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Analysis of the dependence of the tensile behaviour of a short fibre reinforced polyamide upon fibre volume fraction, length and orientation

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Abstract

The tensile properties of different grades of a short fibre reinforced polyamide, having various contents of glass fibres, were analysed. The fibre length distributions, obtained by optical microscope observations of the fibres extracted from the matrix, were statistically analysed and compared. The tensile behaviour of the different materials was modelled by applying micro-mechanical models, taking into account the fibre length distributions. The effect of the fibre content on the fibre orientation distribution is discussed on the basis of the analysis of the reconstructions of the internal fibre structure by micro tomography using synchrotron light. The relationship between the degree of anisotropy and the orientation factors used in the models is discussed.

Keywords: Short fibre reinforced polymers; modelling; fibre length; micro-tomography

1. Introduction

The tensile behaviour of injection moulded, short fibre reinforced polymers (SFRP) mainly depends upon fibre volume fraction. However, the relationship between the mechanical properties of the composite and those of the fibres, the matrix and interfaces is complex and it also depends upon the statistical distributions of the fibre lengths and the fibre orientations. The existing models take into account these factors in different manners. Some models [1] take into account the full fibre length and orientation distributions. However, the latter is considerably more difficult to obtain than the fibre length distribution. Therefore other models, like the Bowyer-Bader approach [2] are more often applied because consider only the fibre length distribution and summarize the effect of fibre orientation by orientation factors. The Bowyer-Bader model is an evolution of the Kelly-Tyson [3] model, and was further improved by
Thomason [4, 5], who proposed a method for easily identifying of the micro-mechanical parameters and the fibre orientation factor.

In this paper we investigated the relationship between the fibre content, varying from 10% to 30% by weight, and the tensile properties of a short glass fibre reinforced polyamide. By analysing the fibre length distribution and by applying the Thomason model, the micro-mechanical parameters of the Bowyer-Bader and the orientation factors were identified and the tensile modulus and strength predicted. Similarly, the tensile modulus only was predicted by applying the Cox model. The assumption of a constant value of the orientation factor, equal for all grades of reinforced polyamide considered in this study, was discussed in the light of the analysis of the internal fibre micro-structure, as obtained by applying a method [6] we developed for the analysis of samples micro tomographic reconstructions.

2. Experimental

Four grades of polyamide 6, unreinforced (PA6) and reinforced with various contents of short glass fibres (10%, 20% and 30% by weight, 4.68%, 9.95% and 15.92% by volume, identified as PA6GF10, PA6GF20 and PA6GF30, respectively) were tested and analysed. Standard ISO 527-2 type 1A specimens were injection moulded and tested in the Dry As Moulded (DAM) state. Samples were extracted from the specimens for the analysis of the fibre lengths and for micro-tomography.

2.1. Tensile tests

Tensile tests were conducted using an MTS Alliance RF150 of 150 kN capacity, equipped with an additional load cell of 10 kN for higher accuracy. Tests were conducted at constant crosshead speed (5 mm/min) and strain measured using a 50 mm base extensometer. Stress-strain curves are reported in Fig.1. The relevant properties of tensile curves are reported in Table 1.

Table 1. Tensile properties of the different grades of polyamide

<table>
<thead>
<tr>
<th>Material</th>
<th>$E$, Young’s Modulus (MPa)</th>
<th>$\sigma_{\text{M}}$, Maximum stress (MPa)</th>
<th>$\varepsilon_{\text{b}}$, Strain at break (%)</th>
</tr>
</thead>
<tbody>
<tr>
<td>PA6</td>
<td>2760</td>
<td>61</td>
<td>141</td>
</tr>
<tr>
<td>PA6GF10</td>
<td>4640</td>
<td>100</td>
<td>3</td>
</tr>
<tr>
<td>PA6GF20</td>
<td>6590</td>
<td>134</td>
<td>4</td>
</tr>
<tr>
<td>PA6GF30</td>
<td>9025</td>
<td>154</td>
<td>4</td>
</tr>
</tbody>
</table>

Fig. 1. Tensile stress-strain curves of the four grades of reinforced polyamide (plot of the PA6 curve interrupted)
2.2. Analysis of fibre length distributions

Fibre samples were obtained from specimens by hot hydrolysis of the matrix. The fibres were then separated by filtration, washed and dried using an air circulating oven. Images of about 1000 fibres for each material sample were digitally acquired using an optical microscope and processed using image analysis software. Fibre length values were statistically analysed, by applying a method described in [7]. The fibre length distributions thus obtained are reported in the histograms of Figure 2. Values of \( L_w \), the weight-average fibre length, are reported below each histogram. It clearly appears that fibres are shorter as the fibre content increases, presumably because of fibre breaking during injection moulding.

\[
\begin{align*}
\text{(a)} & : L_w = 473 \, \mu m \\
\text{(b)} & : L_w = 354 \, \mu m \\
\text{(c)} & : L_w = 276 \, \mu m 
\end{align*}
\]

Fig. 2. Histograms of the fibre length distributions of: (a) PA6GF10, (b) PA6GF20 and (c) PA6GF30

2.3. Micro tomography

Material samples of dimensions 3 x 4 x 10 mm\(^3\), extracted from the central part of the tensile specimens (cross section is 4 x 10 mm\(^2\)), were analysed using the method described in [6]. The internal structure was reconstructed by micro-tomography at the SYRMEP beamline of Trieste Synchrotron. Several Volumes of Interests (VOI) of 80 x 80 x 80 voxel\(^3\) size (approximately 0.7 x 0.7 x 0.7 mm\(^3\)), whose position in the cross section is shown in Fig. 3, were analysed using the Quant3D software [8], which calculates the values of the Mean Intercept Length (MIL) and provides the components of the MIL fabric tensor.

\[
\text{The MIL fabric tensor is a second order, symmetric tensor. Its first eigenvector defines the direction of preferred fibre orientation and the eigenvalues (} T_1, T_2, T_3, \text{ in descending order) can be used to define an index of anisotropy } I_A, \text{ which is a measure of the distribution around the preferred orientation (} I_A = 0 \text{ randomly distributed, } I_A = 1 \text{ for perfectly aligned fibres)}
\]

\[
I_A = 1 - T_3/T_1 \tag{1}
\]
The first eigenvectors was always found parallel to the longitudinal axis of the specimen (i.e. in flow direction). Values of $IA$ for the three grades of reinforced PA6 are reported in the graphs of Fig.4. The average value across the sample section is reported below each graph. It clearly appears that the fibre volume fraction affects the values of $IA$, thus suggesting an increasing degree of alignment of the fibres along the longitudinal axis with increasing fibre content.

$$IA_{\text{average}} = 0.36 \quad IA_{\text{average}} = 0.51 \quad IA_{\text{average}} = 0.53$$

Fig. 4. Contour plots of the values of $IA$ of: (a) PA6GF10, (b) PA6GF20 and (c) PA6GF30

### 3. Modelling and discussion

The Bowyer-Bader model relates the tensile strength of a SFRP composite to the fibre length distribution, through the relationship

$$\sigma_M = \eta_0 \left( \sum_{i=0}^{L_c} \frac{l_i v_i}{d} + \sum_{i=L_c}^{L_s} \eta_0' v_i \left( 1 - \frac{L_c}{2l_i} \right) + (1 - v_f) \sigma'_m \right)$$

where $\tau$ is the matrix-fibre interfacial strength, $L_c$ is the fibre critical length, $\eta_0$ is an orientation factor, $d$ is the average fibre diameter (11 $\mu$m), $\sigma_{uf}$ is the ultimate tensile stress of fibres, $\sigma'_m$ is the stress carried by the matrix at the fibre failure strain (61 MPa, see Fig.1), $l_i$ and $l_j$ are the length of fibres having volume fraction $v_i$ and $v_j$, respectively. Subscripts $i$ and $j$ refers to fibres of sub-critical and super-critical length, respectively. A modified version – using a larger number of points for data fitting – of the iterative method proposed by Thomason was applied for the identification of the micro-mechanical parameters $\tau$ and $L_c$, and the orientation factor $\eta_0$. When applied to the three grades of polyamide independently, it yielded the values reported in Table 2.

<table>
<thead>
<tr>
<th>Material</th>
<th>$L_c$</th>
<th>$\tau$</th>
<th>$\eta_0$</th>
<th>$\sigma_M$</th>
<th>$\sigma'_m$</th>
</tr>
</thead>
<tbody>
<tr>
<td>PA6GF10</td>
<td>151</td>
<td>57</td>
<td>0.61</td>
<td>100</td>
<td>89</td>
</tr>
<tr>
<td>PA6GF20</td>
<td>126</td>
<td>69</td>
<td>0.61</td>
<td>122</td>
<td>120</td>
</tr>
<tr>
<td>PA6GF30</td>
<td>146</td>
<td>59</td>
<td>0.64</td>
<td>148</td>
<td>146</td>
</tr>
</tbody>
</table>

It must be noted that, since the method does not impose the equality of the predicted tensile strength and the experimental one, but it only imposes a best fit at several points of the stress-strain curve, the predicted $\sigma_M$ does not always match closely the experimental value. The value of the orientation factor increases with fibre content, in agreement with the trend of $IA$. However, it also predicted different values...
of \( \tau \) and \( L_c \), whereas \( \tau \) and \( L_c \) are not expected to vary with fibre volume fraction because they are related only to the properties of the fibre-matrix interface. The orientation factor only is supposed to change with fibre content. Thus, only one set of parameters was used, that of PA6GF30, and the model was applied to the remaining materials, PA6GF20 and PA6GF10. The value of the orientation factor was varied in order to match the experimental values of the tensile strength with a tolerance of 1%. The values of tensile strength thus obtained are reported in Table 3, where it clearly appears that the orientation factor has to be considerably increased for the PA6GF10 in order to match the experimental value of tensile strength. This is in contrast with the trend of \( I_A \) with fibre content reported in Fig. 4. However, in the case of PA6GF20 and PA6GF30, it is confirmed that the difference between the fibre orientation factors is negligible, as it was for the values of \( I_A \).

Table 3. Values of the tensile strength predicted by the Bowyer-Bader model (\( L_c = 146 \, \mu m \) and \( \tau = 59 \, MPa \))

<table>
<thead>
<tr>
<th>Material</th>
<th>( \eta_0 )</th>
<th>( \sigma_M )</th>
<th>( \sigma_{M,B} )</th>
</tr>
</thead>
<tbody>
<tr>
<td>PA6GF10</td>
<td>0.83</td>
<td>100</td>
<td>99</td>
</tr>
<tr>
<td>PA6GF20</td>
<td>0.70</td>
<td>122</td>
<td>122</td>
</tr>
<tr>
<td>PA6GF30</td>
<td>0.69</td>
<td>148</td>
<td>148</td>
</tr>
</tbody>
</table>

Similar controversial conclusions can be drawn if the variation of the elastic modulus with fibre content is analysed. It was shown [9] that the longitudinal elastic modulus can be predicted with good accuracy by the Cox-Krenchel shear lag model, when the weigh average fibre length \( L_w \) is used to calculate the fibre stress ratio as \( s = L_w / d \). The expression of the tensile modulus is

\[
E_c = \eta_0 \nu_f E_f \left( 1 - \frac{\tanh(n s)}{n s} \right) + \left( 1 - \nu_f \right) E_m
\]

(3)

where \( n \) is expressed by

\[
n = \sqrt{2E_m \left[ E_f (1 + \nu_m) \ln \left( \frac{1}{\nu_f} \right) \right]} \]

(4)

Table 4. Values the tensile moduli predicted by the Cox-Krenchel model, using optimal \( \eta_0 \)

<table>
<thead>
<tr>
<th>Material</th>
<th>( \eta_0 )</th>
<th>( E ) (MPa)</th>
<th>( E_{C,K} ) (MPa)</th>
</tr>
</thead>
<tbody>
<tr>
<td>PA6GF10</td>
<td>0.66</td>
<td>4635</td>
<td>4657</td>
</tr>
<tr>
<td>PA6GF20</td>
<td>0.65</td>
<td>6587</td>
<td>6591</td>
</tr>
<tr>
<td>PA6GF30</td>
<td>0.69</td>
<td>9025</td>
<td>9039</td>
</tr>
</tbody>
</table>

In this case the values of the orientation factors \( \eta_0 \) were determined by imposing that the predicted values of the elastic moduli equalled the experimental ones. The results obtained using these optimal values of \( \eta_0 \) are reported in Table 4. In this case, PA6GF30 requires a slightly higher value of \( \eta_0 \) than the other two materials, but PA6GF10 requires a value of \( \eta_0 \) higher than that of PA6GF20, which is in contrast with the trend of \( I_A \) values.
Concluding remarks

From the discussion of the results presented above, the following conclusions can be drawn:

- micro-mechanical models, like the Bowyer-Bader and the Cox-Krenchel model, can predict with good accuracy the tensile strength and modulus, respectively, of short fibre reinforced polymers, taking into account the fibre length distribution, provided a fibre orientation factor is introduced;
- micro-tomography can provide information about the fibre orientation distribution, and a method based on the MIL concept can describe it using synthetic parameters like the index of anisotropy, which was found to be lower for PA6GF10 than for PA6GF20 and PA6GF30, thus suggesting a lower degree of alignment of the fibres along the longitudinal direction for very low fibre content;
- the application of the micro-mechanical models required the use of orientation factors, whose values varied with fibre content; however the trend, i.e. decreasing values of the orientation factor with increasing fibre content, were in contrast with the observed fibre orientation distribution;
- a good correlation between observed fibre distribution and values of the orientation factor was found for the PA6GF20 and PA6GF30 only, thus suggesting that the micro-mechanical models considered herein are more suitable for materials having a larger fibre content, for whom a correspondence between the original assumption of fibres having the same orientation, underlying both models before the introduction of orientation factors, can be verified.

Acknowledgements

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References