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Density perturbations in warm inflation and COBE normalization

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Abstract

Starting from a gauge invariant treatment of perturbations an analytical expression for the spectrum of long wavelength density perturbations in warm inflation is derived. The adiabatic and entropy modes are exhibited explicitly. As an application of the analytical results, we determined the observational constraint for the dissipation term compatible with COBE observation of the cosmic microwave radiation anisotropy for some specific models. In view of the results the feasibility of warm inflation is discussed.

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1. Introduction

Inflation [1] is the better description of the early stages of the Universe, where besides providing a satisfactory solution for the main problems of the standard Cosmology, also predicts a mechanism to generate the inhomogeneities for the structure formation. The advent of more precise observational data given by COBE [2], MAXIMA [3], etc. has so far confirming the basic predictions of the inflation. Nonetheless, since the first model of inflation was proposed, we have witnessed a myriad of alternative models, as for instance, new inflation, chaotic inflation, extended inflation, hybrid inflation, warm inflation, etc. A direct way of deciding which model must be discarded is confronting their theoretical predictions with the appropriate observational data [4,5]. In this context, I shall discuss the warm inflation [6–13] (WI) whose

important feature is that thermal fluctuations during inflation play a dominant role in producing the initial spectrum of perturbations. In WI radiation is continuously generated through a dissipation mechanism resulting from the interaction of the inflaton with other fields. Therefore, it is expected that at the end of inflation enough radiation for a smooth transition to the next phase, and the reheating phase is no longer necessary.

Warm inflation can be understood as a two-field inflation model. In this case, entropy perturbations arise from the variation of the effective equation of state relating the total pressure with the total energy density. As a consequence, contrary to the single-field inflation models, the curvature perturbation [14,15] ζ has a non-trivial evolution after the perturbation crosses outside the horizon, and the way in which it evolves in super-horizon scales depends on the details of the model itself. We expect that COBE normalization provides observational limits on the dissipation term Γ once, as we have already shown [13], the dissipation

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term plays a crucial role in producing the entropy perturbations. Recently, de Oliveira and Joras [13] investigated cosmological perturbations in WI together with the first determination of the observational limits on the dissipation term.¹ Moreover, in this determination it was assumed a very small dissipation in the sense of allowing, as a good approximation, the conservation of ζ . Under such circumstances, we found $\Gamma \sim 10^{-16} m_{\text{Pl}}$ for two distinct models, being compatible with other estimations [7,10] which the only criterion was the requirement that at the end of inflation there is enough radiation for the next phase. On the other hand, it will very important to determine the observational limits on Γ with no assumption whether the dissipation is small or not when compared with the Hubble parameter. Then, this observational test will constitute a definitive solution of the theoretical dispute [8,12] concerning the feasibility of WI.

This Letter is divided as follows. In Section 2 the basic equations that describe WI, and the equation for the perturbations are presented. Taking into account the slow-roll conditions, an approximate solution of these equations is derived where the contribution of adiabatic and entropy modes are shown explicitly. In Section 3, we present a procedure to normalize the amplitude of density perturbations in WI using the COBE data. A specific application of COBE normalization is performed in Section 4. Finally, in Section 5 is devoted to discuss the main results and conclusions.

2. Adiabatic and entropy modes in warm inflation

The basic equations that describe the homogeneous background dynamics of interacting radiation and scalar fields in flat Friedmann–Robertson–Walker spacetimes are [6]

$$H^2 = \frac{8\pi}{3m_{\text{Pl}}^2} \left(\rho_{\text{r}} + \frac{1}{2} \dot{\phi}^2 + V(\phi) \right), \quad (1)$$

$$\ddot{\phi} + (3H + \Gamma) \dot{\phi} + V'(\phi) = 0, \quad (2)$$

¹ Berera and Fang [6] have established an expression for the range of Γ in the weak dissipative regime just by considering $T_{\text{r}} > H$, T_{r} being the temperature of the thermal bath, without any treatment on the evolution of cosmological perturbations.

$$\dot{\rho}_{\text{r}} + 4H\rho_{\text{r}} = \Gamma\dot{\phi}^2. \quad (3)$$

Here $H = \dot{a}/a$ is the Hubble parameter, m_{Pl} is the Planck mass, dot and prime denote derivative with respect to the cosmological time t and the scalar field, respectively; ρ_{r} is the energy density of the radiation field. The dissipative term Γ is responsible for the decay of the scalar field into radiation during the slow-roll phase. In general, Γ can be assumed as a function of ϕ of the type $\Gamma = \Gamma_m \phi^m$, reflecting the types of interactions between the scalar field ϕ and other fields [6,7]. In addition, m must be even and $\Gamma_m > 0$ to guarantee the positiveness of Γ as demanded by the second law of thermodynamics. We remark that there is a theoretical controversy about the introduction of the dissipative term in Eq. (2) during the slow-roll [8,12], and therefore on the feasibility of WI. Our approach here will be to assume it as part of the WI scenario and, from a consistent treatment of cosmological perturbations, allow that observation dictates the result of this issue.

Warm inflation is characterized by the accelerated growth of the scale factor driven by the potential term $V(\phi)$ that dominates over other energy terms in Eq. (1), or

$$H^2 \simeq \frac{8\pi}{3m_{\text{Pl}}^2} V(\phi). \quad (4)$$

Also, the $\ddot{\phi}$ term can be neglected in Eq. (2), yielding

$$\dot{\phi} \simeq -\frac{V'(\phi)}{3H + \Gamma}, \quad (5)$$

and the radiation density is kept approximately constant given by

$$\rho_{\text{r}} \simeq \frac{\Gamma\dot{\phi}^2}{4H}. \quad (6)$$

Several authors have studied the dynamics of WI in which the basic issue was to present WI as a viable description of the early universe. In this context, it was shown that even for a tiny dissipation, in the sense of the ratio $\alpha \equiv \frac{\Gamma}{3H} \ll 1$, enough radiation is found at the end of inflation for a smooth transition to the radiation era. In particular, for α of order 10^{-9} is in agreement with observational data [13].

We consider inhomogeneous perturbations of the FRW background described by the metric in the longitudinal gauge [14,15]

$$ds^2 = (1 + 2\Phi) dt^2 - a^2(t)(1 - 2\Psi)\delta_{ij} dx^i dx^j, \quad (7)$$

where $a(t)$ is the scale factor, $\Phi = \Phi(t, \mathbf{x})$ and $\Psi = \Psi(t, \mathbf{x})$ are the metric perturbations. The spatial dependence of all perturbed quantities are of the form of plane waves $e^{i\mathbf{k}\cdot\mathbf{x}}$, k being the wave number, so that the perturbed field equations regarding now only their temporal parts (we omit the subscript k) are

$$\dot{\Phi} + H\Phi = \frac{4\pi}{m_{\text{Pl}}^2} \left(-\frac{4}{3k} \rho_r a v + \dot{\phi} \delta\phi \right), \quad (8)$$

$$\begin{aligned} (\delta\phi)'' + (3H + \Gamma)(\delta\phi)' + \left(\frac{k^2}{a^2} + V'' + \dot{\phi}\Gamma' \right) \delta\phi \\ = 4\dot{\phi}\dot{\Phi} + (\dot{\phi}\Gamma - 2V')\Phi, \end{aligned} \quad (9)$$

$$\begin{aligned} (\delta\rho_r)' + 4H\delta\rho_r + \frac{4}{3}ka\rho_r v \\ = 4\rho_r\dot{\Phi} + \dot{\phi}\Gamma'\delta\phi + \Gamma\dot{\phi}(2(\delta\phi)' - 3\dot{\phi}\Phi), \end{aligned} \quad (10)$$

$$\dot{v} + \frac{\Gamma\dot{\phi}^2}{\rho_r} v + \frac{k}{a} \left(\Phi + \frac{\delta\rho_r}{4\rho_r} + \frac{3\Gamma\dot{\phi}}{4\rho_r} \delta\phi \right) = 0, \quad (11)$$

where $\delta\rho = \delta\rho_r + \dot{\phi}(\delta\phi)' - \dot{\phi}^2\Phi + V'\delta\phi$ and $\delta p = \frac{1}{3}\delta\rho_r + \dot{\phi}(\delta\phi)' - \dot{\phi}^2\Phi - V'\delta\phi$ are the perturbations of the total energy density and pressure, respectively; v originates from the decomposition of the velocity field as $\delta U_i = -\frac{iak}{k} v e^{i\mathbf{k}\cdot\mathbf{x}}$ (see Bardeen [14]). Also, due the fact that the perturbation of the total energy-momentum tensor does not give rise to anisotropic stress ($\delta T_j^i \propto \delta_j^i$), $\Psi = \Phi$. Warm inflation can be considered as a hybrid-like inflationary model since two basics fields, the inflaton and the radiation fields, are interacting during the slow-roll phase. Therefore, isocurvature or entropy perturbations are generated, besides the adiabatic ones. We have shown [13] that the entropy perturbations are directly related to the dissipation term. In what follows, we will obtain an approximate solution for the long wavelength perturbations valid during the slow-roll phase, in which the adiabatic and the entropy modes are exhibited explicitly.

During the slow-roll phase, it can be assumed that the perturbed quantities do not vary strongly (see Appendix A). This means, that in Eq. (8) $H\Phi \gg \dot{\Phi}$; in Eq. (9), $(\delta\phi)'' \ll (\Gamma + 3H)(\delta\phi)'$, and so on. Following these approximations, together with the slow-roll conditions provided by Eqs. (4), (5) and (6), $\delta\rho_r$ and v are expressed as

$$\frac{\delta\rho_r}{\rho_r} \simeq \frac{\Gamma'}{\Gamma} \delta\phi - 3\Phi, \quad (12)$$

$$v \simeq -\frac{k}{4aH} \left(\Phi + \frac{\delta\rho_r}{4\rho_r} + \frac{3\Gamma\dot{\phi}}{4\rho_r} \delta\phi \right). \quad (13)$$

Using these equations, the metric perturbation (Eq. (8)) can be written as

$$\Phi \simeq \frac{4\pi\dot{\phi}}{m_{\text{Pl}}^2 H} \left(1 + \frac{\Gamma}{4H} + \frac{\Gamma'\dot{\phi}}{48H^2} \right) \delta\phi. \quad (14)$$

Note that in the case of null dissipation, we recover the standard relation between the metric and scalar field perturbations, $\Phi \simeq \frac{4\pi\dot{\phi}}{m_{\text{Pl}}^2 H} \delta\phi$, that describes the adiabatic mode [16,17].

The equation of motion for $\delta\phi$ reads now as

$$(\Gamma + 3H)(\delta\phi)' + (V'' + \dot{\phi}\Gamma')\delta\phi \simeq (\dot{\phi}\Gamma - 2V')\Phi, \quad (15)$$

where it will be useful to introduce an auxiliary function χ by

$$\chi = \frac{\delta\phi}{V'} \exp\left(\int \frac{\Gamma'}{\Gamma + 3H} d\phi \right), \quad (16)$$

that generalizes the procedure due to Starobinski and Polarski [17] in the realm of double inflationary models. Substituting Eq. (16) into (15), we obtain the following equation for χ

$$\begin{aligned} \frac{\chi'}{\chi} + \frac{9}{8} \frac{(\Gamma + 2H)}{(\Gamma + 3H)^2} \left(\Gamma + 4H - \frac{\Gamma'V'}{12H(\Gamma + 3H)} \right) \frac{V'}{V} \\ \simeq 0. \end{aligned} \quad (17)$$

We separate two situations: $\Gamma = \Gamma_0 = \text{const}$, and the variable dissipation term $\Gamma = \Gamma_m \phi^m$, $m = 2, 4, \dots$. Considering the first case, Eq. (17) can be integrated exactly, yielding

$$\delta\phi \simeq \frac{C_1 \dot{\phi}}{H} \left(1 + \frac{\Gamma_0}{3H} \right)^{5/4} \exp\left[\frac{\Gamma_0}{4(\Gamma_0 + 3H)} \right], \quad (18)$$

where C_1 is a constant of integration. It will be instructive to expand this expression in power series of Γ_0 ,

$$\delta\phi \simeq \frac{C_1 \dot{\phi}}{H} \left(1 + \frac{\Gamma_0}{2H} + \frac{\Gamma_0^2}{36H^2} + \dots \right). \quad (19)$$

Notice that the zeroth order term is the same found in single field inflationary models [16,17], which is related the adiabatic mode. The remaining terms describe the effect of dissipation to the fluctuation of the scalar field, or in another words, the entropy mode. The metric perturbation is obtained straightforwardly

from Eqs. (14) and (18),

$$\Phi \simeq -\frac{C_1 \dot{H}}{H^2} \left(1 + \frac{\Gamma_0}{4H}\right) \left(1 + \frac{\Gamma_0}{3H}\right)^{1/4} \times \exp\left[\frac{\Gamma_0}{4(\Gamma_0 + 3H)}\right]. \quad (20)$$

Again, expanding it power series of Γ_0 , the role of the dissipation in producing entropy mode becomes evident

$$\Phi \simeq -\frac{C_1 \dot{H}}{H^2} \left(1 + \frac{5\Gamma_0}{12H} + \frac{\Gamma_0^2}{72H^2} + \dots\right). \quad (21)$$

For the second case, the integration of (17) can be performed only after specifying the potential $V(\phi)$.

An important relation between the density of matter fluctuations, $\frac{\delta\rho}{\rho}$, and the metric perturbation can be derived in WI after taking into account that, during the slow-roll phase, $\frac{\delta\rho}{\rho} \simeq \frac{V'}{V}\delta\phi$, and Eq. (14),

$$\frac{\delta\rho}{\rho} \simeq -\frac{8}{3} \frac{(\Gamma + 3H)}{(\Gamma + 4H + \frac{\Gamma'\dot{\phi}}{12H})} \Phi. \quad (22)$$

In the absence of the dissipation, we recover the usual relation $\frac{\delta\rho}{\rho} \simeq -2\Phi$ valid for single-field inflationary models. On the other limit, for instance, high constant dissipation, $\Gamma_0 \gg H$, it follows that $\frac{\delta\rho}{\rho} \simeq -\frac{8}{3}\Phi$. The above relation together with the integration of Eq. (17) will be of fundamental importance for the COBE normalization as we are going to show next.

3. COBE normalization

In super-cooled inflation the overall dynamics of perturbations can be reduced to a single conservation law for the gauge invariant curvature perturbation on comoving hypersurfaces, ζ , defined by [14,15]

$$\zeta \equiv \frac{2\rho}{3H(\rho + p)} (H\Phi + \dot{\Phi}) + \Phi. \quad (23)$$

During inflation, we have $H\Phi \gg \dot{\Phi}$ such that, together with Eqs. (18), (20) and (22) and $\Gamma = 0$, it follows that the curvature perturbation can be approximated as $\zeta \simeq \frac{\delta\rho}{\rho+p}$ [16,18]. In order to illustrate the importance of this quantity, consider a perturbation corresponding the size of a galaxy, λ_{gal} . Accordingly, it can be shown that this perturbation leaves the horizon about 50 e-folds before the end of inflation. Then,

$\zeta_{50} \simeq (\frac{\delta\rho}{\rho+p})_{50}$, with the subscript 50 indicating that the curvature perturbation is evaluated at 50 e-folds before the end of inflation, is frozen until the perturbation re-enters the Hubble horizon in the radiation or matter dominated eras. When this happens, it can be established that $(\frac{\delta\rho}{\rho+p})_{50} \propto (\frac{\delta\rho}{\rho})_{\text{horizon},\lambda}$ is the amplitude of density perturbations on the scale λ_{gal} when the perturbations crosses back inside the horizon during the post-inflation epoch, where the constant of proportionality depends whether $\rho + p = n\rho$, $n = 1, 4/3$ for matter, radiation era, respectively. For instance, for single-field inflation models, it can be show that [4,5] $(\frac{\delta\rho}{\rho})_{\text{horizon},\lambda} \simeq (\frac{V^{3/2}}{m_{\text{pl}}^2 V'})_{50}$, where quantum fluctuations are the source of the perturbations of the scalar field. The Sachs–Wolfe effect [19] establishes, roughly speaking, that the metric perturbation, which in turn is related to the density perturbations, determines the temperature anisotropy of the CBR (cosmic microwave background), or

$$\frac{\delta T}{T} \simeq \left(\frac{\delta\rho}{\rho}\right)_{\text{horizon},\lambda}. \quad (24)$$

The COBE satellite performed reliable measurements of the anisotropies of temperature, and therefore, became a powerful tool of testing inflation models by normalizing the amplitude of perturbations, as well as determining the spectral index of the spectrum of perturbations.

Under the influence of entropy perturbations, the conservation of ζ in super-horizon scales is no longer valid, and in the specific case of WI, the rate of variation of ζ is depends directly on the dissipation term [13]. An important observational test for WI is the determination of the magnitude of the dissipation term compatible with COBE data, implying that it can be decided if strong dissipative regime characterized by $\Gamma \gg H$, weak dissipative regime $\Gamma \ll H$, or even dissipation of the order of the Hubble parameter is the most viable model for WI. Indeed, this task was partially accomplished by de Oliveira and Joras [13] by assuming ab initio weak dissipation such that the effect of the entropy perturbation could be neglected. Under this assumption it was possible to determine a lower bound of the dissipation term taking into account that thermal fluctuations are mainly responsible for the production of the inhomogeneities of the in-

flaton, and the COBE data, where it was found that $\Gamma \approx 10^{-16} m_{\text{pl}}$.

Here, we derive a convenient expression for ζ_{end} necessary for COBE normalization, in which it is assumed that the dissipation is constant and vanishes after inflation. This last condition is physically reasonable, since dissipation results from the interaction between the inflaton and other fields, and at the end of inflation the inflaton field has transferred the enough energy to radiation to start a new phase without the necessity of reheating. The next step is to obtain a suitable expression for the curvature perturbation ζ in WI. For this, we may consider that during the slow-roll phase, Eq. (23) is reduced to

$$\zeta \simeq \frac{2\rho}{3(\rho + p)} \Phi, \quad (25)$$

due to the quasi-static evolution of the perturbations in this phase. This equation together with Eq. (22) yield,

$$\zeta \simeq -\frac{(\Gamma_0 + 4H)}{4(\Gamma_0 + 3H)} \frac{\delta\rho}{\rho + p}. \quad (26)$$

In the limit $\Gamma_0 = 0$, as well as for $\Gamma_0 \gg H$, $\zeta \propto \frac{\delta\rho}{\rho + p}$, suggesting that the effect of the dissipation is to change this proportionality. Now, we are in conditions trace out a procedure to apply the COBE normalization.

Consider again a perturbation in WI corresponding to a scale of astrophysical interest that corresponds to the size of a galaxy. This perturbation will cross outside the Hubble radius at approximately 50 e-folds before the end of inflation [13]. As previously discussed, the presence of entropy perturbations are responsible for the variation of ζ in super-horizon scales, and such a variation takes place until the end of inflation, since $\Gamma_0 = 0$ after inflation. The curvature perturbation ζ_{end} associated to this perturbation is kept constant until the moment when the perturbation re-enters the Hubble horizon during the post-inflation epoch to become the necessary fluctuations for structure formation. The amplitude of the resulting spectrum is normalized by COBE data. As one can see, we need to know ζ_{end} instead of ζ_{50} due to the variation of curvature perturbation. To determine the former, we calculate the ratio

$$K \equiv \frac{\zeta_{\text{end}}}{\zeta_{50}} \simeq \left(\frac{\rho}{\rho + p} \right)_{\text{end}} \left(\frac{\rho}{\rho + p} \right)_{50}^{-1} \frac{\Phi_{\text{end}}}{\Phi_{50}}, \quad (27)$$

where we have taken into account Eq. (25). The ratio between the metric perturbation follows from

the approximate solution (20), for which it can be shown that for $\Gamma_0 = 0$, $K = 1$, as expected. Then, the expression for ζ_{end} as given by

$$\zeta_{\text{end}} = K \zeta_{50} \simeq -K \left[\frac{(\Gamma_0 + 4H)}{4(\Gamma_0 + 3H)} \frac{\delta\rho}{(\rho + p)} \right]_{50}, \quad (28)$$

that must be normalized by COBE data. Notice that $\delta\rho \simeq V'(\phi)\delta\phi$, with the fluctuations of the scalar field being due to thermal interactions with the radiation field [6]. In the next section we apply the COBE normalization to determine the dissipation term.

4. Worked examples: high dissipation and COBE data

Let us consider the case of quadratic potential $V(\phi) = \frac{1}{2}m^2\phi^2$. It will be useful to introduce adimensional quantities such as

$$\gamma_0 = \frac{\Gamma_0}{m}, \quad x = \frac{\sqrt{4\pi}}{m_{\text{pl}}}\phi, \quad \alpha = \frac{\Gamma_0}{3H} \simeq \frac{\gamma_0}{\sqrt{3}x}, \quad (29)$$

where in the last expression it is assumed that $H^2 \simeq \frac{8\pi}{3m_{\text{pl}}^2} V(\phi)$. The regime of high dissipation is characterized by $\alpha \gg 1$. The end of WI is achieved when $\epsilon_{\text{wi}} \simeq \frac{m_{\text{pl}}^2}{4\pi\alpha} \left(\frac{H'}{H} \right)^2 = 1$ for $x = x_e$, where ϵ_{wi} is the generalized slow-roll parameter for WI [13]. The beginning of WI at $x = x_*$ can be determined from the condition of 60 e-folds necessary for a successful inflation. Then, both conditions render

$$x_e \simeq \frac{\sqrt{3}}{\gamma_0}, \quad x_* \simeq \frac{61\sqrt{3}}{\gamma_0}. \quad (30)$$

To guarantee that the regime of high dissipation holds since the beginning of WI, it is necessary that $\gamma_0^2 \gg 183$. Also, if x_{50} corresponds the scalar field evaluated at 50 e-folds before the end of inflation, it can be shown that

$$x_{50} \simeq \frac{51\sqrt{3}}{\gamma_0}. \quad (31)$$

According to the discussion of the last section, we need to know K , which can be determined from the approximate solution for Φ given by Eq. (20) together with the slow-roll conditions (4)–(6). The derived

expression is found to be

$$K \simeq \left(\frac{\alpha_e}{\alpha_{50}} \right)^{5/4} \approx 1.36 \times 10^2, \quad (32)$$

where the values of α at the end of inflation and at 50 e-folds before the end of inflation are given by Eqs. (30) and (31). This value of K is the maximum reached by considering very high dissipation. The next step is to consider Eq. (28) assuming $\delta\rho \simeq V'(\phi)\delta\phi$, with the fluctuations of the scalar field being generated by thermal interactions with radiation, instead of quantum fluctuations. Then, following Berera and Taylor [11], it is established that

$$(\delta\phi)^2 \simeq \frac{(\Gamma_0 H)^{1/2} T_\Gamma}{2\pi^2}, \quad (33)$$

where T_Γ is the temperature of the thermal bath. These fluctuations are evaluated at 50 e-folds before the end of inflation. After performing this calculation, together with Eqs. (29)–(31), and the condition $\alpha \gg 1$, we arrived to a simple expression for ζ_{end}

$$|\zeta_{\text{end}}| \simeq 93.45 \left(\frac{m}{m_{\text{Pl}}} \right)^{3/4} \gamma_0^{3/4}. \quad (34)$$

COBE normalization tells us that $|\zeta_{\text{end}}| \simeq 7.33 \times 10^{-6}$, implying

$$\Gamma_0 \simeq 3.36 \times 10^{-10} m_{\text{Pl}}, \quad (35)$$

which, in view of our approximation, this result can be interpreted as the upper bound of the dissipation term. In order to assure the regime of high dissipation during the whole WI, it is necessary that $\gamma_0 \gg 14$. This restriction together with the obtained upper bound of Γ_0 implies that

$$m \ll 2.40 \times 10^{-11} m_{\text{Pl}}. \quad (36)$$

This constraint is a very small bound for the mass of the inflaton, even if compared with the value obtained by COBE normalization in super-cooled inflation [4], which is $m \sim 10^{-6} m_{\text{Pl}}$.

We have considered another potential $V(\phi) = \lambda^4 \phi^4$ and variable dissipation term $\Gamma = \Gamma_2 \phi^2$, with Γ_2 being a constant of dimension m_{Pl}^{-1} . In this case, high dissipation is characterized by

$$\alpha \simeq \frac{m_{\text{Pl}} \Gamma_2}{\sqrt{24\pi} \lambda^2} \gg 1. \quad (37)$$

The scalar field evaluated at the end, the beginning and at 50 e-folds before the end of WI, correspond, respectively, to

$$x_e \simeq \frac{2}{\sqrt{\alpha}}, \quad x_* \simeq \sqrt{61} x_e, \quad x_{50} \simeq \sqrt{51} x_e. \quad (38)$$

For the sake of completeness, the end of WI occurs for $\epsilon \simeq \frac{m_{\text{Pl}}^2}{\pi\alpha} \frac{1}{\phi^2}$, and the remaining values are determined from $N \simeq \frac{\pi\alpha}{m_{\text{Pl}}^2} (\phi_*^2 - \phi_N^2)$, by setting $N = 50, 60$. We need to know δx in order to determine the metric perturbation from Eq. (14), and consequently K (see Eq. (25) which is valid for any Γ). After integrating Eq. (17) imposing $\alpha \gg 1$, the perturbation of the scalar field is given by

$$\delta x \simeq 4\lambda^4 \chi_0 x^{-7/2} \exp\left(-\frac{3}{4\alpha^2 x^2}\right), \quad (39)$$

where χ_0 is the constant of integration. Taking into account this result and the corresponding expression for Φ , we obtain

$$K \simeq \left(\frac{3\alpha x_e^2 - 1}{3\alpha x_{50}^2 - 1} \right) \frac{x_{50}}{x_e} \frac{(\delta x)_e}{(\delta x)_{50}} \approx 1.24 \times 10^2. \quad (40)$$

Notice that the maximum value of growth of the curvature perturbation is approximately the same as found in the previous case. The final step for the COBE normalization is to consider Eq. (33) inserted into

$$\zeta_{\text{end}} = K \zeta_{50} \simeq -K \left[\frac{(\Gamma + 4H + \frac{\Gamma'\dot{\phi}}{12H})}{4(\Gamma + 3H)} \frac{V'\delta\phi}{(\rho + p)} \right]_{50},$$

together with the value of K given by Eq. (39). Hence, after a direct calculation

$$|\zeta_{\text{end}}| \simeq 3.21 \times 10^3 \lambda^{3/2}. \quad (41)$$

Contrary to the previous case, the dissipation term does not appear explicitly. COBE normalization imposes that

$$\lambda^4 \simeq 10^{-24}. \quad (42)$$

5. Discussion

In this Letter, we have integrated the equations for the metric and scalar field perturbation in WI. The contributions of the adiabatic and entropy modes were exhibited explicitly showing the central role played by

the dissipation in producing the entropy mode. Another very important result was the generalization of the relation between the total density of matter perturbations and metric perturbations given by Eq. (22). Using the analytical expressions, a general procedure to apply COBE normalization was proposed, since in WI a conservation equation for the curvature perturbation, ζ , is no longer valid.

As an application of the COBE normalization, we have considered two distinct examples, namely, quadratic potential $V(\phi) = \frac{1}{2}m^2\phi^2$ with constant dissipation, and quartic potential $V(\phi) = \lambda^4\phi^4$ with variable dissipation term. The common aspect shared by these two examples is the assumption of high dissipative regime guaranteed by $\frac{\Gamma}{3H} \gg 1$. In the first case, COBE normalization determines directly the upper bound of the dissipation term as $\Gamma_0 \simeq 3.36 \times 10^{-10}m_{\text{Pl}}$, with the mass of the inflaton given by $m \ll 2.40 \times 10^{-11}m_{\text{Pl}}$. Thus, considering the previous result concerning lower bound of the dissipative term [13], the following constraint is obtained

$$\Gamma_{\text{min}} \leq \Gamma_0 \leq \Gamma_{\text{max}}, \quad (43)$$

where $\Gamma_{\text{min}}/m_{\text{Pl}} \approx 10^{-16}$ and $\Gamma_{\text{max}}/m_{\text{Pl}} \approx 10^{-10}$. On the other hand, for the case of quartic potential, COBE normalization imposes that $\lambda^4 \simeq 10^{-24}$, which represents a stronger fine tuning if compared with 10^{-14} found in super-cooled inflation. In spite of representing a drawback for very high dissipative regime in WI in this specific example, it is necessary further analysis considering other situations in order to give a final word about the feasibility of the very high dissipative regime of WI as far as COBE normalization is concerned. In this way, a possible direction of investigation is to treat other cases characterized by distinct potentials and by not imposing a priori any kind of high/low dissipative regime to verify the consequences of COBE normalization.

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Appendix A

It is well established that the slow-roll phase $\dot{H} \ll H^2$. This condition has a direct consequence on the evolution of the perturbed quantities as we are going to illustrate integrating Eq. (8). The general solution of Eq. (8) can be written as

$$\Phi = \frac{\Phi_0}{a} + \frac{1}{a} \int \Delta a dt, \quad (\text{A.1})$$

where Φ_0 is the initial value of the metric perturbation and $\Delta = \frac{4\pi}{m_{\text{Pl}}^2}(-\frac{4}{3k}\rho_{\text{rav}} + \dot{\phi}\delta\phi)$ for convenience. The non-decreasing mode is given by the integral, that can be expanded in power series [15,16] like

$$\Phi = \frac{\Delta}{H} - \left(\frac{\Delta}{H}\right)' H^{-1} + \left[\left(\frac{\Delta}{H}\right)'' H^{-1}\right]' H^{-1} + \dots \quad (\text{A.2})$$

Then, it is acceptable to approximate the above expression considering only the first term of the series, since during the slow-roll, $\frac{\dot{H}}{H^2} \ll 1$ and $\frac{\dot{\Delta}}{H^2} \ll 1$. As we can see, this approximation is equivalent to assume $H\Phi \ll \dot{\Phi}$. Applying this procedure to the other perturbed equations, we arrive to the Eqs. (12)–(14) and (15).

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