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Application of Monte Carlo Method in Tolerance Analysis

Huiwen Yan, Xiangji Wu, Jiangxin Yang*

The State Key Lab of Fluid Power Transmission and Control, Zhejiang University, Hangzhou, China;

* Corresponding author. Tel.: +86-571-87953198; fax: +86-571-87951145. E-mail address: yangjx@zju.edu.cn.

Abstract

Monte-Carlo simulation is the most popular and simplest method for nonlinear statistical tolerance analysis. Random values for every part are got according to the part distributions, and the value of the response function is computed for each set of part values. A sample of response function values is thus got, and the moments of the sample are computed using the standard statistical formula. At present, the researches on assembly tolerance analysis have been focused on dimension tolerance and hardly take geometric tolerance into consideration. In this paper, geometric tolerance is treated as dimension tolerance whose nominal value is zero, and the Monte Carlo Simulation Method is applied to tolerance analysis including geometric tolerance. Finally, a case of a top column assembly verifies the validity and accuracy of the method.

Keywords: Tolerance analysis; Geometric tolerance; Monte-Carlo Simulation Method;

1. Introduction

Tolerance analysis has become more and more important and has received considerable attention in manufacturing industries and thus in many literatures with increasingly fierce market competition. The final target of tolerance analysis is to not only improve the performance of product, but also reduce the cost of product.

So far, a large amount of researchers have devoted themselves to mathematical models and methods of tolerance analysis. Many well-known models or methods are introduced in several review literatures for tolerance analysis [1-4]. The variational model has its roots in parametric geometric modeling, where geometry can be modelled by mathematical equations that allow shape and size attributes to be changed and controlled through a reduced set of parameters [5-6]. The vector loop model adopts a graph-like schematization where any relevant linear dimension in the assembly is represented by a vector, and an associated tolerance is represented as a small variation of such a vector [7-11]. The tolerance map (T-Map) represents all possible variations of size, form, position, and orientation for a target feature [12]. The small displacement torsor (SDT) model uses six small displacement vectors to represent the position and orientation of an ideal surface in relation to another ideal surface in a kinematic way [13-15]. The matrix model aims at deriving the explicit mathematical representation of geometry of each tolerance region through displacement matrices [16-18].

Monte Carlo simulation [19-20] is probably the simplest statistical tolerance analysis method and it takes into consideration the probabilistic behaviour of the manufacturing process. Its general procedure is:

1. Use a generator to randomly generate n sets of manufactured dimensions in an assembly with specified component distributions.
2. Get a sample of assembly functions employing the n sets of manufactured dimensions.
3. Estimate the assembly performance parameters, such as mean, standard deviation and reject rate of the assembly.

So far few research at home and abroad on effect of geometric tolerance on assembly quality, and the geometric tolerance may have great effect in some cases, resulting from rigid body effects.

This paper aims at including geometric tolerance in the assembly tolerance analysis with the Monte Carlo simulation method. The paper is organized as six parts. In the following part, geometric tolerance is illustrated. The effect of geometric tolerance on the tolerance analysis is discussed in Section 3; Section 4 will discuss how to include geometric tolerance of components in Monte Carlo simulations. In the Section 5, a case of a top column assembly verifies the validity and accuracy of the method. The conclusions are summarized in the Section 6.
2. Geometric Tolerance

The geometric tolerance can be defined into four main groups, i.e., form tolerance, orientation tolerance, location tolerance, and runout tolerance, according to GB/T 1182-2008[21].

1. A form tolerance states how far an actual surface or feature is permitted to vary from the desired form implied by the drawing. It includes straightness, flatness, circularity, profile of a line and profile of a surface.

2. An orientation tolerance states how far an actual surface or feature is permitted to vary relative to a datum or datums. It consists of parallelism, perpendicularity, angularity, profile of a line and profile of a surface.

3. A location tolerance states how far an actual size feature is permitted to vary from the perfect location implied by the drawing as related to a datum, ordatums, or other features. This category includes position, concentricity, Symmetry, profile of a line and profile of a surface.

4. A runout tolerance states how far an actual surface or feature is permitted to vary from the desired form implied by the drawing during full (360°) rotation of the part on a datum axis. A runout can be either a circular runout or a total runout. Tolerance zone is the space limited by one or several geometrically perfect lines or surfaces, and characterized by a linear dimension, called a tolerance.

According to the characteristic to be tolerated and the manner in which it is dimensioned, the tolerance zone is one of the following:

1) the space within a circle;
2) the space between two concentric circles;
3) the space between two equidistant lines or two parallel straight lines;
4) the space within a cylinder;
5) the space between two coaxial cylinders
6) the space between two equidistant surfaces or two parallel planes;
7) the space within a sphere.

3. The Effect of Geometric Tolerance on the Tolerance Analysis

There are three main sources of variations in a mechanical assembly. Two of them are the result of the natural variation in manufacturing processes and the third is from assembly processes and procedures. These three sources are

1) dimensional variation,
2) geometric feature variation and
3) variation due to kinematic adjustments at assembly time.

The geometric feature tolerance can be treated as dimension tolerance whose nominal value is zero and added to the related contact surfaces. And its direction is mainly determined by the kinematic joint type and the geometric characteristic. The effect of the geometric feature tolerances associated with each of the joins may result in translational variation or rotational variation.

Fig. 1 illustrates how a flatness tolerance zone of the low plane can affect two mating parts differently when viewed in 2-D. The cylinder on the left illustrates a translational variation, while the block on the right exhibits the rotational variation, due to the same geometric feature variation. The translational variation for the cylinder in Fig. 1 is related to the flatness tolerance, while the rotational variation for the block is determined by not only the flatness tolerance but the contact length of the block, in this case, the horizontal dimension of the block.

\[
\Delta \alpha = \pm \frac{1}{2} T
\]

\[
\Delta \beta = \pm \tan^{-1}\left(\frac{T}{L}\right)
\]

where T is the flatness tolerance, L is the contact length of the block, and the rotational variation caused by the flatness in a planar joint, and \( \Delta \beta \) is the rotational variation resulted from the flatness in a planar joint and the contact length of the block.

4. Components with Geometric Tolerance in Monte Carlo Simulations

The effect of geometric feature tolerances can be treated as dimension tolerances whose value can be got by the Formula (1) or (2). So the Monte Carlo Simulation can be used to estimate the effect of both the dimensional tolerances and geometric feature tolerances.
The traditional relationship between the input parameters and output variables in a mechanical assembly can be rewritten as:

\[ Y = f(X_1, \ldots, X_m, X_{m+1}, \ldots, X_{m+n}) \]  

(3)

where the \( X_1, \ldots, X_m \), the manufactured dimension, are random variables, typically derived from the drawing dimensions, the \( X_{m+1}, \ldots, X_{m+n} \) representing the effect of geometric feature variations are also random variables and \( Y \), the output variable, is the assembly function to be controlled. In this paper, all the \( X_i \) are assumed to be independent and follow Gaussian distributions. After a sample of \( Y \) is got, the first moments of \( Y \) can be determined by the following equations:

\[ \bar{Y} = \frac{1}{n} \sum_{i=1}^{n} Y_i \]  

(4)

\[ \sigma_1^2 = \frac{1}{n} \sum_{i=1}^{n} (Y_i - \bar{Y})^2 \]  

(5)

\[ \mu_3 = \frac{1}{n} \sum_{i=1}^{n} (Y_i - \bar{Y})^3 \]  

(6)

\[ \mu_4 = \frac{1}{n} \sum_{i=1}^{n} (Y_i - \bar{Y})^4 \]  

(7)

\[ \beta_1 = \frac{\mu_3}{\sigma^3} \]  

(8)

\[ \beta_2 = \frac{\mu_4}{\sigma^4} \]  

(9)

where the \( \beta_1 \) and \( \beta_2 \) are the coefficients of skewness and kurtosis respectively. Once the moments have been got, we may compute a tolerance range for the response function \( Y \) that encompasses a given fraction of the assembly yield, or we may compute the acceptance fraction for a given tolerance range.

5. Case Study

As an example[22] to demonstrate how to include geometric feature tolerances in the variation estimation of kinematic or assembly variables and prediction of assembly rejects employing the Monte Carlo Simulation Method, a top column assembly is studied. Fig. 2 shows the assembly with geometric feature tolerance. In this case study, effort will be focused on the entire top column assembly model which is used to mount. The assembly consists of a handle which press on the column. The design requirement for the assembly is to have adequate interference between the column and the box. So the critical feature is the magnitude of the variation on dimension \( r_3 \) when the tolerances of the components including geometric feature variations are considered.

\[ r_3 = r_5 + r_6 + \alpha_1 + \alpha_2 + \alpha_3 + (\alpha_4 + \gamma) \cos(\theta) - r_7 - r_2 \]  

(10)

Then adopt the simulation method, and its basic steps are illustrated in the following:

(1) Set the sample size \( n \).

(2) Generate \( n \) sets of manufactured dimensions and geometric dimensions. For each manufactured dimension or geometric dimension following Gaussian distribution \( N(\text{mean}, \sigma) \), mean is its nominal dimension and sigma is one sixth of its tolerance.

(3) Get a sample of assembly functions. For each set of manufactured dimension and geometric dimension, an assembly function can be obtained using the Formula(10).

(4) Estimate the assembly performance parameters, such as mean, standard deviation, employing the Formula (4-9).

(5) Change the sample size \( n \), and repeat the step (2-5) until the sample size \( n \) is large enough to guarantee the analysis accuracy. So The relationships between first four order moments and sample size \( n \) can be obtained as shown in the Fig. 3. According to the Fig. 3, each relationship is periodically oscillating centered at a value. So we can set first four order moments equal to their mean values. The result is listed as below:

- Mean=34.943 mm
- Standard deviation=0.0694 mm
- Coefficient of Skewness=0.0017

Table 2 shows the geometric information for the assembly. Dimensions with a given tolerance are the manufactured variables. Table 3 lists all the geometric feature tolerances applied.

<table>
<thead>
<tr>
<th>Symbol</th>
<th>Nominal Dim</th>
<th>Tolerance (±)</th>
</tr>
</thead>
<tbody>
<tr>
<td>t1</td>
<td>30mm</td>
<td>0.05mm</td>
</tr>
<tr>
<td>t2</td>
<td>118mm</td>
<td>0.14mm</td>
</tr>
<tr>
<td>t3</td>
<td>155mm</td>
<td>0.10mm</td>
</tr>
<tr>
<td>t6</td>
<td>5mm</td>
<td>0.02mm</td>
</tr>
<tr>
<td>t7</td>
<td>40mm</td>
<td>0.15mm</td>
</tr>
<tr>
<td>α</td>
<td>55</td>
<td>10</td>
</tr>
</tbody>
</table>

Table 3. Geometric feature data of the top column assembly.

<table>
<thead>
<tr>
<th>Symbol</th>
<th>Name</th>
<th>Nominal Dim</th>
<th>Tolerance Band</th>
</tr>
</thead>
<tbody>
<tr>
<td>α1</td>
<td>Profile</td>
<td>0</td>
<td>0.040</td>
</tr>
<tr>
<td>α2</td>
<td>Planar</td>
<td>0</td>
<td>0.030</td>
</tr>
<tr>
<td>α3</td>
<td>Roundness</td>
<td>0</td>
<td>0.024</td>
</tr>
<tr>
<td>α4</td>
<td>Concentricity</td>
<td>0</td>
<td>0.050</td>
</tr>
</tbody>
</table>

The assembly function can be written as below according to Fig. 2:

\[ r_3 = r_5 + r_6 + \alpha_1 + \alpha_2 + \alpha_3 + (\alpha_4 + \gamma) \cos(\theta) - r_7 - r_2 \]  

(10)
Coefficient of Kurtosis=3.0017

(6) Obtain the results of tolerance analysis. Because the coefficient of skewness of $r_3$ is close to zero and the coefficient of kurtosis of $r_3$ is close to three, the distribution of $r_3$ can be regarded as normal distribution and the tolerance of $r_3$ can be set to be 6 standard deviation.

Table 4. The result of tolerance analysis.

<table>
<thead>
<tr>
<th>Symbol</th>
<th>Nominal Dimension</th>
<th>Upper Deviation</th>
<th>Low Deviation</th>
<th>Tolerance</th>
</tr>
</thead>
<tbody>
<tr>
<td>$r_3$</td>
<td>35mm</td>
<td>-0.265mm</td>
<td>+0.151mm</td>
<td>0.416mm</td>
</tr>
</tbody>
</table>

Fig. 3. The relationships between first four order moments and sample size $n$.

6. Conclusion

In this paper, assembly tolerance analysis including geometric tolerance is studied. There are three main sources of variations in a mechanical assembly. Two of them are the result of the natural variation in manufacturing processes and the third is from assembly processed and procedures. These three sources are 1) dimensional variation, 2) geometric feature variation and 3) variation due to kinematic adjustments at assembly time. Geometric tolerance is treated as dimension tolerance whose nominal value is zero, and the Monte Carlo Method is applied to tolerance analysis including geometric tolerance.

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References


