Higher order corrections to color superconducting gaps

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Abstract

We find a (nonlocal) gauge where the wavefunction renormalization constant does not get any corrections for all momenta in the hard-dense loop approximation. In this gauge, we solve the Schwinger–Dyson equations for the diquark condensate in dense QCD to calculate the Cooper-pair gap. We determine not only the exponent but also the prefactor of the gap in a gauge independent way. We find that the higher order corrections increase the gap only by about 1.6 times to the leading order gap at Coulomb gauge.
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Matter at extreme density is known to be a color superconductor [1], quarks forming Cooper pairs to open a gap at the Fermi momenta. The properties of color superconductor are intensively studied recent years at asymptotic density [2], where quarks interact weakly due to asymptotic freedom in quantum chromodynamics (QCD). However, the study beyond the leading order has been hindered by calculational difficulties.

Since superconductors are characterized by the Cooper-pair gap or the minimum energy to excite a pair of a quark and a hole at the Fermi momenta, it is quite important to determine the size of the gap accurately. By solving the Schwinger–Dyson (SD) equations, the gap at the asymptotic density has been found to be

$$\Delta_0 \simeq b \frac{\mu}{g_s^5} \exp\left(-\frac{c}{g_s}\right), \tag{1}$$

where the $1/g_s$ power in the exponent is due to the long-range interaction in dense QCD. The constant c in the exponent is gauge-independent and is determined by the long-range interaction mediated by the magnetic gluons, unscreened at high baryon density. In the hard-dense loop (HDL) approximation, it is known to be $c = 3\pi^2/\sqrt{2}$ [3]. The short-range interaction mediated by the Debye-screened electric gluons contributes only to the prefactor. Among the negative fifth power of the coupling in the prefactor, the third is due to the screened electric gluons,

while the magnetic gluons contribute the remaining. In the leading order analysis of the SD equation [4,5] the numerical prefactor b is found to be

$$b = 2^7 \pi^4 \left(\frac{2}{N_f} \right)^{5/2} e^{3\xi/2+1},$$

(2)

where N_f is the number of quark flavors and ξ is the gauge-fixing parameter.

The result Eq. (2) is gauge-dependent, since all subleading contributions that lead to logarithmic divergences as in the BCS superconductivity are not taken into account. Being a physical observable, the gap Δ_0 should be gauge-independent. However, if one sums the contributions partially, the result is often gauge-dependent. We therefore need to calculate the subleading corrections to obtain the gauge-independent gap. Some of the subleading corrections like the finiteness of the quasiquark life in medium [6], the running effect of the strong coupling [7], and the quark self-energy [8,9] have been studied.

In this Letter we determine accurately the contributions to the prefactor, coming from the vertex corrections and the wavefunction renormalization for quarks, since these are the remaining gauge-dependent contributions to the prefactor and, if summed up, we should get a gauge-independent prefactor.

We first try to find a (nonlocal) gauge [10] where the quark wavefunction is not renormalized for all momenta, $Z(p) = 1$. In this gauge we then solve the SD equations to find the Cooper-pair gap, paying special attention to the Ward–Takahashi identity. This way of calculating higher order corrections in the SD analysis has been proved extremely useful in dynamical mass generation of $(2 + 1)$-dimensional quantum electrodynamics and in others, since the higher order corrections vanish exactly [11,12].

Unlike ordinary electron superconductors, without additional interaction the quark–quark scattering is attractive in the color antitriplet channel, where the color flux energy is lowered. By the Cooper theorem, diquark operators of opposite momenta therefore develop a condensate in quark matter, breaking the color gauge symmetry,

$$\langle \psi(x) \bar{\psi}_c(x) \rangle = K(p_F),$$

(3)

where the charge-conjugated field is defined as $(\psi_c)_i(x) = C_{ij} \bar{\psi}_j(x)$. The matrix C satisfies $C^{-1} \gamma_\mu C = -\gamma_\mu^{\mathrm{T}}$ and i, j are Dirac indices.

To calculate the condensate, we introduce a Nambu–Gorkov field $\Psi(x) \equiv (\psi(x), \psi_c(x))^{\mathrm{T}}$. The inverse propagator for the Nambu–Gorkov field is then given as

$$S^{-1}(p) = -i \begin{pmatrix} a(p)[(p_0 + \mu)\gamma^0 + b(p)\vec{p}] & -\Delta(p) \\ -\gamma^0 \Delta^\dagger(p)\gamma^0 & a(p)[(p_0 - \mu)\gamma^0 + b(p)\vec{p}] \end{pmatrix},$$

(4)

where $a(p), b(p)$ denote the wave function renormalization constants and $\Delta(p)$ denotes the Cooper-pair gap at a momentum p. Notice that the time and spatial components of quark wavefunction renormalize differently since the Lorentz symmetry is broken in quark matter.

We study the Schwinger–Dyson equation for the Nambu–Gorkov propagator, following the notations in [4]. The gap equation in dense QCD takes a following form (see Fig. 1)

$$\Delta(p_0) = \frac{g_s^2}{c^2} \int dq_0 \frac{\Delta(q_0)}{\sqrt{q_0^2 + \Delta^2}} \left[(1 + \eta) \ln \left(\frac{\mu}{|p_0 - q_0|} \right) + \ln b + \zeta \right].$$
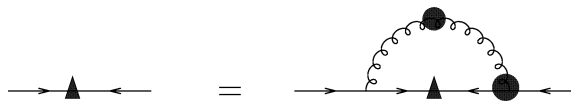
(5)

Fig. 1. The triangle denotes the Cooper-pair gap and the blobs higher-order corrections.
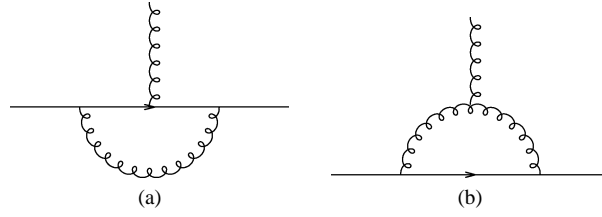
Fig. 2. The solid line denotes quarks and the curly lines gluons.

The first term in Eq. (5) comes from the magnetic gluons and gives the leading contribution. The constant term, $\ln b$, is due to the Debye screened electric gluons and the gauge fixing terms. The higher-order terms, denoted as η and ζ, are in general given in powers of coupling constant and energy as $g_s^l \left[\frac{\Delta, q_0}{\mu}\right]^m \left[\ln\left(\frac{\Delta, q_0}{\mu}\right)\right]^n$. In our calculation $q_0 \sim \Delta$. The higher-order corrections are in general suppressed by powers of coupling constant. However, since the gap depends on the coupling, the logarithmic corrections might make the higher-order corrections comparable to the leading term. We examine the two-loop corrections to the gap equation in detail.

In general, the SD equations are infinitely coupled equations for the 1PI functions. However, some of them are related by Ward–Takahashi (WT) identities. The identity we are going to use in solving the gap equation is one that relates the quark two-point functions with the vertex functions,

$$\partial_z^\mu \langle j_\mu^a(z)\psi(x)\bar{\psi}(y)\rangle = \langle \partial^\mu j_\mu^a(z)\psi(x)\bar{\psi}(y)\rangle - \delta(z-x)\langle T^a\psi(x)\bar{\psi}(y)\rangle + \delta(z-y)\langle \psi(x)\bar{\psi}(y)T^a\rangle. \tag{6}$$

The identity reads in the momentum space as

$$\begin{aligned}
&T^a a(p)\left[(p_0+\mu)\gamma^0 + b(p)\vec{\not{p}}\right] - a(p')\left[(p'_0+\mu)\gamma^0 + b(p')\vec{\not{p}}'\right]T^a \\
&= (p-p')_\mu \Lambda^\mu(p,p')T^a + \Gamma^a(p,p';-p-p'),
\end{aligned} \tag{7}$$

where

$$\Gamma^a(p,p';k)\delta(k+p+p') = \int_{z,x,y} e^{i(z\cdot k + x\cdot p + y\cdot p')}\langle \partial^\mu j_\mu^a(z)\psi(x)\bar{\psi}(y)\rangle. \tag{8}$$

The one-loop vertex correction has two parts (see Fig. 2).

As in quantum electrodynamics, the first part (Fig. 2(a)) is related to the correction to the wavefunction renormalization constant as

$$(p-p')_\mu \Lambda^{(a)\mu}(p,p') = a(p)\left[(p_0+\mu)\gamma^0 + b(p)\vec{\not{p}}\right] - a(p')\left[(p'_0+\mu)\gamma^0 + b(p')\vec{\not{p}}'\right],$$

where we suppressed the color indices. We see that $\Lambda^{(a)\mu} = \gamma^\mu$ for the nonlocal gauge where the wavefunction renormalization constants are $a(p) = 1 = b(p)$. (This result is similar to the condition obtained by Kugo and Mitchard in the SD analysis of chiral symmetry breaking in QCD, using vector Ward identities [13].) Therefore, if we find a gauge where $a(p) = 1 = b(p)$, not only the corrections from the wavefunction renormalization but also those from the vertex corrections from Fig. 2(a) are absent. This simplifies significantly the calculation in the higher order corrections to the Cooper-pair gap, since the vertex correction to the gap, coming from the second diagram, Fig. 2(b), does not contribute to the prefactor, b, in Eq. (1). To see this, we note the leading contributions to the vertex comes from when both the internal and the external gluon lines are magnetic. Furthermore, the main contribution to the gap occurs when the external lines carry small momenta. For small external gluon and quark momenta $(q, |\vec{p}| - \mu, |\vec{p}'| - \mu \sim \Delta)$, the vertex correction due to the diagram Fig. 2(b) is given as, using the high density effective theory of QCD [5],

$$I_\mu^a \simeq g_s^3 \int_l \frac{l\cdot V}{l_\parallel^2 + \Delta^2} \frac{\gamma_0 f^{abc}T^bT^c[(c_1 l_\parallel^i + c_2 l_\perp^i + c_3 p^i + c_4 p'^i)g_{\mu i} + c_5(2l-p-p')_\mu]}{[|\vec{l}-\vec{p}|^2 + \pi M^2|l_0 - p_0|/(2|\vec{l}-\vec{p}|)][|\vec{l}-\vec{p}'|^2 + \pi M^2|l_0 - p_0'|/(2|\vec{l}-\vec{p}'|)]}, \tag{9}$$

where c_1 and c_5 are $1 + O(\Delta^2/l_\perp^2)$ while all other c_i's are $O(\Delta^2/l_\perp^2)$. The screening mass M is given as $g_s\mu\sqrt{N_f}/(2\pi)$ for N_f light quarks in the hard-dense-loop (HDL) approximation. Therefore, we find upon integration the correction becomes

$$I_\mu^a \sim i g_s^3 \gamma_0 V_\mu T^a \left(\frac{\Delta}{M}\right)^2 \ln(\Delta/\mu), \tag{10}$$

which is indeed negligible compared to the constant term, $\ln b$, in the gap equation (5).

Now, we look for a nonlocal gauge where the wave function constants remain unrenormalized, $a(p) = 1 = b(p)$. At this gauge, the vertex is bare, $\Lambda^\mu = \gamma^\mu$, and the SD equation leads to an equation for the (nonlocal) gauge-fixing parameter, given as

$$\int d^4q \, D_{\mu\nu}(q-p) \operatorname{Tr}\left(\gamma^0 \gamma^\mu \Lambda_q^+ \gamma^0 \gamma^\nu \Lambda_p^-\right) \frac{(q_0 + |\vec{q}| - \mu)}{q_0^2 - (|\vec{q}| - \mu)^2 - \Delta^2} = 0, \tag{11}$$

where the antiquarks are projected out by the positive and negative energy state projectors, defined as

$$\Lambda_p^\pm \equiv \frac{1}{2}\left(1 \pm \frac{\vec{\alpha} \cdot \vec{p}}{|\vec{p}|}\right). \tag{12}$$

The (Higgsed) gluon propagator is given in the HDL approximation as[1]

$$D_{\mu\nu}(k) = A O_{\mu\nu}^{(1)} + B O_{\mu\nu}^{(2)} + C O_{\mu\nu}^{(3)}, \tag{13}$$

where in the weak coupling limit, $|k_0| \ll |\vec{k}|$,

$$A = \frac{|\vec{k}|}{|\vec{k}|^3 + M_0^2\Delta + \pi M^2 |k_4|/2}, \qquad B = \frac{1}{k_4^2 + \vec{k}^2 + 2M^2}, \qquad C = \frac{\xi}{k_4^2 + \vec{k}^2}, \tag{14}$$

where M_0 is Higgs-like gluon mass $\sim g_s\mu/(2\pi)$ [4]. The polarization tensors are defined as [4]

$$O^{(1)} = P^\perp + \frac{(u \cdot k)^2}{(u \cdot k)^2 - k^2} P^u, \qquad O^{(2)} = P^\perp - O^{(1)}, \qquad O^{(3)} = P^\parallel, \tag{15}$$

where $u_\mu = (1, \vec{0})$ and

$$P_{\mu\nu}^\perp = g_{\mu\nu} - \frac{k_\mu k_\nu}{k^2}, \qquad P_{\mu\nu}^\parallel = \frac{k_\mu k_\nu}{k^2}, \qquad P_{\mu\nu}^u = \frac{k_\mu k_\nu}{k^2} - \frac{k_\mu u_\nu + u_\mu k_\nu}{(u \cdot k)} + \frac{u_\mu u_\nu}{(u \cdot k)^2} k^2. \tag{16}$$

Contracting the tensors, we get

$$O_{\mu\nu}^1 \operatorname{Tr}\left(\gamma^0 \gamma^\mu \Lambda_q^+ \gamma^0 \gamma^\nu \Lambda_p^-\right) = -2(1+t)\frac{q^2 + p^2 - qp(1+t)}{q^2 + p^2 - 2qpt},$$

$$O_{\mu\nu}^2 \operatorname{Tr}\left(\gamma^0 \gamma^\mu \Lambda_q^+ \gamma^0 \gamma^\nu \Lambda_p^-\right) = 1 - t,$$

$$O_{\mu\nu}^3 \operatorname{Tr}\left(\gamma^0 \gamma^\mu \Lambda_q^+ \gamma^0 \gamma^\nu \Lambda_p^-\right) = -(1-t)\frac{(p+q)^2}{q^2 + p^2 - 2pqt}, \tag{17}$$

where $p = |\vec{p}|$, $q = |\vec{q}|$, and $t = \vec{p} \cdot \vec{q}/(pq)$.

Now, we assume the (nonlocal) gauge fixing parameter $\xi(k)$ has only temporal dependence, that is, $\xi(k) \simeq \xi(k_0)$, and would like to have (11) satisfied already at angular integration. This gives

$$D + E + F = 0, \tag{18}$$

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[1] For two flavor color superconductors, some of the gluons are not Higgsed. However, since the Meissner effect is subleading, its inclusion does not affect our calculation.

where

$$D = q^2 \int\limits_{-1}^{1} dt\, A(p_4 - q_4, \vec{p} - \vec{q}) \left[-2(1+t) \frac{q^2 + p^2 - qp(1+t)}{q^2 + p^2 - 2qpt} \right]$$

$$\approx \frac{2}{3} \ln \left[\frac{(2\mu)^3}{(q-p)^3 + M_0^2 \Delta + \pi M^2 |p_4 - q_4|/4} \right],$$

$$E = q^2 \int\limits_{-1}^{1} dt\, B(p_4 - q_4, \vec{p} - \vec{q})(1-t) \approx 1,$$

$$F = q^2 \int\limits_{-1}^{1} dt\, C(p_4 - q_4, \vec{p} - \vec{q}) \left[-(1-t) \frac{(p+q)^2}{q^2 + p^2 - 2pqt} \right] \approx -\xi \ln \left[\frac{(2\mu)^2}{(p-q)^2 + |p_4 - q_4|^2} \right]. \tag{19}$$

In this calculation, we take $p, q \simeq \mu$, since the quarks are near the Fermi surface. We find the solution of Eq. (18)

$$\xi \approx \frac{\frac{2}{3} \ln \frac{(2\mu)^3}{M_0^2 \Delta + \pi M^2 |p_4 - q_4|/2}}{\ln \frac{(2\mu)^2}{|p_4 - q_4|^2}} \approx \frac{1}{3}, \tag{20}$$

with $|p_4 - q_4| \sim \Delta$.

Plugging Eq. (20) into the leading-order expression for the gap Eq. (2), we get

$$\Delta_0 = 2^7 \pi^4 \left(\frac{2}{N_f} \right)^{5/2} e^{3/2} \exp \left(-\frac{3\pi^2}{\sqrt{2}\, g_s} \right). \tag{21}$$

We see that the vertex and the wavefunction correction increase the gap by about two thirds, compared to the leading-order gap at the Coulomb gauge [14,15],

$$\Delta_0 = e^{0.5} \Delta_0(\xi = 0). \tag{22}$$

In conclusion, we find that the wavefunction renormalization vanish at a nonlocal gauge-fixing parameter $\xi \simeq 1/3$. By the Ward–Takahashi identity, the QED-like vertex correction vanishes at this gauge as well. Since the remaining one-loop vertex corrections are suppressed for small external momenta, which has a major contribution to the gap equation, both the wavefunction correction and the vertex correction to the prefactor of the gap vanishes at this nonlocal gauge. As the gap is independent of the choice of gauge, we conclude that the next-to-leading order corrections due to the vertex renormalization and the wavefunction renormalization increase the leading-order gap at the Coulomb gauge by about two thirds.

Acknowledgements

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