0.045	r_9	0.072	0.044
$\overline{m}_4$	0.048	0.043	r_{10}	0.043	0.030
$\overline{m}_5$	0.072	0.045	r_{11}	0.003	0.025
$\overline{m}_6$	0.015	0.041	r_{12}	0.018	0.025
$\overline{m}_7$	−0.059	0.043	r_{13}	−0.052	0.031
$\overline{m}_8$	−0.178	0.073	r_{14}	0.003	0.025
$\overline{m}_9$	−0.035	0.044	r_{15}	0.001	0.025
χ^2/dof	0.46		r_{16}	0.001	0.025
$BR(\%)$	10.652	0.156	r_{17}	−0.028	0.025
$\mathbf{10^3 \lvert V_{cb} \rvert}$	**42.11**	**0.74**	r_{18}	−0.001	0.025

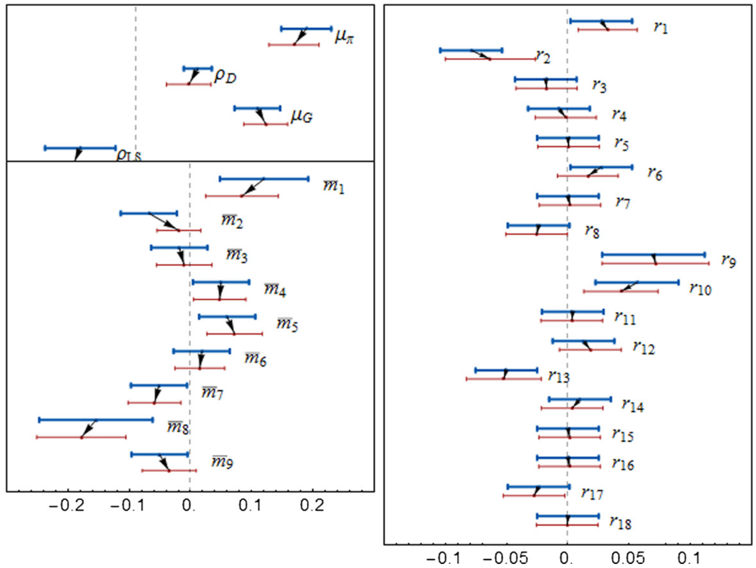

Fig. 1. Shifts in the OPE parameters from the LLSA using the 2014 fit results (upper ranges) to the current fit including higher-order corrections (lower ranges). Error bars represent the error in the priors and the resulting fit error, respectively.

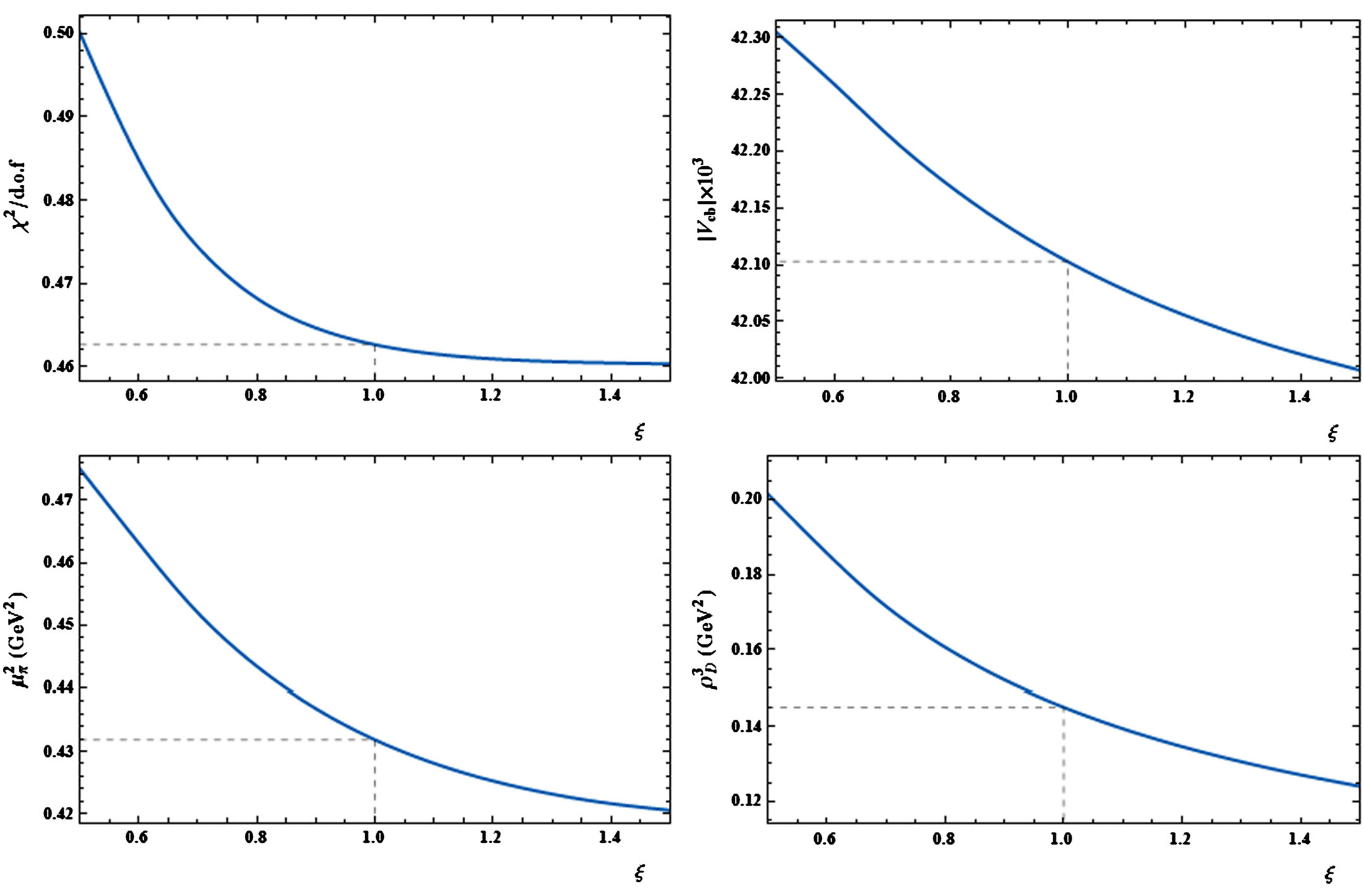

Fig. 2. Dependence of the fit results as a function of the LLSA uncertainty.

ically sensitive to some of the $\overline{m}_i$, see Eqs. (11) in the Appendix. Using the fit results we compute the total semileptonic width, also reported in the Appendix, and comparing it to the BR in Table 2 divided by τ_B, we get $|V_{cb}|$. The value of $|V_{cb}|$ is remarkably close to that obtained in [6] and the quality of the fit is very good, $\chi^2/dof = 0.46$, but somewhat higher than in [6].

To verify the stability of the fit with respect to the choices we made for the LLSA uncertainty, we varied this uncertainty by a multiplicative factor ξ. The results are shown in Fig. 2: $|V_{cb}|$ changes very little. Of course, increasing the uncertainty on the higher-order matrix elements too much is equivalent to ignoring the LLSA completely, which would be unwise. We can therefore

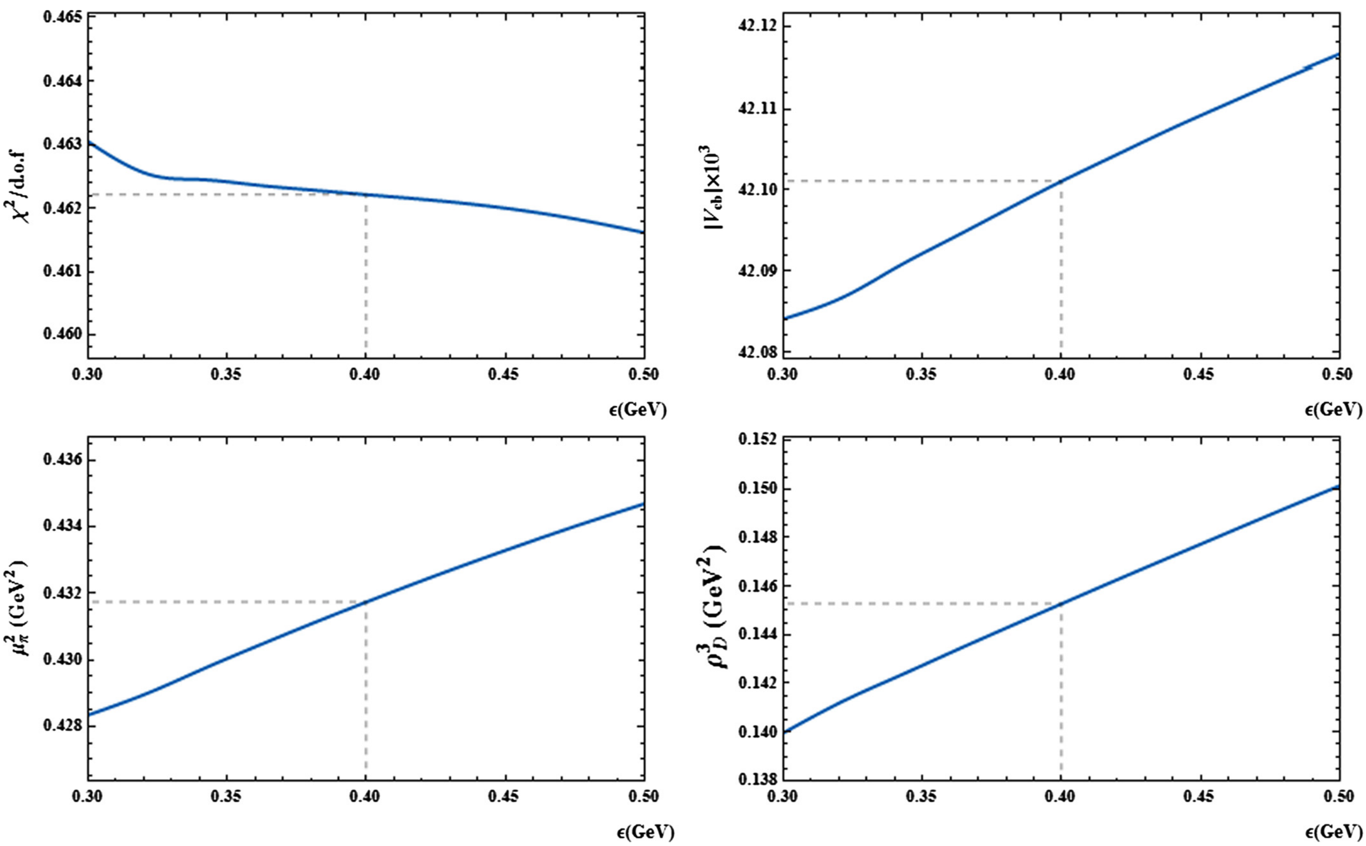

Fig. 3. Dependence of the fit results as a function of the P-wave excitation energy ϵ.

estimate the uncertainty related to the assumptions on the LLSA error by varying ξ between 0.7 and 1.3, obtaining the relative variations on the main parameters

$$\delta^\xi V_{cb} = ^{+0.2\%}_{-0.2\%}, \quad \delta^\xi \mu_\pi^2 = ^{+4.7\%}_{-2.0\%}, \quad \delta^\xi \mu_G^2 = ^{+1.0\%}_{-0.9\%},$$
$$\delta^\xi \rho_D^3 = ^{+18.2\%}_{-10.0\%}, \qquad \delta^\xi \rho_{LS}^3 = ^{-1.3\%}_{+0.9\%}. \tag{9}$$

We will include this uncertainty in the final error on $|V_{cb}|$. We also vary ϵ over the range 0.4 ± 0.1 GeV to gauge the related uncertainty. The dependence of the parameters on the choice of excitation energy can be seen in Fig. 3, and the resulting relative uncertainties are

$$\delta^\epsilon V_{cb} = ^{+0.04\%}_{-0.04\%}, \quad \delta^\epsilon \mu_\pi^2 = ^{+0.7\%}_{-0.8\%}, \quad \delta^\epsilon \mu_G^2 = ^{-0.4\%}_{+0.3\%},$$
$$\delta^\epsilon \rho_D^3 = ^{+3.3\%}_{-3.6\%}, \qquad \delta^\epsilon \rho_{LS}^3 = ^{+0.3\%}_{-0.4\%}, \tag{10}$$

which are mostly negligible.

We also repeated the default fit in two slightly different ways: *i*) adding the PDG constraint on m_b [34] after a scheme conversion, $m_b^{kin} = 4.550(42)$ GeV, which leads to $|V_{cb}| = 42.10(73)\ 10^{-3}$; *ii*) changing, in addition to that, the m_c constraint into $m_c(2\,\text{GeV}) = 1.091(14)$ GeV, obtained evolving the result of [32] to 2 GeV. This leads to a somewhat better convergence of the perturbative series for the semileptonic width [17]; in this case $|V_{cb}| = 42.00(64)\ 10^{-3}$ and $\chi^2/dof = 0.44$. The results of all these fits are remarkably consistent with each other.

6. The fourth hadronic moment

The central hadronic moments are sensitive probes of power corrections. For instance, $\mathcal{O}(1/m_b^{4,5})$ affect h_3 in a significant way and one could expect even higher moments to be able to constrain the higher power contributions in a useful way. As DELPHI has measured $h_{4,5}$ without a cut on the lepton energy [31], we have computed h_4 to explore the possibility of including them in the fit, despite the high correlation with lower hadronic moments. The result, in GeV^8, is

$$h_4 = 0.15_{tree} + 15.97_{pert} + 4.23_{\mu_\pi^2} + 1.81_{\alpha_s\mu_\pi^2} - 0.16_{\mu_G^2}$$
$$+ 0.74_{\alpha_s\mu_G^2} + 2.31_{\rho_D^3} - 0.10_{\rho_{LS}^3} + 3.80_{m_i} - 4.91_{r_i},$$

where we have evaluated the different contributions using Table 2. Perturbative contributions are largely dominant, diluting any possible $\mathcal{O}(1/m_b^{4,5})$ effect and amplifying the uncertainty. In fact, the inclusion of DELPHI's h_4 in the fit has negligible impact on $|V_{cb}|$ and the OPE parameters.

7. Summary

We have studied the effect of higher power corrections on the fits to inclusive semileptonic B decays which determine $|V_{cb}|$. Because of the large number of new parameters at $\mathcal{O}(1/m_b^{4,5})$, we used the LLSA to provide loose constraints on the higher power matrix elements and performed a new global fit to the semileptonic moments. The higher power corrections have a minor effect on $|V_{cb}|$ and on the expectation values of the lower dimensional operators, and we observe a good convergence of the heavy mass expansion. There is a -0.25% reduction in $|V_{cb}|$

$$10^3\,|V_{cb}| = 42.11(53)(50)(07)(10) = 42.11(74),$$

where the four errors are, respectively, the parametric error from the fit, the theoretical error on the semileptonic width, and those due to the τ_B uncertainty, and to $\delta^{\xi}, \delta^{\epsilon}$. The bottom mass determination from the fit is $m_b^{kin} = 4.546(21)$ GeV. A slightly more precise alternative fit makes use of $\overline{m}_c$ at a lower scale, 2 GeV, and of the PDG average for m_b, leading to

$$10^3\,|V_{cb}| = 42.00(50)(39)(07)(10) = 42.00(64).$$

After the implementation of various higher order effects the inclusive determination of V_{cb} appears robust. Further improvements may come from the calculation of $\mathcal{O}(\alpha_s/m_b^3)$ and $\mathcal{O}(\alpha_s^3)$ effects, from lattice QCD determination of some of the non-perturbative parameters, and from new [35] and more precise measurements at Belle-II.

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Appendix A

The $\mathcal{O}(1/m_b^4)$ corrections to the h_i for $E_{cut} = 1$ GeV and $m_{c,b}$ from Table 2 are (in units GeV^{2,4,6})

$$\begin{aligned}
\delta h_1 &= 0.01\overline{m}_1 + 0.28\overline{m}_2 + 0.54\overline{m}_3 - 0.40\overline{m}_4 - 0.04\overline{m}_5 \\
&\quad - 0.21\overline{m}_6 - 0.01\overline{m}_7 - 0.08\overline{m}_8 + 0.00\overline{m}_9 \\
\delta h_2 &= 0.6\overline{m}_1 - 3.3\overline{m}_2 - 2.0\overline{m}_3 - 0.0\overline{m}_4 + 0.2\overline{m}_5 \\
&\quad + 0.9\overline{m}_6 + 0.8\overline{m}_7 + 1.0\overline{m}_8 - 0.2\overline{m}_9 \qquad (11) \\
\delta h_3 &= -9.5\overline{m}_1 + 27.2\overline{m}_2 - 0.8\overline{m}_3 + 3.6\overline{m}_4 + 0\overline{m}_5 \\
&\quad + 1.5\overline{m}_6 - 3.3\overline{m}_7 - 4.2\overline{m}_8 + 0.6\overline{m}_9.
\end{aligned}$$

The total semileptonic width can be written as

$$\begin{aligned}
\Gamma = \Gamma_0 \Bigg[& z(r)\left(1 - \frac{\mu_\pi^2 - \mu_G^2}{2m_b^2} - \frac{\rho_D^3 + \rho_{LS}^3}{2m_b^3} + \frac{\frac{1}{8}\overline{m}_1 + \frac{1}{3}\overline{m}_4 + \frac{1}{4}\overline{m}_8}{m_b^4}\right) \\
& - 2(1-r)^4 \left(\frac{\mu_G^2}{m_b^2} - \frac{\rho_D^3 + \rho_{LS}^3}{m_b^3} + \frac{16}{9}\frac{\overline{m}_9}{m_b^4}\right) \\
& + d(r)\left(\frac{\rho_D^3}{m_b^3} - \frac{2\overline{m}_4 + \frac{2}{3}\overline{m}_9}{m_b^4}\right) + \sum_{i=2,3,5,6} h_i(r)\frac{\overline{m}_i}{m_b^4} \qquad (12) \\
& + \sum_{i=1}^{18} k_i(r)\frac{r_i}{m_b^5} + \sum_{i=1}^{2} a_i(r)\left(\frac{\alpha_s}{\pi}\right)^i + \sum_{i=\pi,G} f_i(r)\frac{\alpha_s}{\pi}\frac{\mu_i^2}{m_b^2} + \ldots \Bigg],
\end{aligned}$$

where $\Gamma_0 = A_{ew} G_F^2 (m_b^{kin})^5 |V_{cb}|^2/192\pi^3$, $A_{ew} = 1.014$, $r = (\overline{m}_c(3\ \text{GeV})/m_b^{kin})^2$, $z(r) = 1 - 8r + 8r^3 - r^4 - 12r^2 \ln r$, $d(r) = 2(17 - 16r - 12r^2 + 16r^3 - 5r^4 + 12\ln r)/3$, and h_i, k_i, a_i and f_i are listed in Table 3 for a specific r value. Using the values of the parameters given in Table 2 one gets

$$\begin{aligned}
\frac{\Gamma}{z(r)\Gamma_0} &= 1 - 0.116_{\alpha_s} - 0.030_{\alpha_s^2} - 0.042_{1/m^2} - 0.002_{\alpha_s/m^2} \\
&\quad - 0.030_{1/m^3} + 0.005_{1/m^4} + 0.005_{1/m^5}. \qquad (13)
\end{aligned}$$

Table 3
Higher-order contributions to the semileptonic width evaluated at $r = 0.0472$.

h_2	−2.65	k_6	75.20	k_{15}	−34.41
h_3	−11.20	k_7	−20.17	k_{16}	−17.33
h_5	3.12	k_8	4.26	k_{17}	−0.23
h_6	−2.94	k_9	19.91	k_{18}	18.00
k_1	−1.25	k_{10}	59.21	a_1	−1.17
k_2	−91.12	k_{11}	−23.57	a_2	−4.26
k_3	120.83	k_{12}	−26.13	f_π	0.95
k_4	−131.94	k_{13}	26.56	f_G	−2.10
k_5	20.88	k_{14}	5.25		

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