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Determination of distances and sizes of visible objects using a plane transparent glass plate

Arvind Kumar Pandey [a,b], Umesh Yadava [b,*]

[a] Lauhar, Bharthaw, Ballia 221717, India
[b] Department of Physics, DDU Gorakhpur University, Gorakhpur 273009, India

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ABSTRACT

A novel approach for the determination of distances and sizes of the different visible objects has been proposed. Accordingly, an experimental set up has been designed. This method is cheap, handy and easy to use. Observations are conducted with the help of a plane transparent glass plate and a travelling microscope. Distances and sizes calculated with this method are found to be in good agreement with those of the reported values.

Introduction

Determination of distances of nearby objects or distant objects and height of buildings, mountains *etc.* have been the subject of immense interest among scientists [1,2]. The most commonly used methods are parallax method, sextant method, radar method *etc.* [3,4] which are very complex, time consuming and costly. Now a days, the use of inclinometer application on smartphones is very popular for the determination of height [5,6]. We know that the apparent size of an object on a glass plate depends on the distance of the object from the observer. The apparent size on the glass plate becomes shorter with the movement of the object farther from the observer. We have used this principle to find out new simple formulae to determine the distances of the objects from the observer and also sizes of the objects. Experiments have also been performed to verify these formulae, which is discussed in this communication.

Principles and methods

We have used a plane transparent glass plate to view the object. The apparent size of the object on the glass plate has been sketched, the dimensions of which are measured with the help of a travelling microscope. We have performed a number of experiments whose results are listed in Tables 1–11. From tables, we have derived several relations among different quantities. The apparent length, perimeter and the area on the glass plate are denoted by L_g, P_g and A_g respectively, whereas the actual length, perimeter and front area of the object are denoted by L_o, P_o and A_o respectively. The variable quantities are (i) distance of object from the observer (d), (ii) distance of glass plate from the observer (x), (iii) angle between the plane of object and the line joining the middle point of the object from observer (φ) and (iv) the angle between the normal of the front area of the surface of object and the line joining the middle point of object from the observer (θ) (Fig. 1).

Distance determination when size of the object is known

Based on the observations as shown in Tables 1–11, we have extracted the following rules:

Rule 1. Observations from Tables 1–4 demonstrate that

i. $L_g \propto L_o$
ii. $L_g \propto x$
iii. $L_g \propto \text{Sin}\ \varphi$
iv. $L_g \propto 1/d$

Thus we can conclude that

$$L_g = c\frac{L_o x \sin\varphi}{d} = \frac{L_o x \sin\varphi}{d} \quad (1)$$

where c is the proportionality constant, found experimentally equal to 1.

* Corresponding author.
E-mail address: u_yadava@yahoo.com (U. Yadava).

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Table 1
Variation of L_g with L_o keeping other variants constant (x = 10.0 cm, d = 100.0 cm and φ = 90.0°).

L_o (cm)	L_g (cm)
10.0	1.040
12.0	1.209
14.0	1.405
16.0	1.606
18.0	1.807
20.0	2.008
22.0	2.209
24.0	2.399
26.0	2.613
28.0	2.825
30.0	3.008

Table 2
Variation of L_g with x keeping other variants constant (L_o = 20.0 cm, d = 100.0 cm, φ = 90.0°).

x (cm)	L_g (cm)
10.0	2.029
12.0	2.403
14.0	2.804
16.0	3.205
18.0	3.603
20.0	4.045
22.0	4.419
24.0	4.801
26.0	5.200
28.0	5.608
30.0	6.016

Table 3
Variation of L_g with $\sin\varphi$ keeping other variants constant (L_o = 20.0 cm, d = 100.0 cm, x = 10.0 cm).

Φ (°)	Sin φ	L_g (cm)
0.0	0.000	0.011
15.0	0.259	0.520
30.0	0.500	1.004
45.0	0.707	1.420
60.0	0.866	1.733
75.0	0.966	1.909
90.0	1.000	2.007

Table 4
Variation of L_g with d keeping other variants constant (L_o = 20.0 cm, φ = 90.0°, x = 10.0 cm).

d (cm)	$1/d$ (cm^{-1})	L_g (cm)
10.0	0.1000	20.002
20.0	0.0500	10.034
30.0	0.0330	6.661
40.0	0.0250	5.022
50.0	0.0200	4.007
60.0	0.0167	3.333
70.0	0.0143	2.852
80.0	0.0125	2.505
90.0	0.0111	2.223
100.0	0.0100	2.006

Table 5
Variation of P_g with P_o keeping other variants constant (d = 100.0 cm, θ = 0.0°, φ = 90.0°, x = 10.0 cm).

P_o	P_g
40.0	4.100
48.0	4.820
56.0	5.603
64.0	6.404
72.0	7.208
80.0	8.023
88.0	8.801
96.0	9.621
104.0	10.404
112.0	11.205
120.0	12.014

Table 6
Variation of P_g with x keeping other variants constant (P_o = 80.0 cm, d = 100.0 cm, θ = 0.0°, φ = 90.0°).

x (cm)	P_g (cm)
10.0	8.004
12.0	9.601
14.0	11.202
16.0	12.834
18.0	14.404
20.0	16.005
22.0	17.611
24.0	19.202
26.0	20.803
28.0	22.416
30.0	24.022

Table 7
Variation of P_g with d keeping other variants constant (P_o = 80.0 cm, x = 10.0 cm, θ = 0.0°, φ = 90.0°).

d (cm)	$1/d$ (cm^{-1})	P_g (cm)
10.0	0.1000	80.020
20.0	0.0500	40.024
30.0	0.0330	26.663
40.0	0.0250	20.011
50.0	0.0200	16.021
60.0	0.0167	13.333
70.0	0.0143	11.424
80.0	0.0125	10.027
90.0	0.0111	8.881
100.0	0.0100	8.010

Table 8
Variation of A_g with A_o keeping other variants constant (x = 10.0 cm, d = 100.0 cm, θ = 0.0°).

A_o (cm^2)	A_g (cm^2)
100.0	1.011
144.0	1.442
196.0	1.963
256.0	2.562
324.0	3.245
400.0	4.004
484.0	4.843
576.0	5.765
676.0	6.768
784.0	7.847
900.0	9.005

Table 9
Variation of A_g with x keeping other variants constant (A_o = 400.00 cm^2, d = 100.0 cm, θ = 0.0°).

x (cm)	X^2 (cm^2)	A_g (cm^2)
10.0	100.00	4.0281
12.0	144.00	5.7636
14.0	196.00	7.8449
16.0	256.00	10.2455
18.0	324.00	12.9646
20.0	400.00	16.0072
22.0	484.00	19.3683
24.0	576.00	23.0464
26.0	676.00	27.0436
28.0	784.00	31.361
30.0	900.00	36.002

Table 10
Variation of A_g with $\cos\theta$ keeping other variants constant (A_o = 400.00 cm^2, d = 100.0 cm, x = 10.0 cm).

θ	$\cos\theta$	A_g (cm^2)
0.0	1.000	4.1140
15.0	0.966	3.8641
30.0	0.866	3.4624
45.0	0.707	2.8278
60.0	0.500	2.0231
75.0	0.259	1.0364
90.0	0.000	0.0149

Table 11
Variation of A_g with d keeping other variants constant (A_o = 400.00 cm^2, x = 10.0 cm, θ = 0.0°).

d (cm)	d^2 (cm^2)	$1/d^2$ (10^{-4} cm^{-2})	A_g (cm^2)
20.0	400.00	25.00	100.0255
40.0	1600.00	6.25	25.0048
60.0	3600.00	2.78	11.1229
80.0	6400.00	1.563	6.2524
100.0	10000.00	1.000	4.0245
120.0	14400.00	0.694	2.7766
140.0	19600.00	0.510	2.0407
160.0	25600.00	0.391	1.5643
180.0	32400.00	0.309	1.2362
200.0	40000.00	0.250	1.0098

Rule 2. From Tables 5–7 we observe that

i. $P_g \propto P_o$
ii. $P_g \propto x$
iii. $P_g \propto d^{-1}$

Thus, we have

$$P_g = c\frac{P_o x}{d} = \frac{P_o x}{d} \tag{2}$$

Here, c the proportionality constant is also determined equal to 1.

Rule 3. From Tables 8–11, we observe that

i. $A_g \propto A_o$
ii. $A_g \propto x^2$
iii. $A_g \propto \cos\theta$
iv. $A_g \propto d^{-2}$

Thus,

$$A_g = c\frac{A_o x^2 \cos\theta}{d^2} = \frac{A_o x^2 \cos\theta}{d^2} \tag{3}$$

Here, also $c = 1$ from experiments.

Thus Eqs. (1)–(3) can be used to calculate the distance of the object from the observer when length of the object, perimeter of the object or area of the object is known.

If the length of the object (L_o) is known then Eq. (1) is utilised. The glass plate is placed at a certain distance from the eye (say at x = 10.0 cm) (For simplicity we have taken as φ = 90.0°). The apparent size of the object is sketched on the glass plate (L_g) which is measured with the help of a travelling microscope. Thus knowing the quantities like L_o, L_g, x and $\sin\varphi$ one can calculate the distance of the object from the observer (d). When the perimeter of the object is known, Eq. (2) is used to determine the distance of the object from the observer, whereas Eq. (3) is utilised if the front area of the object is known.

Above formulae are also suited for the determination of larger distances as the distance of sun or moon from the earth. From this method, the apparent diameter of the sun on the glass plate has been observed equal to 0.232 cm (during sunset) when the glass plate is placed at a distance of 25.0 cm from the eye of the observer. Taking the diameter of the sun 1.39×10^6 km we found, using Eq. (1), the distance of the sun from the earth is equal to 1.500×10^8 km which is in quite agreement with the reported values [7,8].

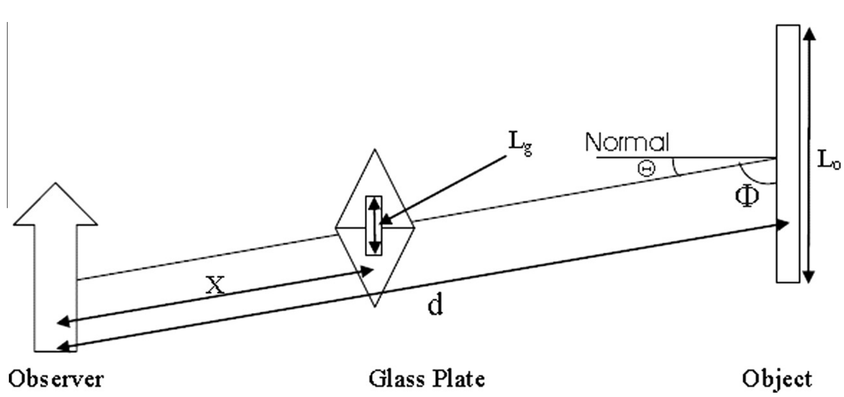

Fig. 1. Schematic diagram and variables of observation when size of object is known.

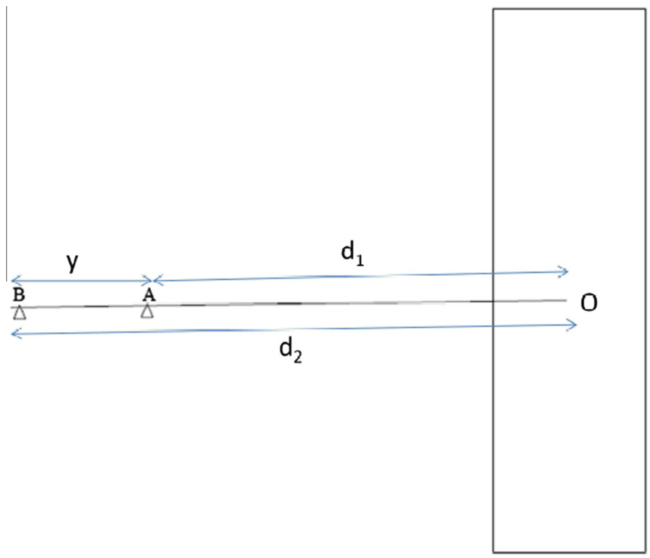

Fig. 2. Schematic diagram and variables of observation when size of object is unknown.

Distance determination when size of object is unknown

To determine the distance, when the size of the object is unknown, the schematic diagram is shown in Fig. 2. O is the middle point on the visible object. Suppose L_o, P_o and A_o represent the real length, perimeter and area of the object respectively. Observations are taken from two points A and B at distances d_1 and d_2 from the object, respectively. At first point A, L_{g1}, P_{g1} and A_{g1} are the apparent length, perimeter and area on the glass plate respectively while at the second point of observation B, L_{g2}, P_{g2} and A_{g2} are the apparent length, perimeter and area on the plane transparent glass plate respectively. Let y be the distance between the two points A and B, x the distance between eye and glass plate.

Determination of length and distance

Using Eq. (1), distances d_1 and d_2, at positions A and B, from the object can be determined whose difference gives the value of y,

$$y = d_2 - d_1 = L_o.x\frac{\sin\varphi_2 L_{g1} - \sin\varphi_1 L_{g2}}{L_{g2}L_{g1}}$$

If d_1 and d_2 are large distances in comparison to L_o then $\varphi_1 = \varphi_2 = 90°$. Therefore from (1), we have

$$L_o = \frac{yL_{g2}L_{g1}}{x(L_{g1} - L_{g2})} \tag{4}$$

Using Eq. (4), the values of d_1 and d_2 can be found out from Eq. (1) which is given by

$$d_1 = \frac{yL_{g2}}{L_{g1} - L_{g2}} \text{ and } d_2 = \frac{yL_{g1}}{L_{g1} - L_{g2}} \tag{5}$$

Determination of perimeter and distance

In a similar way, the perimeter of the object and distances d_1 and d_2 can be determined by taking observations at points A and B respectively. These formulae can be given by

$$P_o = \frac{yP_{g2}P_{g1}}{x(P_{g1} - P_{g2})} \tag{6}$$

$$d_1 = \frac{yP_{g2}}{P_{g1} - P_{g2}} \text{ and } d_2 = \frac{yP_{g1}}{P_{g1} - P_{g2}} \tag{7}$$

Determination of area and distance

Area and distance of the object can also be determined in a similar way by taking observations at positions A and B. Formulae can be given by

$$A_o = \frac{y^2 A_{g2}A_{g1}}{x^2\cos\theta\left(\sqrt{A_{g1}} - \sqrt{A_{g2}}\right)^2} \tag{8}$$

$$d_1 = \frac{y\sqrt{A_{g2}}}{\sqrt{A_{g1}} - \sqrt{A_{g2}}} \text{ and } d_2 = \frac{y\sqrt{A_{g1}}}{\sqrt{A_{g1}} - \sqrt{A_{g2}}} \tag{9}$$

Corollaries

Determination of the perimeter of the object

Having $\theta = 0°$ and $\varphi = 90°$, we can find using Eq. (1) and (2)

$$P_o = \frac{L_o P_g}{L_g} \tag{10}$$

Here L_o can be taken as an arbitrary length at the middle point on the object and L_g being the corresponding length of image on the glass plate. Thus with the help of Eq. (10), by measuring L_o, L_g and P_g, the perimeter of the object could be determined easily.

Determination of the front area of the object – I

Using Eq. (1) and (3) we have

$$A_o = \frac{L_o^2 A_g \sin^2\varphi}{L_g^2\cos\vartheta} \tag{11}$$

Thus front area of the object could also be determined with the help of Eq. (11).

Determination of the front area of the object – II

Using Eq. (2) and (3) having $\theta = 0.0°$ we find,

$$A_o = \frac{P_o^2 A_g}{P_g^2} \tag{12}$$

Thus with the help of Eq. (11), (12) the area of the object could be determined if the perimeter of the object is known.

Conclusion

A more convenient method for the determination of distances and sizes of visible objects has been developed and accordingly experiments have been designed. More accurate we measure the dimensions of the image on the glass plate, more accurate will be the result. Using these formulae, the height of Building, area of fields *etc.* could also be determined easily.

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