



Full one-loop corrections to SUSY Higgs boson decays into charginos

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Received 24 May 2004; accepted 6 July 2004

Available online 22 July 2004

Editor: G.F. Giudice

Abstract

We present the decay widths of the heavier Higgs bosons (H^0, A^0) into chargino pairs in the minimal supersymmetric standard model, including full one-loop corrections. All parameters for charginos are renormalized in the on-shell scheme. The importance of the corrections to the chargino mass matrix and mixing matrices is pointed out. The full corrections are typically of the order of 10%.

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1. Introduction

The Minimal Supersymmetric Standard Model (MSSM) [1] is considered the most attractive extension of the Standard Model. This model contains two Higgs scalar doublets, implying the existence of five physical Higgs bosons [2]; two CP-even neutral bosons (h^0, H^0), one CP-odd boson A^0 , and two charged bosons H^\pm . For the verification of the MSSM, detection and precision studies of these Higgs bosons are necessary.

The decay modes of the heavier Higgs bosons (H^0, A^0) are in general complicated [3,4], especially

if $\tan\beta$, the ratio of the vacuum expectation values of the two Higgs scalars, is not much larger than one. For example, they may decay into pairs of the SUSY particles [3] such as squarks, sleptons, charginos, and neutralinos. In this Letter, we focus our attention on the decays into charginos,

$$(H^0, A^0) \rightarrow \tilde{\chi}_i^+ + \tilde{\chi}_j^-, \quad (1)$$

with $i, j = (1, 2)$. Existing numerical analyses [3–5] at tree-level have shown that the decays (1) have in general non-negligible branching ratios. These decays are also interesting because they are generated by gaugino–higgsino–Higgs boson couplings [2] at tree-level and very sensitive to the components of charginos. Detailed studies of these decays would therefore provide useful information about the

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chargino sector, complementary to the pair production processes $e^+e^- \rightarrow \tilde{\chi}_i^+ \tilde{\chi}_j^-$ [6].

Since the masses and mixing matrices of the charginos are expected to be precisely determined at future colliders [7–9], it is interesting to study the radiative corrections to the decays (1). The one-loop corrections involving quarks and squarks in the third generation were calculated in Ref. [10]. However, for the masses and mixings of the charginos, the corrections from quark–squark loops [11] and those from the other loops [12,13] are shown to be numerically comparable. It is therefore necessary to include the other loop corrections to the decays (1).

In this Letter, we study the widths of the decays (1) including full one-loop corrections and present numerical results for the $i = j = 1$ case. We adopt the on-shell renormalization scheme for the chargino sector, following Refs. [11,13]. We also show numerical results for the one-loop corrected widths of the crossed-channel decay

$$\tilde{\chi}_2^\pm \rightarrow \tilde{\chi}_1^\pm + h^0, \quad (2)$$

which has been studied at tree-level [14].

2. Tree-level widths

The tree-level widths for the decay $H_k^0 \rightarrow \tilde{\chi}_i^+ \tilde{\chi}_j^-$, with $H_{\{1,2,3\}}^0 \equiv \{h^0, H^0, A^0\}$ and $i, j = (1, 2)$, are given by [3]

$$\begin{aligned} \Gamma^{\text{tree}}(H_k^0 \rightarrow \tilde{\chi}_i^+ \tilde{\chi}_j^-) &= \frac{g^2}{16\pi m_{H_k^0}^3} \kappa(m_{H_k^0}^2, m_i^2, m_j^2) \\ &\times [(m_{H_k^0}^2 - m_i^2 - m_j^2)(F_{ijk}^2 + F_{jik}^2) \\ &\quad - 4\eta_k m_i m_j F_{ijk} F_{jik}], \end{aligned} \quad (3)$$

with $\kappa(x, y, z) \equiv ((x - y - z)^2 - 4yz)^{1/2}$. η_k represents the CP eigenvalue of H_k^0 ; $\eta_{1,2} = 1$ for the (h^0, H^0) decays and $\eta_3 = -1$ for the A^0 decays. We use the abbreviation $m_i \equiv m_{\tilde{\chi}_i^\pm}$. In this Letter, we assume that the contributions of CP violation and generation mixings of the quarks and squarks are negligible.

The chargino-Higgs boson couplings gF_{ijk} , defined by the interaction Lagrangian

$$\begin{aligned} \mathcal{L}_{\text{int}} &= -gH_a^0 \overline{\tilde{\chi}_i^+} (F_{ija} P_R + F_{jia} P_L) \tilde{\chi}_j^+ \\ &\quad + igH_c^0 \overline{\tilde{\chi}_i^+} (F_{ijc} P_R - F_{jic} P_L) \tilde{\chi}_j^+, \end{aligned} \quad (4)$$

with $a = 1, 2, c = 3, 4$, are given by [2]

$$gF_{ijk} = \frac{g}{\sqrt{2}} (e_k V_{i1} U_{j2} - d_k V_{i2} U_{j1}). \quad (5)$$

The would-be Nambu–Goldstone boson $H_4^0 \equiv G^0$ is included here for later convenience. The mixing matrices (U, V) for the charginos are determined by diagonalizing the chargino mass matrix X as

$$\begin{aligned} X &= \begin{pmatrix} M & \sqrt{2}m_W \sin \beta \\ \sqrt{2}m_W \cos \beta & \mu \end{pmatrix} \\ &= U^\dagger \begin{pmatrix} m_{\tilde{\chi}_1^+} & 0 \\ 0 & m_{\tilde{\chi}_2^+} \end{pmatrix} V. \end{aligned} \quad (6)$$

Here M and μ are the mass parameters of the SU(2) gaugino and higgsino states, respectively. We choose U and V to be real. The effect of the mixings of H_k^0 is represented by e_k and d_k , which take the values

$$\begin{aligned} e_k &= (-\sin \alpha, \cos \alpha, -\sin \beta, \cos \beta)_k, \\ d_k &= (-\cos \alpha, -\sin \alpha, \cos \beta, \sin \beta)_k. \end{aligned} \quad (7)$$

We also show the widths of the decays $\tilde{\chi}_2^+ \rightarrow \chi_1^+ H_k^0$ at the tree-level [14]

$$\begin{aligned} \Gamma^{\text{tree}}(\tilde{\chi}_2^+ \rightarrow \chi_1^+ H_k^0) &= \frac{g^2}{32\pi m_{\tilde{\chi}_2^+}^3} \kappa(m_2^2, m_1^2, m_{H_k^0}^2) \\ &\times [(m_2^2 + m_1^2 - m_{H_k^0}^2)(F_{12k}^2 + F_{21k}^2) \\ &\quad + 4\eta_k m_1 m_2 F_{12k} F_{21k}]. \end{aligned} \quad (8)$$

3. One-loop corrections

We calculate the full one-loop corrections to the decay widths (3).

The one-loop correction to the coupling F_{ijk} is expressed as

$$\begin{aligned} F_{ijk}^{\text{corr.}} &= F_{ijk} + \Delta F_{ijk} \\ &= F_{ijk} + \delta F_{ijk}^{(v)} + \delta F_{ijk}^{(w)} + \delta F_{ijk}^{(c)}, \end{aligned} \quad (9)$$

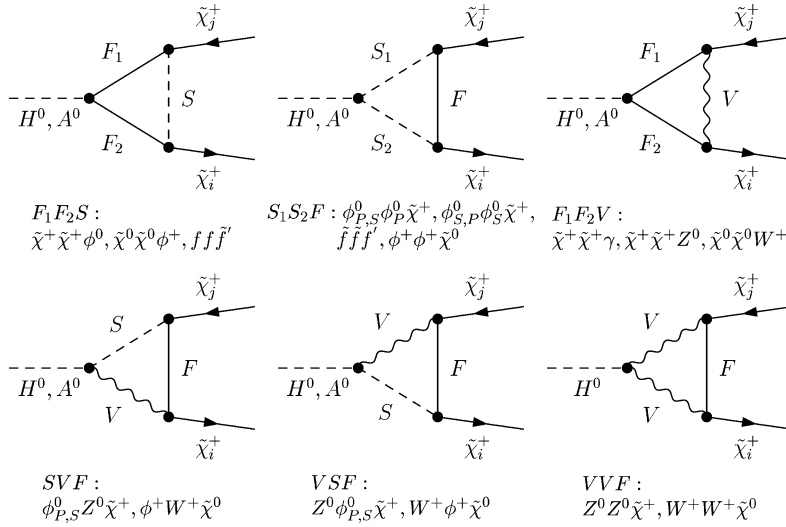


Fig. 1. One-loop vertex corrections to the $H_k^0 \rightarrow \tilde{\chi}_i^+ \tilde{\chi}_j^-$ decays, $\phi^0 = \{\phi_S^0, \phi_P^0\} = \{h^0, H^0, A^0, G^0\}$, $\phi^+ = \{H^+, G^+\}$.

where $\delta F_{ijk}^{(v)}$, $\delta F_{ijk}^{(w)}$, and $\delta F_{ijk}^{(c)}$ are the vertex correction, the wave function correction, and the counter terms for the parameters in Eq. (5), respectively.

The vertex correction $\delta F_{ijk}^{(v)}$ comes from the diagrams listed in Fig. 1. In this Letter we do not show the analytic forms of these diagrams.

The wave-function correction $\delta F_{ijk}^{(w)}$ is expressed as

$$\delta F_{ijk}^{(w)} = \frac{1}{2} [\delta Z_{lk}^{H^0} F_{ijl} + \delta Z_{i'i}^{+L} F_{i'jk} + \delta Z_{j'j}^{+R} F_{ij'k}], \quad (10)$$

with the implicit summations over $l = 1, 2$ for $k = 1$ or 2 , $l = 3, 4$ for $k = 3$, and $i', j' = (1, 2)$. The correction terms $\delta Z^{+(L,R)}$ for the chargino wave-functions are given by

$$\begin{aligned} \delta Z_{ii}^{+L} = & -\text{Re} \{ \Pi_{ii}^{\tilde{\chi}^+ L} (m_i^2) \\ & + m_i [m_i \dot{\Pi}_{ii}^{\tilde{\chi}^+ L} (m_i^2) + m_i \dot{\Pi}_{ii}^{\tilde{\chi}^+ R} (m_i^2) \\ & + 2 \dot{\Pi}_{ii}^{\tilde{\chi}^+ S,L} (m_i^2)] \}, \quad (11) \end{aligned}$$

$$\begin{aligned} \delta Z_{pi}^{+L} = & \frac{2}{m_p^2 - m_i^2} \\ & \times \text{Re} \{ m_i^2 \Pi_{pi}^{\tilde{\chi}^+ L} (m_i^2) + m_i m_p \Pi_{pi}^{\tilde{\chi}^+ R} (m_i^2) \\ & + m_p \Pi_{pi}^{\tilde{\chi}^+ S,L} (m_i^2) + m_i \Pi_{pi}^{\tilde{\chi}^+ S,R} (m_i^2) \}, \quad (12) \end{aligned}$$

where $p \neq i$ and

$$\begin{aligned} \Pi_{ij}^{\tilde{\chi}}(p) = & \Pi_{ij}^{\tilde{\chi}^+ L} (p^2) \not{p} P_L + \Pi_{ij}^{\tilde{\chi}^+ R} (p^2) \not{p} P_R \\ & + \Pi_{ij}^{\tilde{\chi}^+ S,L} (p^2) P_L + \Pi_{ij}^{\tilde{\chi}^+ S,R} (p^2) P_R, \quad (13) \end{aligned}$$

are the self-energies of the charginos. δZ^{+R} are obtained from Eqs. (11), (12) by the exchange $L \leftrightarrow R$. The CP symmetry relation $\text{Re} \Pi_{ii}^{\tilde{\chi}^+ S,L} = \text{Re} \Pi_{ii}^{\tilde{\chi}^+ S,R}$ is used in Eq. (11). The corrections δZ^{H^0} for the Higgs bosons are

$$\delta Z_{kk}^{H^0} = -\text{Re} \dot{\Pi}_{kk}^{H^0} (m_{H_k^0}^2), \quad k = 1, 2, 3, \quad (14)$$

$$\begin{aligned} \delta Z_{ab}^{H^0} = & \frac{2}{m_{H_a^0}^2 - m_{H_b^0}^2} \text{Re} \Pi_{ab}^{H^0} (m_{H_b^0}^2), \\ a, b = & (1, 2), \quad a \neq b, \quad (15) \end{aligned}$$

$$\delta Z_{43}^{H^0} = -\frac{2}{m_{A^0}^2} \text{Re} \Pi_{43}^{H^0} (m_{A^0}^2). \quad (16)$$

The Higgs boson self-energies $\Pi^{H^0}(k^2)$ in Eqs. (14)–(16) include momentum-independent contributions from the tadpole shifts [15,16] and leading higher-order corrections. The latter contribution is relevant for the corrections to $(m_{h^0}, m_{H^0}, \alpha)$. For the A^0 decays, Eq. (16) already includes the contribution from the A^0 – Z^0 mixing in addition to the A^0 – G^0 mixing, using the Slavnov–Taylor identity, $\Pi_{43}^{H^0}(m_{A^0}^2) =$

$i \frac{m_{A^0}^2}{m_{Z^0}^2} \Pi_{AZ}(m_{A^0}^2)$. The explicit forms of the self energies $\Pi^{\tilde{\chi}}(p^2)$, $\Pi_{ab}^{H^0}(p^2)$, and $\Pi_{AZ}(m_{A^0}^2)$ are shown, for example, in Refs. [17,18].

To obtain ultraviolet finite corrections, we further need the counter term contribution $F_{ijk}^{(c)}$ from the renormalization of the parameters in the tree-level couplings Eq. (5). The chargino mixing matrices (U, V) are renormalized in the on-shell scheme, as described in Refs. [11,13]. In this scheme, extending Ref. [19] for quark and lepton mixings, the counter terms for (U, V) are determined such as to cancel the anti-Hermitian parts of the chargino wavefunction corrections Eq. (12). As a result, after including $(\delta V, \delta U)$ into Eq. (12), $\delta Z_{ii}^{+L,R}$ are modified as $(\delta Z_{ii}^{+L,R} + \delta Z_{ii'}^{+L,R})/2$. The counter term of β for A^0 decays is fixed by the condition [15,16] that the renormalized A^0 - Z^0 mixing self energy $\Pi_{A^0 Z^0}(p^2)$ vanishes at $p^2 = m_{A^0}^2$. Inclusion of this counter term $\delta\beta$ cancels the half of $\delta Z_{43}^{H^0}$ in Eq. (16). As usual, we use the pole mass m_{A^0} and on-shell $\tan\beta$ as inputs for the Higgs boson sector.

Since the zero-momentum contribution $\Pi_{kl}^{H^0}(0)$ to the masses and mixing angle of (h^0, H^0) are often very large, we calculate (m_{h^0}, m_{H^0}) and the effective mixing angle α_{eff} , which is defined to cancel the zero-momentum part of $\Pi_{ab}^{H^0}(p^2)$ in Eq. (15), by Feyn-Higgs [20], which includes the leading higher-order corrections, and use these values both for the tree-level and corrected widths. After the inclusion of the corresponding counterterm $\delta\alpha$, Eq. (15) is modified as

$$\delta Z_{ab}^{H^0} \rightarrow \frac{2}{m_{H_a^0}^2 - m_{H_b^0}^2} \text{Re}[\Pi_{ab}^{H^0}(m_{H_b^0}^2) - \Pi_{ab}^{H^0}(0)],$$

$$a, b = (1, 2), \quad a \neq b \quad (17)$$

with the $\overline{\text{DR}}$ renormalization scale $Q = m_Z$ for $\Pi_{ab}^{H^0}(p^2)$.

Our calculation is performed in the $\xi = 1$ gauge. Although the on-shell mixing matrices generally depend on the gauge parameter [21,22], our (U, V) may be understood as the ones improved by the pinch technique [23,24]. We ignore here very small differences of the on-shell β between the $\xi = 1$ results and improved ones by the pinch technique (see Refs. [24,25] for the case of CP-even Higgs bosons).

For the renormalization of the SU(2) gauge coupling g in Eq. (5), two schemes are used. In both the W - and Z -pole masses m_W and m_Z are input parameters. The Weinberg angle is defined by $\cos\theta_W = m_W/m_Z$ [26], and therefore

$$\frac{\delta \sin\theta_W}{\sin\theta_W} = \frac{\cos^2\theta_W}{\sin^2\theta_W} \left(\frac{\delta m_Z}{m_Z} - \frac{\delta m_W}{m_W} \right). \quad (18)$$

In the $\alpha(m_Z)$ scheme we use as input the $\overline{\text{MS}}$ running electromagnetic coupling $\alpha(m_Z)$ ($= e^2(m_Z)/(4\pi)$). We have

$$g = \frac{e(m_Z)}{\sin\theta_W} \quad \text{and} \quad \frac{\delta g}{g} = \frac{\delta e}{e} - \frac{\delta \sin\theta_W}{\sin\theta_W}, \quad (19)$$

with δe given, e.g., in [27,28], δm_Z and δm_W in [18].

In the other scheme, called here the G_F scheme, the Fermi constant G_F for the muon decay is input parameter,

$$g = \left[\frac{8G_F m_W^2}{\sqrt{2}} \right]^{1/2} \quad \text{and}$$

$$\frac{\delta g}{g} = \delta Z_e - \frac{1}{2} \Delta r - \frac{\delta \sin\theta_W}{\sin\theta_W} \quad (20)$$

δZ_e is the renormalization constant for the electric charge in the Thomson limit [29]. The term Δr includes the full one-loop MSSM correction [30] and the leading two-loop QCD corrections [31].

The corrected widths are

$$\Gamma^{\text{corr}} = \Gamma^{\text{tree}} + \frac{g^2}{16\pi m_{H_k^0}^3} \kappa(m_{H_k^0}^2, m_i^2, m_j^2)$$

$$\times \left[(m_{H_k^0}^2 - m_i^2 - m_j^2) \right.$$

$$\times 2 \text{Re}(F_{ijk} \Delta F_{ijk} + F_{jik} \Delta F_{jik})$$

$$- 4\eta_k m_i m_j$$

$$\times \text{Re}(F_{ijk} \Delta F_{jik} + F_{jik} \Delta F_{ijk}) \left. \right]$$

$$+ \Gamma(H_k^0 \rightarrow \tilde{\chi}_i^+ \tilde{\chi}_j^- \gamma). \quad (21)$$

The process $H_k^0 \rightarrow \tilde{\chi}_i^+ \tilde{\chi}_j^- \gamma$ with real photon emission is included to cancel the infrared divergence by virtual photon loops.

One has to be careful in using the on-shell mixing matrices (U, V) and masses m_i ($i = 1, 2$) in the numerical analysis. When the gauge and Higgs boson sectors are fixed, the chargino sector is fixed by two independent parameters. Here we follow the method proposed in Refs. [11,13]: we fix the chargino sector

by taking $M \equiv X_{11}$ and $\mu \equiv X_{22}$, where the on-shell mass matrix X is defined to give the on-shell masses m_i and on-shell mixing matrices (U, V) by diagonalization. Note that, for given values of the on-shell M and μ , the one-loop corrected on-shell masses m_i and mixing matrices (U, V) are shifted [11,13] from the values obtained by the tree-level mass matrix X^{tree} composed by the input parameters, the on-shell $M, \mu, \tan\beta$, and the pole mass m_W . This is due to the shift of the off-diagonal elements of X from their tree-level values and related to the deviation of the gaugino couplings from the corresponding gauge couplings by SUSY-breaking loop corrections [32]. These shifts of m_i and (U, V), in addition to the “conventional” corrections shown in Eq. (21), have to be taken into account for a proper treatment of the loop corrections. (A slightly different scheme for the chargino sector was proposed in Ref. [12]. Apart from the different definition of the renormalized M and μ , their method is equivalent to ours.)

The full one-loop corrections were calculated using the packages FeynArts, FormCalc, and LoopTools [33]. For the contributions of the quarks, leptons, and their superpartners, we also checked the consistency with Ref. [10], both analytically and numerically.

4. Numerical results

We present numerical results for the tree-level and one-loop widths of the decays $A^0 \rightarrow \tilde{\chi}_1^+ \tilde{\chi}_1^-$, $H^0 \rightarrow \tilde{\chi}_1^+ \tilde{\chi}_1^-$, and $\chi_2^+ \rightarrow \chi_1^+ h^0$. The SUSY parameter set SPS1a of the Snowmass Points and Slopes in Ref. [34] is chosen as reference point; for the trilinear breaking terms A_t, A_b and A_τ we use the $\overline{\text{DR}}$ running values given at the scale of the mass of the decaying particle, $A_t = -487$ GeV, $A_b = -766$ GeV, $A_\tau = -250$ GeV. All other parameters are taken on-shell, $M = 197.6$ GeV, $M' = 98$ GeV, $\mu = 353.1$ GeV, $\tan\beta = 10$, and $m_{A^0} = 393.6$ GeV. The soft breaking sfermion mass parameters, for the first and second generation are $M_{\tilde{Q}_{1,2}} = 558.9$ GeV, $M_{\tilde{U}_{1,2}} = 540.5$ GeV, $M_{\tilde{D}_{1,2}} = 538.5$ GeV, $M_{\tilde{L}_{1,2}} = 197.9$ GeV, $M_{\tilde{E}_{1,2}} = 137.8$ GeV, and for the third one, $M_{\tilde{Q}_3} = 512.2$ GeV, $M_{\tilde{U}_3} = 432.8$ GeV, $M_{\tilde{D}_3} = 536.5$ GeV, $M_{\tilde{L}_3} = 196.4$ GeV, $M_{\tilde{E}_3} = 134.8$ GeV. In all figures, these values are used, if not specified otherwise.

For the standard model parameters, we take $\alpha(m_Z) = 1/127.922$, $m_Z = 91.1876$ GeV, $m_W = 80.423$ GeV, the on-shell parameters $m_t = 174.3$ GeV, and $m_\tau = 1.777$ GeV. For the bottom mass, our input is the $\overline{\text{MS}}$ value $m_b(m_b) = 4.2$ GeV. For the values of the Yukawa couplings of the third generation quarks (h_t, h_b), we take the running ones at the scale of the decaying particle mass.

In the G_F scheme for the renormalization of g , we use $G_F = 1.16639 \times 10^{-5}$ GeV $^{-2}$ instead of $\alpha(m_Z)$.

We compare three cases: the “naive” tree-level width $\Gamma^{\text{naive tree}}$, the tree-level width already including the loop corrections to the chargino mass matrix Γ^{tree} , and the full one-loop width Γ^{corr} .

In Fig. 2 we show the tree-level and corrected widths in (a) of $A^0 \rightarrow \tilde{\chi}_1^+ \tilde{\chi}_1^-$ as functions of m_{A^0} , and in (b) of $H^0 \rightarrow \tilde{\chi}_1^+ \tilde{\chi}_1^-$ as functions of m_{H^0} . The tree-level branching ratios of these decays at $m_{A^0} = 393.6$ GeV (where $m_{H^0} = 394.1$ GeV) are, using HDECAY program [35], $\text{Br}(A^0 \rightarrow \tilde{\chi}_1^+ \tilde{\chi}_1^-) = 21\%$ and $\text{Br}(H^0 \rightarrow \tilde{\chi}_1^+ \tilde{\chi}_1^-) = 4\%$, which are not negligible. We see that the full one-loop corrections amount up to $\sim -12\%$. In Fig. 2(c) the individual contributions to Fig. 2(a) relative to the naive tree-level width are exhibited. The dash-dotted line show the (s)fermion loop contribution (loops with quarks, leptons, and their superpartners) through the correction to the chargino mass matrix, while the dotted line shows the full correction to the mass matrix. The solid (dashed) line shows the total correction $\Gamma^{\text{corr.}}/\Gamma^{\text{naive tree}} - 1$ including full ((s)fermion) one-loop contributions. This figure shows that the (s)fermion loop corrections and other corrections are of comparable order, both for the chargino mass matrix and for the conventional corrections (21).

A comparison of two renormalization schemes for fixing g , the $\alpha(m_Z)$ scheme and the G_F scheme, is shown in Fig. 3 for the decay $A^0 \rightarrow \tilde{\chi}_1^+ \tilde{\chi}_1^-$ as functions of m_{A^0} . The difference between these two schemes is below 1%, scaling with the one-loop correction part, and mainly a higher order effect.

Since the Higgs boson couplings to charginos are very sensitive to the gaugino–higgsino mixing, it is interesting to study the dependence of the decay widths on the gaugino and higgsino components of $\tilde{\chi}_1^\pm$. Fig. 4 shows the tree-level and one-loop corrected widths of $A^0 \rightarrow \tilde{\chi}_1^+ + \tilde{\chi}_1^-$ as functions of μ for fixed M . One

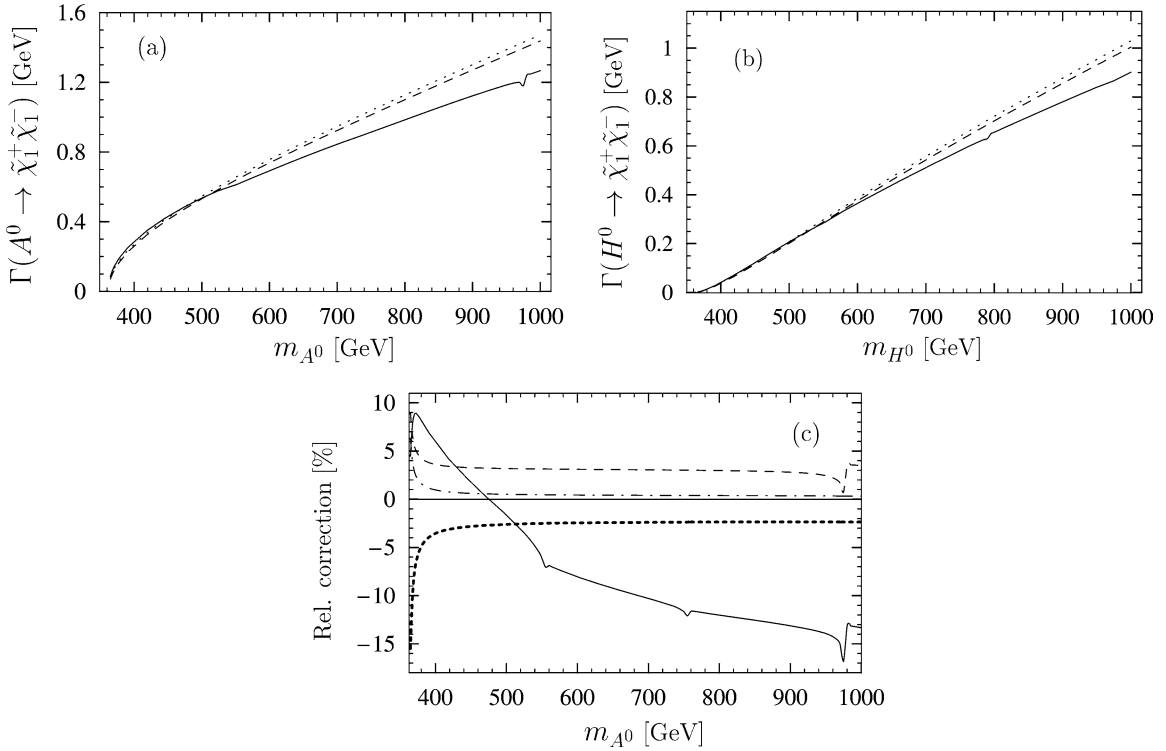


Fig. 2. Naive tree-level (dotted), tree-level (dashed) and one-loop corrected (solid) widths of the decays $A^0 \rightarrow \tilde{\chi}_1^+ + \tilde{\chi}_1^-$ as functions of m_{A^0} (a), and $H^0 \rightarrow \tilde{\chi}_1^+ + \tilde{\chi}_1^-$ as functions of m_{H^0} (b), in the $\alpha(m_Z)$ schemes for the renormalization of the SU(2) gauge coupling g . The individual loop contributions to (a) are shown in (c), for explanation see the text.

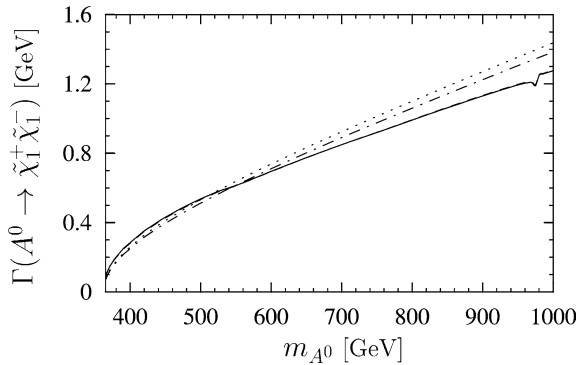


Fig. 3. Comparison of the results using the $\alpha(m_Z)$ scheme or the G_F scheme for the decay widths of $A^0 \rightarrow \tilde{\chi}_1^+ \tilde{\chi}_1^-$. The dotted and the solid (dash-dotted and dashed) lines denote the tree-level and one-loop corrected line in the $\alpha(m_Z)$ (G_F) scheme.

can see that in the region where the light chargino $\tilde{\chi}_1^+$ becomes a pure wino the width gets very small. The correction grows from $\sim -1\%$ for $\mu \sim 120$ GeV to

20% for $\mu \sim 600$ GeV. The μ dependence of the decay width $H^0 \rightarrow \tilde{\chi}_1^+ + \tilde{\chi}_1^-$ is not shown because its behavior is similar to that shown in Fig. 4.

Fig. 5 shows the tree-level and one-loop corrected widths of $A^0 \rightarrow \tilde{\chi}_1^+ + \tilde{\chi}_1^-$ as functions of M for fixed μ . In the whole range of this figure $\tilde{\chi}_1^+$ is gaugino-like. In (a), for increasing M the decay widths decreases due to phase space. The correction, see (b), gets up to 30% near the threshold. Again, the $H^0 \rightarrow \tilde{\chi}_1^+ + \tilde{\chi}_1^-$ is not shown because of a similar behavior.

Fig. 6 shows the decay widths for $A^0 \rightarrow \tilde{\chi}_1^+ + \tilde{\chi}_1^-$ in (a) and $H^0 \rightarrow \tilde{\chi}_1^+ + \tilde{\chi}_1^-$ in (b) as functions of $\tan \beta$. The correction is in the range of $\sim 10\%$ and the dependence on $\tan \beta$ is small. We examined the difference of the renormalization scheme taking the $\overline{\text{DR}}$ value for $\tan \beta$ at the scale $Q = 454.7$ GeV as input parameter instead of the on-shell $\tan \beta$. For these processes the difference is small, e.g., in the Fig. 6(a) it is about 0.5% for low and 0.2% for large $\tan \beta$, respectively.

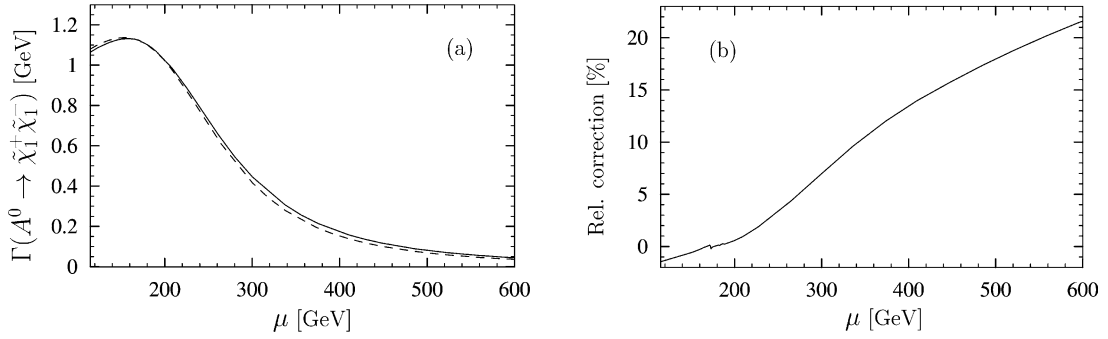


Fig. 4. Tree-level (dotted) and one-loop corrected (solid) widths of the decays $A^0 \rightarrow \tilde{\chi}_1^+ + \tilde{\chi}_1^-$ (a) and (b) the correction of this process relative to the tree-level width as a function of μ .

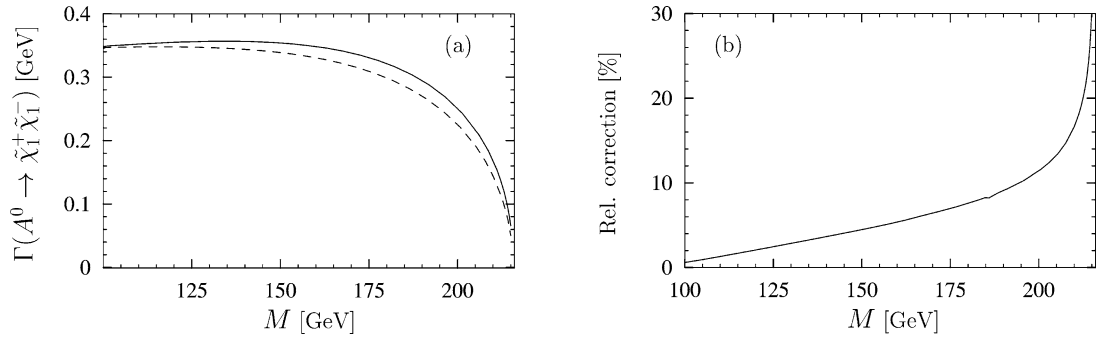


Fig. 5. Tree-level (dotted) and one-loop corrected (solid) widths of the decay $A^0 \rightarrow \tilde{\chi}_1^+ + \tilde{\chi}_1^-$ (a) and (b) the corrections of this process relative to the tree-level widths as a function of M .

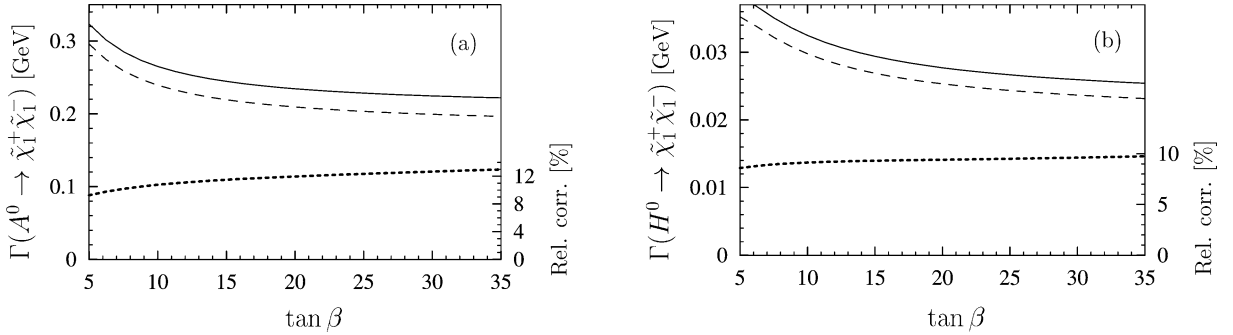


Fig. 6. Tree-level (dashed), one-loop corrected (solid) width and the correction (dotted) relative to the tree-level width for the decays $A^0 \rightarrow \tilde{\chi}_1^+ + \tilde{\chi}_1^-$ (a) and $H^0 \rightarrow \tilde{\chi}_1^+ + \tilde{\chi}_1^-$ (b) as a function of $\tan \beta$.

Fig. 7 shows the corrections to the decay widths for $A^0 \rightarrow \tilde{\chi}_1^+ + \tilde{\chi}_1^-$ in (a) and $H^0 \rightarrow \tilde{\chi}_1^+ + \tilde{\chi}_1^-$ in (b) relative to the naive tree-level width as functions of $m_{\tilde{Q}}$. The SUSY breaking mass terms for all sfermions ($M_{\tilde{Q}_i}, M_{\tilde{U}_i}, M_{\tilde{D}_i}, M_{\tilde{L}_i}, M_{\tilde{E}_i}$) ($i = 1, 2, 3$) are taken to be equal to $m_{\tilde{Q}}$, while the other parameters are un-

changed. The relative corrections $\Gamma^{\text{tree}}/\Gamma^{\text{naive tree}} - 1$ (dashed lines), stemming from the shift of the chargino mass matrix by the renormalization, are negative. The remaining conventional corrections shown in Eq. (21) (dotted lines) are positive. The total correction $\Gamma^{\text{corr.}} - \Gamma^{\text{naive tree}}$ (solid lines) is positive and in the range of

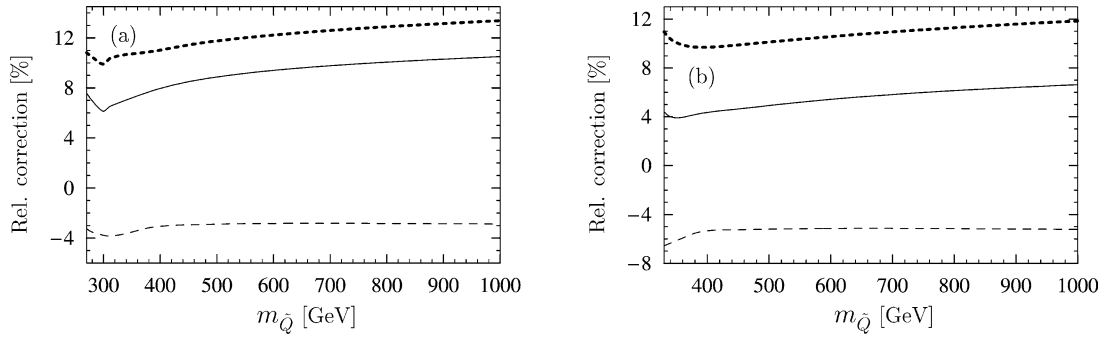


Fig. 7. Correction of the full one-loop corrected (solid), the tree-level (dashed), and the conventional one-loop corrected width (dotted) for the decays $A^0 \rightarrow \tilde{\chi}_1^+ + \tilde{\chi}_1^-$ (a) and $H^0 \rightarrow \tilde{\chi}_1^+ + \tilde{\chi}_1^-$ (b) relative to the naive tree-level width as a function of $m_{\tilde{Q}}$. (Note that the tree-level already includes the correction due to the chargino mass matrix renormalization.)

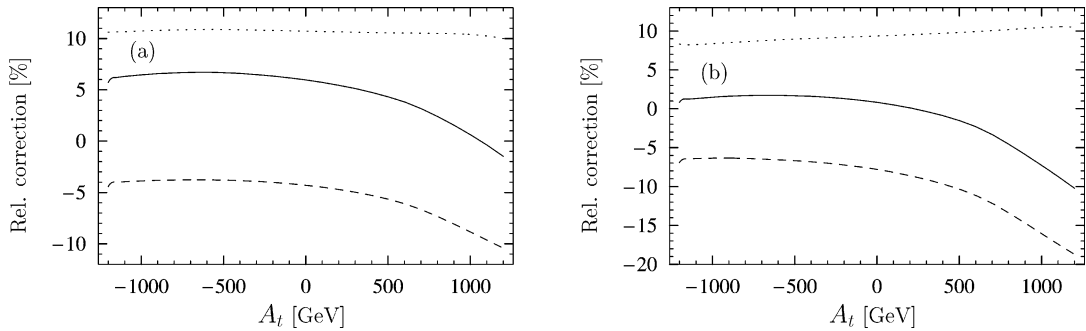


Fig. 8. Relative corrections for the decays $A^0 \rightarrow \tilde{\chi}_1^+ + \tilde{\chi}_1^-$ (a) and $H^0 \rightarrow \tilde{\chi}_1^+ + \tilde{\chi}_1^-$ (b) as a function of A_t . The dashed lines denote $\Gamma^{\text{tree}}/\Gamma^{\text{naive tree}} - 1$, the solid lines denote $\Gamma^{\text{corr.}}/\Gamma^{\text{naive tree}} - 1$ and the dotted lines $\Gamma^{\text{corr.}}/\Gamma^{\text{tree}} - 1$.

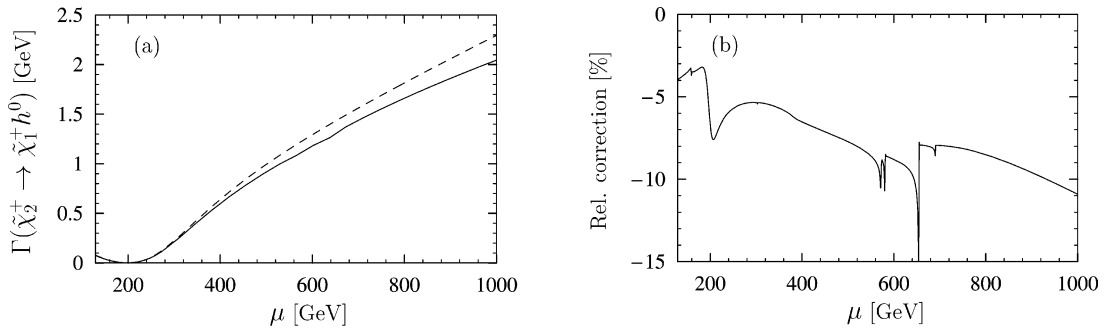


Fig. 9. The tree-level and one-loop corrected widths of the decay $\tilde{\chi}_2^+ \rightarrow \tilde{\chi}_1^+ h^0$ for varying μ . The dotted and solid lines correspond to the tree-level and loop-corrected widths, respectively.

6–11% in (a) and 4–7% in (b). The corrections become quite insensitive to $m_{\tilde{Q}}$ for large $m_{\tilde{Q}}$. The total correction consists of the $m_{\tilde{Q}}$ dependent (s)fermion contribution and the remaining contribution, the latter of which is $\sim 7.8\%$ for (a) and $\sim 9.6\%$ for (b). Again,

these two types of loop corrections are of comparable order.

Fig. 8 shows the corrections to the decay widths for $A^0 \rightarrow \tilde{\chi}_1^+ + \tilde{\chi}_1^-$ in (a) and $H^0 \rightarrow \tilde{\chi}_1^+ + \tilde{\chi}_1^-$ in (b) as a function of $A_t = A_b = A_\tau$, with the

other parameters unchanged. The dashed lines denote $\Gamma^{\text{tree}}/\Gamma^{\text{naive tree}} - 1$. They show the effect due to the chargino mass matrix renormalization. The solid lines show the total correction in terms of the naive tree level width, $\Gamma^{\text{corr.}}/\Gamma^{\text{naive tree}} - 1$. The dotted lines stand for $\Gamma^{\text{corr.}}/\Gamma^{\text{tree}} - 1$. This is the total correction in terms of the tree-level result, where the chargino mass matrix renormalization effect is already included. One sees that $\Gamma^{\text{tree}}/\Gamma^{\text{naive tree}} - 1$ and $\Gamma^{\text{corr.}}/\Gamma^{\text{naive tree}} - 1$ are much stronger dependent on A_t compared to $\Gamma^{\text{corr.}}/\Gamma^{\text{tree}} - 1$. This shows that the A_t dependence of the corrected widths comes mainly from the shifts of the masses and mixing matrices of the charginos.

Finally, Fig. 9 shows the width of the crossed channel decay $\tilde{\chi}_2^+ \rightarrow \tilde{\chi}_1^+ h^0$, as a function of μ . The total correction is in the range of -5% to -10% . In Fig. 9(b) a few pseudo thresholds are seen due to opening decay channels into loop particles, such as $\tilde{\chi}_2^+ \rightarrow t\tilde{b}_1^*$ at $\mu \sim 650$ GeV.

5. Conclusions

We have calculated the full one-loop corrections to the decays $(H^0, A^0) \rightarrow \tilde{\chi}_i^+ + \tilde{\chi}_j^-$ ($i, j = 1, 2$). All parameters in the chargino mass matrix X and mixing matrices (U, V) are renormalized in the on-shell scheme. The importance of the corrections to these matrices, in addition to the conventional corrections (vertex and wave-function corrections with counter terms), was emphasized. We have studied the dependence of the corrections on the SUSY parameters. The corrections to the widths of the decays $(H^0, A^0) \rightarrow \tilde{\chi}_1^+ + \tilde{\chi}_1^-$ are of the order of 10%, but can be larger near the thresholds. The corrections from quarks, leptons, and their superpartners were shown to be of similar order of magnitude as the other loop corrections. We also showed that the correction to the decay $\tilde{\chi}_2^+ \rightarrow \tilde{\chi}_1^+ h^0$ can be to $\sim -10\%$.

Acknowledgements

We thank Martin Kincel for collaboration in the early stage of this work. We also thank R.-Y. Zhang for communication about the consistency of our result to that in Ref. [10]. This work was supported by EU under the HPRN-CT-2000-00149 network programme

and the ‘‘Fonds zur F6orderung der wissenschaftlichen Forschung’’ of Austria, project No. P13139-PHY. Y.Y. was also supported by the Grant-in-aid for Scientific Research from the Ministry of Education, Culture, Sports, Science, and Technology of Japan, No. 14740144.

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