New solutions to the strong CP problem

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Abstract

We exhibit a solution to the strong CP problem in which ultraviolet physics renders the QCD θ angle physically unobservable. Our models involve new strong interactions beyond QCD and particles charged under both the new interactions and ordinary color.

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1. Introduction

The CP-violating QCD θ angle is the most mysterious of the fundamental parameters of the standard model [1]. θ is highly constrained by measurements of the neutron electric dipole moment (NEDM): $\theta \lesssim 10^{-10}$. (We work in a basis where $\arg \det M_q = 0$, so that θ is directly related to CP violation and the NEDM.) Theorists have long sought a mechanism to explain why θ is so close to zero.

In this Letter we exhibit a model in which new physics, at possibly very high energy scales, renders θ an unobservable parameter. The main idea is quite

simple. We introduce a massless fermionic field Q which carries both ordinary color and also transforms under another gauge group $SU(N)$. The anomaly associated with the axial transformation of Q receives a contribution from gluons, so that phase rotations of Q are equivalent to shifts of the θ angle. Because Q is massless, there is a symmetry associated with these phase rotations, allowing us to eliminate θ entirely. The $SU(N)$ interactions are necessary to bind Q particles into heavy bound states. These are not observed in low-energy physics, as there is a mass gap which grows with Λ_N, the strong coupling scale of the $SU(N)$ interaction. A novel aspect of our model is that ordinary QCD is embedded in a left–right symmetric group $SU(3)_L \times SU(3)_R$, where here 3 is not due to the number of light quark flavors, but rather the number of colors in QCD. This structure requires a non-standard Higgs sector to generate quark masses as well as an-

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other fermion T which is a color singlet and charged only under the new $SU(N)$ group.

There are other viable solutions to the strong CP problem[2] [2] such as axion models, left–right symmetric models and the ones in which CP is broken spontaneously [3]. In axion models [4–10], additional particles (usually heavy scalars and colored fermions) are added to realize an anomalous Peccei–Quinn symmetry. The observed value of θ QCD is then determined dynamically by the location of the minimum of the instanton-generated potential for the axion field, which is a pseudo-Goldstone boson of the Peccei–Quinn symmetry. Quantum gravity is not believed to exhibit any *exact* global symmetries, and even very small, Planck-suppressed, violations of the Peccei–Quinn symmetry will spoil the axion solution of the strong CP problem [11]. In string theory, it is possible to obtain global symmetries which are exact up to violation by instanton effects. In this sense the axion solution of the strong CP problem is generally considered to be very natural in string theory (see for example [12]). However, the axion mass scale, which is determined by a compactification scale, is generally very close to the four-dimensional Planck scale. This conflicts with some standard cosmological arguments which say that such a scale should be no bigger than about 10^{13} GeV.

In our model we are left with an exactly massless Goldstone boson.[3] This Goldstone boson is not an axion (which would have a non-zero mass), and our solution is not the standard axion solution of the strong CP problem. A simple way to understand why we are left with an exactly massless scalar field is to recall that the new colored and uncolored fermions Q and T have zero mass. This [13,14] prevents the Goldstone boson from acquiring a mass term. Unlike in axion models, the symmetry breaking scale associated with the Goldstone boson can be taken to be arbitrarily large, effectively decoupling it from ordinary particles.

2. Model

The model we consider is described in the accompanying table and figure. (Generalizations are straightforward.) It contains the symmetry groups $SU(N) \times SU(3)_L \times SU(3)_R$ and the additional particles Q and T, which are in the fundamental and adjoint representation of $SU(N)$, respectively. The $SU(3)_c$ color symmetry of QCD is the vector subgroup $SU(3)_V$ of $SU(3)_L \times SU(3)_R$, so that QCD gauge transformations correspond to simultaneous transformations with $U_L = U_R$. Each of $SU(N)$ and $SU(3)_{L,R}$ (or equivalently, $SU(3)_{A,V}$) are gauged. Additional $U(1)$ axial charges and symmetries are listed. The anomaly-free linear combination, denoted $U(1)_{\text{TAF}}$ is gauged. Note that in Table 1 we have suppressed the standard model flavor index $f = 1, \ldots, F = 6$ (i.e., $f = $ up, down, strange, ... top).

Gauging $SU(3)_A$, as well as $U(1)_{\text{TAF}}$, precludes the usual standard model masses for the quarks q. These have the form (suppressing all indices except those of $SU(3)_L \times SU(3)_R$):

$$\mathcal{L}_{\text{quark mass}} = m_q q^c \tilde{q}_c + \text{h.c.}, \tag{1}$$

which under a general $SU(3)_L \times SU(3)_R$ transformation becomes

$$m_q q U_L U_R^\dagger \tilde{q} + \text{h.c.} \tag{2}$$

The quark mass (1) is invariant under $SU(3)_c = SU(3)_V$, transformations, which have $U_L = U_R$, but not under $SU(3)_A$. Hence, the usual Higgs coupling to quarks is forbidden by gauge symmetry; the interaction which gives masses to quarks must involve additional particles charged under both $SU(3)_L$ and $SU(3)_R$.

To overcome this restriction, we postulate a field[4] H which transforms as $(\bar{3}, 3)$ under $SU(3)_L \times SU(3)_R$, carries $U(1)_{\text{TAF}}$ charge $-2F/3$ ($F = N$ to insure gauge anomaly cancellations), and is a singlet under electroweak symmetries. H couples to quarks via the higher dimension operator, see footnote 4

$$H\phi q\tilde{q}, \tag{3}$$

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[2] A massless up quark, $m_u = 0$, does not actually solve the strong CP problem, as emphasized recently by Creutz [2]. This is due to the anomalous non-multiplicative renormalization of m_q which causes the condition $m_u = 0$ to be scale dependent. In our model a bare Q mass breaks the $SU(3)_A$ gauge symmetry, so $m_Q = 0$ is preserved by radiative corrections.

[3] We thank E. Witten for correspondence regarding the presence of the extra Goldstone boson in our model.

[4] The alert reader might wonder why we do not use $\tilde{Q}Q$ in place of H in Eq. (3). The reason is that the Q phase rotation used to eliminate θ QCD would then re-introduce θ into the quark mass matrix. This is also why the $U(1)_{QA}$ symmetry must be violated only by the anomaly and not by other explicit interactions.

Table 1
The table summarizes the symmetries and particle content of the model. The usual quarks q of QCD also have a standard model flavor index $f = 1, \ldots, F = 6$ which we suppressed. All fields are Weyl spinors. There are five independent global $U(1)$'s associated to each Weyl fermion transformation. We can make two independent anomaly free combinations, labeled by $U(1)_V$ and $U(1)_{\mathrm{TAF}}$ in the table while there are still three anomalous $U(1)$ transformations which are the remaining $U(1)$ transformations in the table. The gauge group is $SU(N) \times SU(3)_L \times SU(3)_R \times U(1)_{\mathrm{TAF}}$

	$SU(N)$	$SU(3)_L$	$SU(3)_R$	$U_V(1)$	$U(1)_{\mathrm{TAF}}$	$U(1)_T$	$U(1)_{QA}$	$U(1)_A$
T_C^C	Adj	1	1	0	1	1	0	0
Q_c^C	$\square$	$\bar{\square}$	1	1	$-\frac{N}{3}$	0	1	0
$\tilde{Q}_C^c$	$\bar{\square}$	1	$\square$	-1	$-\frac{N}{3}$	0	1	0
q^c	1	$\square$	1	$-\frac{N}{F}$	$\frac{N^2}{3F}$	0	0	1
$\tilde{q}^c$	1	1	$\bar{\square}$	$+\frac{N}{F}$	$\frac{N^2}{3F}$	0	0	1

where ϕ is the usual Higgs boson.[5] The potential for H must be chosen so that H develops a vacuum expectation value. The Goldstone boson mentioned in the introduction is a linear combination of the phase of H and that of the quark condensate $q\tilde{q}$. In this case the vev of H can be much larger than the electroweak scale since the breaking of electroweak symmetry is solely due to the ordinary Higgs ϕ. Note that we do not allow interactions among H and $Q\tilde{Q}$ leading to a mass term for Q. That is, we assume that the $U(1)_{QA}$ symmetry is only violated by the anomaly (i.e., instanton effects). This is similar to the assumption made about the Peccei–Quinn symmetry in axion models.

The condensate $\langle \tilde{Q}Q \rangle \sim \Lambda_N^3$ spontaneously breaks $SU(3)_L \times SU(3)_R \to SU(3)_V$, where the unbroken subgroup is QCD. The axial subgroup $SU(3)_A$ is spontaneously broken, and the corresponding Goldstone bosons are eaten, leading to massive $SU(3)_A$ gauge bosons. It is dynamically preferred for the H condensate to align with $\langle \tilde{Q}Q \rangle$ in the $SU(3)_L \times SU(3)_R$ internal space, since when they are not aligned color is broken and gluons become massive. As reviewed in [16], the contribution to the vacuum energy from Higgsed gauge bosons is larger than that from massless gauge bosons, which generically leads to groundstates with maximal unbroken gauge symmetry.

We expect the colored excitations associated with rotations of $\langle H \rangle$ relative to $\langle \tilde{Q}Q \rangle$ in the $SU(3)_L \times SU(3)_R$ to have mass much larger than the weak or TeV scale since the H dynamics is unrelated to electroweak breaking. We also expect the T field to condense and spontaneously break $U(1)_{\mathrm{TAF}}$, so that any states containing T constituents have masses at least of order Λ_N.

3. Summary of the UV theory and dynamics

Before describing the low energy theory it is instructive to display explicitly the UV theory corresponding to the fields shown in the table. The fermion kinetic terms are given by:

$$\mathcal{L}_{\mathrm{FKT}} = \bar{Q}_L (i\slashed{\partial}) Q_L + \bar{Q}_R (i\slashed{\partial}) Q_R + \frac{1}{2} \bar{T} (i\slashed{\partial}) T$$
$$+ \bar{\mathcal{Q}}_L (i\slashed{\partial}) \mathcal{Q}_L + \bar{u}_R (i\slashed{\partial}) u_R + \bar{d}_R (i\slashed{\partial}) d_R, \quad (4)$$

with $Q_L = Q$ and $Q_R = \bar{\tilde{Q}}$. Q and $\tilde{Q}$ are left handed Weyl fermions. $\mathcal{Q}_L$ represents the electroweak quark doublet, while $q_L = q$ and $q_R = \bar{\tilde{q}}$ in the notation of the table. The generation index for the ordinary quarks is not explicitly shown. More explicitly, the kinetic terms for the new fermions are:

$$\bar{T} i\gamma^\mu \left(\partial_\mu - i B_\mu^A \hat{T}_{\mathrm{Adj}}^A - i \mathcal{A}_\mu \right) T,$$

$$\bar{\mathcal{Q}}_{L/R} i\gamma^\mu \left(\partial_\mu - i A_{L/R_\mu}^a T^a \right.$$
$$\left. - i B_\mu^A \hat{T}^A \mp i \frac{N}{3} \mathcal{A}_\mu \right) Q_{L/R}. \quad (5)$$

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[5] Another option is to introduce a scalar field H which transforms as $(\bar{3}, 3)$ under $SU(3)_L \times SU(3)_R$, carries $U(1)_{\mathrm{TAF}}$ charge $-2F/3$ ($F = N$ to insure gauge anomaly cancellations), and has the electroweak charges of the ordinary Higgs (i.e., it is a doublet under $SU(2)_L$). Then, quark masses result from an interaction of the form $H q\tilde{q}$. In this case $\langle H \rangle$ must be of order the electroweak scale [15], which is probably ruled out since the Goldstone boson would be detectable.

$A_{L/R}^a$ are the gauge bosons of the color left and color right gauge interactions with $a = 1, \ldots, 8$. B^A, with $A = 1, \ldots, N^2 - 1$ are the gauge bosons strong interaction. $\mathcal{A}$ is the gauge boson of the anomaly free $U(1)_{\text{TAF}}$ gauge symmetry. For the ordinary quarks the kinetic terms are:

$$\bar{\mathcal{Q}}_L i \gamma^\mu \left(\partial_\mu - i A_{L\mu}^a T^a + i \frac{N^2}{3F} \mathcal{A}_\mu - i W_\mu^b \tau^b \right) \mathcal{Q}_L,$$

$$\bar{q}_R i \gamma^\mu \left(\partial_\mu - i A_{R\mu}^a T^a - i \frac{N^2}{3F} \mathcal{A}_\mu \right) q_R. \qquad (6)$$

Here we have indicated for illustration the interactions with the electroweak gauge bosons W^a, but neglected the hypercharge gauge boson. F is the total number of quark flavors. To this Lagrangian one has to add the kinetic term for the new gauge bosons and the modified Yukawa interactions which lead to masses for the ordinary fermions. As mentioned earlier we are now forced to introduce another complex scalar field transforming under the left and right color transformations: $H_{c'}^c$. The resulting Yukawa interactions are:

$$-\frac{\lambda_d}{M} \bar{\mathcal{Q}}_L^{c'} \cdot \phi H_{c'}^c d_{R,c} - \frac{\lambda_u}{M} \epsilon^{\alpha\beta} \bar{\mathcal{Q}}_{L,\alpha}^{c'} \phi_\beta^\dagger H_{c'}^c d_{R,c} + \text{h.c.}$$
$$(7)$$

M is a scale related to the H sector of the theory while ϕ_α is the standard electroweak doublet $\alpha = 1, 2$. Other nicer ways of providing mass to the ordinary quarks can, of course, be explored—a more general structure of the Yukawa couplings in flavor space might be expected. With this choice of the Yukawa sector the leptonic sector of the standard model remains unmodified. It should also be clear that H must condense for the ordinary quarks to acquire a mass.

The $SU(N)$ gauge theory, being vector like, is free from gauge anomalies. Since we independently gauge $SU_{L,R}(3)$ we also need to cancel the associated gauge anomalies. The simplest way is to construct a vector like theory with respect to each gauge group. This can be easily achieved by setting $N = 6$, i.e., equal to the number of ordinary quark flavors. For $N = 6$ (recalling the 6 flavors of quarks q_f) we see that in each vertical column of the table corresponding to a non-Abelian group the particle content is vector like.

As for the summary of the dynamics we recall that our model has four independent scales. The scale of the $SU(N)$ strong dynamics Λ_N, the scale M of the

condensation of H, the electroweak scale and finally the ordinary QCD confining scale. We imagine the first two scales to be much larger than the electroweak scale. Below the $SU(N)$ confining scale, as explained in the previous section, $SU(3)_L \times SU(3)_R \times U(1)_{\text{TAF}}$ breaks spontaneously to $SU(3)_V$. We identify $SU(3)_V$ with ordinary color interactions. The effective low energy theory below the Λ_N scale—but remaining above the electroweak scale—is obtained by replacing the $SU(N)$ UV theory with its low energy chiral perturbation theory. The Goldstone modes (massless colored pions) become longitudinal components of the axial vector bosons and hence, in the end, disappear from the low energy theory. A similar fate is shared by the Goldstone boson associated to the gauged exact $U(1)_{\text{TAF}}$ symmetry. Since $T, Q, \tilde{Q}$ are massless we have no $SU(N)$ theta term. By construction Q carries ordinary color and we stress that, without invoking any chiral rotation of the ordinary quarks, the QCD θ term becomes unphysical (see next sections for a formal proof). This is the main point of our Letter.

At the electroweak scale, having already assumed the condensation of the field H at some energy M less than or of order the confining scale of $SU(N)$, one generates a mass term for the standard fermions via Yukawa interactions. In general the couplings are complex and one might be worried that they regenerate a strong CP phase at low energies. Although the formal proof is provided in a following section we can immediately argue that such a strong CP phase cannot appear. This is *independent* of the mechanism we use to give masses to the ordinary quarks. To demonstrate this we can first perform a non-Abelian chiral rotation which brings all the quarks to the same complex phase. Then we are left with an axial transformation which, due to the quark axial anomaly, potentially leads to a new strong CP phase. However we are free to perform an equivalent axial transformation of the Q quarks to offset the strong CP phase again.

Unfortunately, we were forced to introduce another field H which also carries a new phase. In the absence of the T and Q fields this would lead to a conventional axion field solution to the strong CP problem. However as we shall demonstrate below the would-be axion in our model is exactly massless. It is important to note that our mechanism for rotating away the strong CP phase does not require this extra H field

and the hope is that a model similar to ours can be constructed in which such a field is not needed.

4. The massless Goldstone boson

We now construct the low energy effective theory for the pseudoscalar particles associated with the axial $U(1)$ symmetries in our model. At energies above the electroweak scale we have the two condensates $\langle TT \rangle$ and $\langle Q\tilde{Q} \rangle$. We hence expect two independent pseudoscalars, one from each condensate, which are singlets of the $SU(3)_L \times SU(3)_R$ non-Abelian symmetries:

$$\langle TT \rangle = \left| \langle TT \rangle \right| e^{i\eta_T},$$
$$\det \langle Q\tilde{Q} \rangle = \left| \det \langle Q\tilde{Q} \rangle \right| e^{i\eta_Q}, \qquad (8)$$

by η_T we denote η_T/F_T and η_Q is η_Q/F_Q. The decay constants F_T and F_Q are comparable in size and of the order of the confining scale of the $SU(N)$ gauge theory. Since the confining scale of $SU(N)$ is much larger than the electroweak scale the massive scalar excitations will not appear in the low-energy theory. The instanton-induced effective potential, which preserves the $U(1)_{TAF}$ symmetry, is

$$V_{TAF} = c_1 \left| \langle TT \rangle \right|^N \left| \det \langle Q\tilde{Q} \rangle \right| e^{i(N_{\eta_T} + 3\eta_Q)} + \text{h.c.}, \quad (9)$$

with c_1 a constant. This potential determines the linear combination of pseudoscalars which becomes massive. The orthogonal combination remains massless. It is absorbed in the longitudinal component of the $U(1)_{TAF}$ gauge boson and hence decouples from the low energy physics. The $SU(N)$ θ angle is rendered an unphysical quantity since $T, Q, \tilde{Q}$ are exactly massless.

At much lower energies we include the effects of the ordinary quarks. To give mass to the fermions while preserving the $U(1)_{TAF}$ symmetry we introduced the field H. Note that, even in the presence of a mass term for the quarks, the QCD theta angle is unobservable, since the massless $T, Q, \tilde{Q}$ fields allow us to rotate away *both* the $SU(N)$ and QCD theta angles. (We discuss this issue further in the following subsections.) The phase of H combines with the phase of $q\tilde{q}$ yielding a massless Goldstone boson and a massive pseudo-Goldstone boson. The latter is the η' meson of QCD. To show this one can use either current

algebra techniques [13] or the effective Lagrangian approach [14]. We present here the effective Lagrangian approach. The two new pseudoscalars are related to the phase h of $\langle H \rangle$ and η_q of $\langle q\tilde{q} \rangle$. At very low energies, below the QCD scale, we deduce the following effective potential, where we restrict our attention to pseudoscalar fields (recall $F = 6$ is the number of quark flavors, not a decay constant):

$$V_{low} = c_2 \left| \langle qH\tilde{q} \rangle \right| e^{i(h+\eta_q)}$$
$$+ c_3 \left| \langle \det[qH\tilde{q}] \rangle \right| e^{iF(h+\eta_q)} + \text{h.c.} \qquad (10)$$

The last term is the ordinary instanton-induced fermion determinant. It is clear that the only combination which acquires a mass is $h + \eta_q$. Here h stands for $h/|\langle H \rangle|$ and η_q for η_q/F_π. The linear combination shown in the potential is the standard η' of QCD, while the orthogonal combination $h - \eta_q$ remains massless.

We stress that $h - \eta_q$ is a true massless Goldstone boson, not an axion of the usual type. While the QCD θ angle has become *dynamical*, in the form of the massless linear combination, the presence of the massless fermions $Q, \tilde{Q}$ and T has rendered physics independent of θ, and hence the potential for the massless combination flat. The result is similar to that of an axion model that *also* has a massless quark, except in this case it is the Q and T fields which play the role of the massless quark. In usual axion models, two different linear combinations of the axion field and η_q appear: one induced by the mass term of the quarks and the other due to instantons [14]. This leads to both a massive axion and a massive η', unlike in our model.

5. Axial anomalies

We have three axial currents, and only one is anomaly free with our charge assignments:

$$\partial_\mu J^\mu_{TAF} = 0,$$
$$\partial_\mu J^\mu_{QA} = \frac{1}{16\pi^2} \left[3F_{\mu\nu}\tilde{F}^{\mu\nu} + NG_{\mu\nu}\tilde{G}^{\mu\nu} \right],$$
$$\partial_\mu J^\mu_A = \frac{1}{16\pi^2} \left[6G_{\mu\nu}\tilde{G}^{\mu\nu} \right]. \qquad (11)$$

Here we have denoted with $F^{\mu\nu}$ the $SU(N)$ field strength while the associated gauge field is B^μ. In the
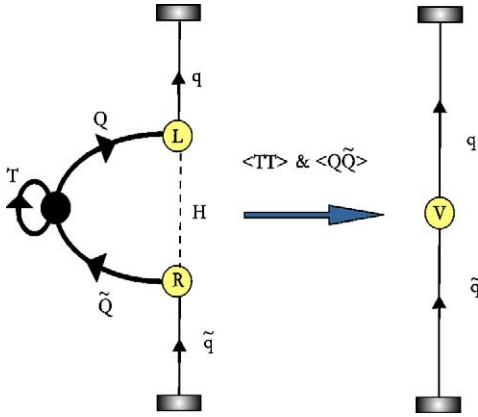
Fig. 1. Schematic representation of the model. The black node represents the $SU(N)$ gauge theory. The left (right) node indicates $SU(3)_{L(R)}$. After condensation of the T and Q fermions the V node is the QCD gauge group. The two squares represents the ungauged $SU(F)$ left and right QCD flavor groups.

anomaly equations $G_{\mu\nu}\tilde{G}^{\mu\nu}$ represents

$$G_{\mu\nu}\tilde{G}^{\mu\nu} = \frac{1}{2}\big(G_{\mu\nu_L}\tilde{G}^{\mu\nu}_L + G_{\mu\nu_R}\tilde{G}^{\mu\nu}_R\big), \qquad (12)$$

where $G^{\mu\nu}_{L,R}$ are the field strengths for the $SU(3)_{L,R}$ gauge groups, whose gauge fields are $A^{\mu}_{L,R}$.

At scales much below the $SU(N)$ confining scale Λ_N the axial gauge bosons can be integrated out. In this limit the field strength $G_{\mu\nu}$ appearing on the rhs of equations (11) is just the usual QCD gluon field strength.

The anomalous Ward identities (11) imply that axial rotations of the T field are equivalent to shifts of the $SU(N)$ θ angle (henceforth denoted θ'). Axial rotations of Q shift θ' as well as the $SU(3)_{L,R}$ angles denoted $\theta_{L,R}$, while q rotations only shift $\theta_{L,R}$. It is clear that we can eliminate θ' by appropriate T rotation. $\theta_{L,R}$ are discussed below.

6. Euclidean functional integral

The partition function of our model is

$$Z = \sum_{\mu\nu_L\nu_R} \int [DB]_\mu [DA_L]_\nu [DA_R]_{\nu'} e^{-S(A_L,A_R,B)}$$
$$\times \det Q \det T \det q \, e^{i\mu\theta' + i\nu_L\theta_L + i\nu_R\theta_R}, \qquad (13)$$

where the fermionic integrals have been performed leaving determinants of the respective Dirac operators.

The measure of the integral has been divided into winding number sectors, where the $SU(3)_{L,R}$ winding numbers are given by $\nu_{L,R} = \frac{1}{16\pi^2}\int d^4x\, G_{\mu\nu_{L,R}} \times \tilde{G}^{\mu\nu}_{L,R}(x)$, and the $SU(N)$ winding number is $\mu = \frac{1}{16\pi^2}\int d^4x\, F_{\mu\nu}\tilde{F}^{\mu\nu}(x)$. It is convenient to define:

$$\nu = \frac{1}{2}(\nu_L + \nu_R), \qquad \nu_A = \frac{1}{2}(\nu_L - \nu_R), \qquad (14)$$
$$\theta = \theta_L + \theta_R, \qquad \theta_A = \theta_L - \theta_R. \qquad (15)$$

The left–right symmetry of our model suggests that $\nu_A = 0$ implying that the physics is insensitive to θ_A. To arrive at the same conclusion we can use the following dynamical argument. Due to the Higgs mechanism, the axial gauge fields $A^{\mu}_L - A^{\mu}_R$ are heavy. At low energies their fluctuations are suppressed, or equivalently: $A^{\mu}_L = A^{\mu}_R$. Consequently, $\nu_L = \nu_R$, which implies that θ_A is an unphysical parameter.

We can then concentrate on the remaining angles: θ' and the usual QCD θ. Here we can use the index theorem relating the number of chiral zero modes of the T Dirac operator to the Pontryagin index:

$$n^T_+ - n^T_- = N\mu. \qquad (16)$$

For the Q operator, we have

$$n^Q_+ - n^Q_- = 3\mu + N\nu,$$

and for the q operator

$$n^q_+ - n^q_- = 6\nu. \qquad (17)$$

Here $n_\pm$ denotes the number of zero modes of chirality $\pm$. Because Q and T are massless, their determinants vanish whenever there are any zero modes—in other words, if *any* of the respective $n_\pm$ are non-zero. The condition of non-vanishing determinants requires $\mu = 0$ and $\nu = 0$. The only topological sectors that can contribute have $\mu = \nu = 0$. This means that the partition function is independent of both θ and θ'—they are unphysical parameters.

Note also that it is sufficient that T and Q are massless to completely rotate away θ and θ'. This is important since it allows, at low energies, the quarks to acquire a mass term without upsetting our results.

7. Conclusions

We described a class of models in which the QCD θ angle is rendered unobservable by new short dis-

tance physics involving a new strong force $SU(N)$. A generic prediction of the models is new colored particles and a non-standard Higgs structure for the quark masses. Interestingly we have an exactly massless pseudoscalar boson which may be relevant for cosmology and is similar to a Majoron, for which constraints from accelerators and astrophysics have been recently analyzed [17].

Acknowledgements

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