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Optimal replenishment and sales team initiatives for pharmaceutical products – A mathematical model

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ABSTRACT

The paper addresses an inventory model of pharmaceutical products where the demand rate of the customers increases with the volume of the initiatives of the sales team. In this model, the deterioration of the product varies depending on on-hand inventory. The volume of sales team initiatives is a control variable. It is dependent on on-hand inventory and vice versa. The profit function of the farm is formulated by the trading of inventory costs, purchasing costs, losses due to deterioration and sales team initiative costs, considering inflation and the time value of the monetary cost and profit parameters. Finally, the profit function is maximized by a variation of the calculus method. A numerical example is given to justify our model.

1. Introduction

Pharmaceutical products are essential commodities in health care management. Many substitutable pharmaceutical products are present in the market. Consequently, the managers of pharmaceutical farms/companies have to think about new strategies to capture market demand. Generally, it is observed that medical representatives act as salesmen of pharmaceutical products. As a result, the sales team should be composed of qualified and knowledgeable pharmacists, doctors and marketing researchers. For this purpose, investment in the sales team and in sample medicines of a particular drug is necessary and constitutes important cost factors for the firms. On the other hand, pharmaceutical products undergo deterioration/decay over time and should not be neglected in the inventory system.

In the present article, we develop a mathematical model that considers sales revenue, inventory holding cost, purchasing/procurement cost, loss due to deterioration and investment in sales team initiatives. Here, the demand for the products in the market has two parts: one part is fixed, which is the demand of fixed customers who have a good relationship with the firm, and other part is an increasing function of the sales team initiatives in which the customers are motivated by the awareness activities that the salesmen use to promote the product. The initiative function of the salesmen is a control variable. The profit function is maximized by a variation of the calculus method by trading off sales revenue, inventory holding cost, loss due to deterioration and cost of sales team initiatives. Finally, optimal values of replenishment and sales team initiatives are calculated to maximize the profit function.

2. Brief review of the literature

In practice, it is observed that perishable pharmaceutical products undergo deterioration over time. Deterioration generally includes spoilage, obsolescence, evaporation and pilferage of the products and increases with time. Many researchers have focused on this topic. Wu et al. (2009) solved an inventory system with non-instantaneous deteriorating items and price-sensitive demands to determine the optimal sales price and length of the replenishment cycle such that the total profit per unit of time of the retailer was maximized. Chang et al. (2010) characterized the optimal solution and obtained theoretical results to determine the optimal order quantity and the optimal replenishment time under a discounted cash-flow (DCF) approach. Jaggi et al. (2011) studied a two-

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warehouse inventory model with a linear trend in demand under inflationary conditions with different rates of deterioration, in which partial backlogging at the owned warehouse was allowed. Widyadana et al. (2011) optimized an inventory model for perishable items with a simplified approach, considering the issue with and without backordering. Ghosh et al. (2011) developed a production- inventory model with backlogging for perishable items with a linear price dependent demand. Sarkar (2012) developed an e*conomic order quantity* model for time with a varying demand and deterioration rate of the items, including trade credit financing and different discount rates on the purchasing costs. Sarkar and Sarkar (2013) extended an inventory model for time varying deteriorating items with stock-dependent demand, allowing time varying backlogging. Sarkar (2013) considered three types of continuous probabilistic deterioration functions in a two-echelon supply chain to analyse a production-inventory model by a simple algebraic approach. Goyal et al. (2013) developed an economic production quantity (EPQ) model for ameliorating/deteriorating items with a ramp type demand, and they minimized the total relevant cost of the system by using a genetic algorithm (GA). Taleizadeh et al. (2015) analysed a Vendor Managed Inventory (VMI) model for a two-level supply chain to obtain the optimal retail price, the replenishment frequency of raw material, the replenishment cycle of the product and the production rate, when both the raw material and the finished product have different deterioration rates.

Research in supply chain management of pharmaceutical products has been extensively applied in the field of health care management. In a pharmaceutical supply chain system, the integration of all performances involved with the flow of products from raw materials to the end customers improves the relationship with the channel members to achieve sustainable competitive opportunities. According to Aptel and Pourjalali (2001), the management of pharmaceutical products is a vital managerial issue in health care industries. Veral and Rosen (2001) showed that one of the long-term benefits of outsourcing could be a reduction in the number of suppliers, which results in lower procurement costs for downstream members of the chain. Nicholson et al. (2004) compared the costs of inventory and service levels for inventory items of an in-house three-echelon distribution network and outsourced a two-echelon distribution network. Kim (2005) studied an integrated supply chain management system, addressing issues related to drugs in health care sectors. Vernon et al. (2005) suggested both analytic and simulation methods for firms to determine the feasible range of prices of products considering licensing risk and developmental Go/No-Go decisions for the payers' use of the cost-effectiveness method. Almarsdóttir and Traulsen (2005) described four types of strategies to curb rising pharmaceutical costs and a taxonomy of strategies for price and profit controls, reimbursement system charges, other fiscal measures, and quality measures. Zhao et al. (2006) discussed the development of software infrastructure for scientists in pharmaceutical industries to help manage information, capture knowledge and provide intelligent decision support for pharmaceutical product formulations. They provided a balanced approach to policymaking in an environment with rising pharmaceutical costs. Lapierre and Ruiz (2007) emphasized scheduling decisions for supply chains of the products instead of multi-echelon inventory systems. Meijboom and Obel (2007) analysed supply chain coordination in the pharmaceutical industry with multi-location and multi-stage operation systems. Chen and Whittemore (2008) suggested that the use of an offsite warehouse to pool resources saved costs on holding inventory by reducing on-hand inventory. Woosley (2009) indicated that hospital administrators and pharmacy managers suffered due to complicated distribution networks for inventory management problems because most hospital administrators and pharmacy managers were doctors without sufficient knowledge of supply chain management. Khan and Shefeeq (2009) showed that mathematical modelling for controlled drug delivery is invaluable in the ongoing struggle to develop new and more effective therapies for the treatment of cancer. Kelle et al. (2012) developed a periodic review inventory model of pharmaceutical products, applying operational and tactical strategies. Uthayakumar and Priyan (2013) investigated a two-echelon pharmaceutical supply chain inventory model for multiple products, considering permissible delays in payment and the inventory lead time under some limitations. Priyan and Uthayakumar (2014) extended the above model for situations in unclear environments. Cárdenas-Barrón et al. (2014) investigated an economic order quantity model in honour of Ford Whitman Harris. Cárdenas-Barrón and Sana (2014, 2015) studied a production inventory model for single and multi-item products in a two-echelon supply chain when end customers' demand varied according to sales team initiatives.

The rest of the paper is organized as follows: Section 3 includes notations that are used to develop the proposed model. The formulation of the model is given in Section 4. Section 5 illustrates the model with numerical examples. Section 6 analyses the sensitivity analysis of the key parameters and provides some managerial insights for the proposed model. Concluding remarks are given in Section 7.

3. Notation

The following notations are used to discuss the model.

$q(t)$ – On-hand inventory at time 't'.
$E(t)$ – The sales team initiatives at time 't', a control variable.
$D(E)$ – Demand rate of the item.
T – The finite time horizon.
R – Replenishment lot size.
θ – Perishable factor ($0 < \theta < 1$), a fraction of the on-hand inventory.
$\delta = (r - i) - r$ is rate of interest per unit currency ($) per unit time, and i is inflation rate per unit currency ($) per unit time.
c_h – Inventory holding cost per unit per unit time.
c_0 – Purchasing cost per unit item.
c_1 – Cost per unit effort of the sales team.
p – Selling price per unit item.
E_0 – Initial level of sales team initiatives.
J – Profit of the objective function.
R^*– Optimum value of replenishment lot size.
E_0^* – Optimum initial value of the initiatives of the sales team.
J^* – Maximum profit of the objective function.

4. Formulation of the model

In this model, the demand rate is

$$D(E) = a_0 + a_1\sqrt{E(t)} \tag{1}$$

where a_0 is fixed demand, which is independent of sales team initiatives, and a_1 is a scale parameter of the demand rate, which is sensitive to sales team initiatives. The demand rate is an increasing function of $E(t)$ that is obvious in practice. Generally speaking, medical representatives are employed to capture the market demand. On the other hand, some pharmaceutical stores employ a medical practitioner to attract customers and boost sales. Under this philosophy, customers/patients benefit from the medical practitioners, and the store owners earn more from selling more items. In this case, the governing differential equation of the on-hand inventory is

$$\frac{dq}{dt} = -\theta q - \left(a_0 + a_1\sqrt{E(t)}\right) \text{ with } q(0) = (R + a_0) \text{ and } q(T) = a_0. \tag{2}$$

Here, the replenishment size is R, and it is received at the end of the cycle length T. At this time, the level of the inventory is a_0 units because fixed demand is met using these units to keep good will with the fixed customers. Quite often, when the lot size (R) is received, it takes some time to prepare to sell them. In this short timeframe, the demand of the genuine customers is met from the stock level a_0. For the generality of the model, it is considered to be a non-zero value and may be considered to be zero, as is done in many articles in the inventory literature. Therefore, the inventory $q(t)$ initially starts at ($t = 0$) with ($R + a_0$) units and is depleted with the demand rate mentioned in Eq. (1). The inventory level reaches level a_0 at time T.

The cost and earning elements of the model at time ‘t’ are as follows:

The cost for sales team initiatives is $c_1E(t)$. This is proportional to the volume of initiatives. The cost for holding inventory is $c_hq(t)$. The loss due to deterioration is $c_0\theta q(t)$. The purchasing cost of R units is c_0R. Earnings from sales items is $p(a_0 + a_1\sqrt{E(t)})$. Therefore, the total profit during the period [0,T], including the time value of money and inflation, is

$$J = \int_0^T e^{-\delta t}\left[p\left(a_0 + a_1\sqrt{E(t)}\right) - c_0R - c_0\theta q(t) - c_hq(t) - c_1E(t)\right]dt = \int_0^T \pi(\dot{q},\ q,\ t)dt \tag{3}$$

where

$$\begin{aligned}\pi(\dot{q},\ q,\ t) =& e^{-\delta t}\left[p\left(a_0 + a_1\sqrt{E(t)}\right) - c_0R - c_0\theta q(t) - c_hq(t) - c_1E(t)\right] = e^{-\delta t} \\ &\times\left\{-\frac{c_1}{a_1^2}(\dot{q}(t))^2 - \left(p + \frac{2a_0c_1}{a_1^2}\right)(\dot{q}(t)) - \frac{2c_1\theta}{a_1^2}(q(t)\dot{q}(t)) - \frac{c_1\theta^2}{a_1^2}(q(t))^2 \right.\\ &\left. - \left(\frac{2c_1a_0\theta}{a_1^2} - p\theta + c_0\theta + c_h\right)q(t) - c_0R - \frac{c_1a_0^2}{a_1^2}\right\}\end{aligned} \tag{4}$$

Proposition 1: The profit function J is concave in the interval [0, T].

Proof: The value of J depends on the path $q = q(t)$ in the interval [0, T]. Let J have an extreme value along the path $q = q_0(t)$ for $t \in [0,T]$. Consider a class of neighbourhood curves $q_\in(t) = q_0(t) + \in x(t)$, where $\in$ is a small constant and $x(t)$ is an arbitrary analytic function of t. Then, the value of J is $J(\in) = \int_0^T \pi_\in(\dot{q}_0(t) + \in\dot{x}(t),\ q_0(t) + \in x(t),\ t)\,dt$. For the concavity of $J(\in)$ we may have $\left.\frac{dJ(\in)}{d\in}\right|_{\in=0} = 0$ and $\left.\frac{d^2J(\in)}{d\in^2}\right|_{\in=0} < 0$.

Now, $\frac{dJ(\in)}{d\in} = \int_0^T\left(\dot{x}(t)\frac{\partial\pi_\in}{\partial\dot{q}} + x(t)\frac{\partial\pi_\in}{\partial q}\right)dt = \int_0^T x(t)\frac{\partial\pi_\in}{\partial q}dt + \left[x(t)\frac{\partial\pi_\in}{\partial\dot{q}}\right]_0^T - \int_0^T x(t)\frac{d}{dt}\left(\frac{\partial\pi_\in}{\partial\dot{q}}\right)dt = \int_0^T x(t)\left\{\frac{\partial\pi_\in}{\partial q} - \frac{d}{dt}\left(\frac{\partial\pi_\in}{\partial\dot{q}}\right)\right\}dt$ because $x(0) = 0 = x(T)$ as q is fixed at the end points $t = 0$ and $t = T$. Because $\left.\frac{dJ(\in)}{d\in}\right|_{\in=0} = 0$,

$$\frac{d}{dt}\left(\frac{\partial\pi}{\partial\dot{q}}\right) - \frac{\partial\pi}{\partial q} = 0 \tag{5}$$

which is a necessary condition for extreme values of J. Differentiating $dJ(\in)/d\in$ with respect to $\in$, we have

$$\frac{d^2J(\in)}{d\in^2} = \int_0^T\left\{\dot{x}^2\frac{\partial^2\pi_\in}{\partial\dot{q}^2} + 2x\dot{x}\frac{\partial^2\pi_\in}{\partial q\partial\dot{q}} + x^2\frac{\partial^2\pi_\in}{\partial q^2}\right\}dt.$$ Now, at $\in = 0$,

$$\left.\frac{d^2J(\in)}{d\in^2}\right|_{\in=0} = \int_0^T\left\{\dot{x}^2\frac{\partial^2\pi}{\partial\dot{q}^2} + 2x\dot{x}\frac{\partial^2\pi}{\partial q\partial\dot{q}} + x^2\frac{\partial^2\pi}{\partial q^2}\right\}dt \tag{6}$$

Now,

$$\frac{\partial\pi}{\partial q} = e^{-\delta t}\left\{-\frac{2c_1\theta}{a_1^2}(\dot{q}) - \frac{2c_1\theta^2}{a_1^2}(q) - \left(\frac{2c_1\theta a_0}{a_1^2} - p\theta + c_0\theta + c_h\right)\right\},$$

$$\frac{\partial\pi}{\partial\dot{q}} = e^{-\delta t}\left\{-\frac{2c_1}{a_1^2}(\dot{q}) - \left(p + \frac{2a_0c_1}{a_1^2}\right) - \frac{2c_1\theta}{a_1^2}(q)\right\},$$

$$\frac{\partial^2\pi}{\partial q\partial\dot{q}} = e^{-\delta t}\left\{-\frac{2c_1\theta}{a_1^2}\right\},$$

$$\frac{\partial^2\pi}{\partial\dot{q}^2} = e^{-\delta t}\left\{-\frac{2c_1}{a_1^2}\right\} \text{ and } \frac{\partial^2\pi}{\partial q^2} = e^{-\delta t}\left\{-\frac{2c_1\theta^2}{a_1^2}\right\}.$$

Substituting these in Eq. (6), we have

$$\left.\frac{d^2J(\in)}{d\in^2}\right|_{\in=0} = -\frac{2c_1}{a_1^2}\int_0^T e^{-\delta t}\{\dot{x} + x\theta\}^2dt < 0.$$

This sufficient condition ensures that J is a concave function in [0,T]. The proof is completed.

Here,

$$\frac{d}{dt}\left(\frac{\partial\pi}{\partial\dot{q}}\right) = \delta e^{-\delta t}\left\{\frac{2c_1}{a_1^2}(\dot{q}) + \left(p + \frac{2a_0c_1}{a_1^2}\right) + \frac{2c_1\theta}{a_1^2}(q)\right\} - e^{-\delta t}\left\{\frac{2c_1}{a_1^2}(\ddot{q}) + \frac{2c_1\theta}{a_1^2}(\dot{q})\right\}$$

Substituting the above derivatives in Eq.(5), we have, after simplification:

$$\ddot{q} - \delta\dot{q} - \theta(\theta + \delta)q = \tau \tag{7}$$

where

$$\tau = \frac{a_1^2}{2c_1}\left\{\delta\left(p + \frac{2a_0c_1}{a_1^2}\right) + \frac{2c_1\theta a_0}{a_1^2} - p\theta + c_0\theta + c_h\right\}.$$

Using the boundary conditions $q(0) = (R + a_0)$ and $q(T) = a_0$, we have the solution of Eq. (7), i.e., the optimal path of $q(t)$, as follows:

$$q(t) = A(R)e^{-\theta t} + B(R)e^{(\theta+\delta)t} - \frac{\tau}{\theta(\theta+\delta)} \tag{8}$$

where

$$A(R) = \left[\frac{(R+a_0)e^{(\theta+\delta)T} - a_0 - \frac{\tau\left(1-e^{(\theta+\delta)T}\right)}{\theta(\theta+\delta)}}{e^{(\theta+\delta)T} - e^{-\theta T}}\right] \text{ and } B(R) = -\left[\frac{(R+a_0)e^{-\theta T} - a_0 - \frac{\tau\left(1-e^{-\theta T}\right)}{\theta(\theta+\delta)}}{e^{(\theta+\delta)T} - e^{-\theta T}}\right]$$

The optimal path of $E(t)$ is

$$E(t) = \frac{1}{a_1^2}\left[a_0 - \frac{\tau\theta}{\theta(\theta+\delta)} + B(R,T)(2\theta+\delta)e^{(\theta+\delta)t}\right]^2 \tag{9}$$

Substituting the optimal paths $q(t)$ and $E(t)$ in Eq. (3), we have

$$J = J_1(R) + J_2(R) + J_3(R) + J_4(R) + J_5(R) + J_6(R) \tag{10}$$

where

$$J_1(R) = -\frac{c_1}{a_1^2}\int_0^T e^{-\delta t}(\dot{q})^2 dt = -\frac{c_1}{a_1^2}\left(\frac{\theta^2A^2}{2\theta+\delta}\right)\left(1-e^{-(2\theta+\delta)T}\right) - \frac{c_1B^2(\theta+\delta)^2}{a_1^2(2\theta+\delta)}\left\{e^{(2\theta+\delta)T}-1\right\} + \frac{2c_1}{a_1^2}\theta AB(\theta+\delta)T,$$

$$J_2(R) = -\left(p+\frac{2a_0c_1}{a_1^2}\right)\int_0^T e^{-\delta t}\dot{q}dt = \left(p+\frac{2a_0c_1}{a_1^2}\right)\left(\frac{\theta A}{\theta+\delta}\right)\left(1-e^{-(\theta+\delta)T}\right) - \left(p+\frac{2a_0c_1}{a_1^2}\right) \times \left(\frac{B(\theta+\delta)}{\theta}\right)\left(e^{\theta T}-1\right),$$

$$J_3(R) = -\frac{2c_1\theta}{a_1^2}\int_0^T e^{-\delta t}q\,\dot{q}dt = \frac{2c_1\theta}{a_1^2}\left(\frac{\theta A^2}{2\theta+\delta}\right)\left(1-e^{-(2\theta+\delta)T}\right) - \frac{2c_1\theta}{a_1^2}AB\delta T - \frac{2c_1\theta A\tau}{a_1^2(\theta+\delta)^2}\left\{1-e^{-(\theta+\delta)T}\right\} - \frac{2c_1\theta B^2(\theta+\delta)}{a_1^2(2\theta+\delta)}\left(e^{(2\theta+\delta)T}-1\right) + \frac{2c_1B\tau}{a_1^2\theta}\left(e^{\theta T}-1\right),$$

$$J_4(R) = -\frac{c_1\theta^2}{a_1^2}\int_0^T e^{-\delta t}q^2\,dt = -\frac{c_1\theta^2A^2}{a_1^2(2\theta+\delta)}\left(1-e^{-(2\theta+\delta)T}\right) - \frac{c_1\theta^2B^2}{a_1^2(2\theta+\delta)}\left(e^{(2\theta+\delta)T}-1\right) - \frac{c_1\tau^2}{a_1^2\delta(\theta+\delta)^2}\left(1-e^{-\delta T}\right) - \frac{2c_1\theta^2ABT}{a_1^2} + \frac{2c_1\tau A}{a_1^2(\theta+\delta)}\left(1-e^{-(\theta+\delta)T}\right) + \frac{2c_1\tau B}{a_1^2(\theta+\delta)}\left(e^{\theta T}-1\right),$$

$$J_5(R) = -\left(\frac{2c_1a_0\theta}{a_1^2} - p\theta + c_0\theta + c_h\right)\int_0^T e^{-\delta t}q\,dt = \left(\frac{2c_1a_0\theta}{a_1^2} - p\theta + c_0\theta + c_h\right)\left\{-\left(\frac{A}{\theta+\delta}\right)\left(1-e^{-(\theta+\delta)T}\right) - \frac{B}{\theta}\left(e^{\theta T}-1\right) + \frac{\tau}{\theta\delta(\theta+\delta)}\left(1-e^{-\delta T}\right)\right\}$$

and

$$J_6(R) = -\left(c_0R + \frac{c_1a_0^2}{a_1^2}\right)\int_0^T e^{-\delta t}\,dt = -\left(c_0R + \frac{c_1a_0^2}{a_1^2}\right)\left(\frac{1-e^{-\delta T}}{\delta}\right).$$

Substituting the above formulas of $J_i^{\prime s}(i = 1,2,\ldots.6)$ in Eq. (10) and simplifying, we have

$$J = -\frac{c_1}{a_1^2}(2\theta+\delta)\left(e^{(2\theta+\delta)T}-1\right)B^2 + A\left\{\left(p+\frac{2a_0c_1}{a_1^2}\right)\left(\frac{\theta}{\theta+\delta}\right) \times \left(1-e^{-(\theta+\delta)T}\right) + \frac{2c_1\delta\tau}{a_1^2(\theta+\delta)^2}\left(1-e^{-(\theta+\delta)T}\right) - \left(\frac{2c_1a_0\theta}{a_1^2} - p\theta + c_0\theta + c_h\right)\left(\frac{1-e^{-(\theta+\delta)T}}{\theta+\delta}\right)\right\} + B\left\{-\left(p+\frac{2a_0c_1}{a_1^2}\right)\left(\frac{\theta+\delta}{\theta}\right)\left(e^{\theta T}-1\right) + \frac{2c_1\tau}{a_1^2\theta(\theta+\delta)} \times (2\theta+\delta)\left(e^{\theta T}-1\right) - \left(\frac{2c_1a_0\theta}{a_1^2} - p\theta + c_0\theta + c_h\right)\left(e^{\theta T}-1\right)\right\} + \left\{-\frac{c_1\tau^2\left(1-e^{-\delta T}\right)}{a_1^2\delta(\theta+\delta)^2} + \left(\frac{2c_1a_0}{a_1^2} - p + c_0 + \frac{c_h}{\theta}\right) \times \left(\frac{\tau}{\delta(\theta+\delta)}\right)\left(1-e^{-\delta T}\right) - \left(c_0R + \frac{c_1a_0^2}{a_1^2}\right)\left(\frac{1-e^{-\delta T}}{\delta}\right)\right\} \tag{11}$$

Proposition 2: $J(R)$ is also concave and a unimodal function of R.
Proof:
Now,

$$\frac{dJ}{dR} = \frac{2c_1}{a_1^2}(2\theta+\delta)\left(e^{(2\theta+\delta)T}-1\right)B\left(\frac{e^{-\theta T}}{e^{(\theta+\delta)T}-e^{-\theta T}}\right) + \left(\frac{e^{(\theta+\delta)T}}{e^{(\theta+\delta)T}-e^{-\theta T}}\right)\left[\left(p+\frac{2a_0c_1}{a_1^2}\right)\left(\frac{\theta}{\theta+\delta}\right) \times \left(1-e^{-(\theta+\delta)T}\right) + \frac{2c_1\delta\tau}{a_1^2(\theta+\delta)^2}\left(1-e^{-(\theta+\delta)T}\right) - \left(\frac{2c_1a_0\theta}{a_1^2} - p\theta + c_0\theta + c_h\right)\left(\frac{1-e^{-(\theta+\delta)T}}{\theta+\delta}\right)\right] - \left(\frac{e^{-\theta T}}{e^{(\theta+\delta)T}-e^{-\theta T}}\right)\left[-\left(p+\frac{2a_0c_1}{a_1^2}\right)\left(\frac{\theta+\delta}{\theta}\right)\left(e^{\theta T}-1\right) + \frac{2c_1\tau}{a_1^2\theta(\theta+\delta)}(2\theta+\delta)\left(e^{\theta T}-1\right) - \left(\frac{2c_1a_0\theta}{a_1^2} - p\theta + c_0\theta + c_h\right)\left(e^{\theta T}-1\right)\right] - \left(\frac{c_0}{\delta}\right)\left(1-e^{-\theta T}\right) \tag{12}$$

and

$$\frac{d^2J}{dR^2}=-\frac{2c_1}{a_1^2}(2\theta+\delta)\left(e^{(2\theta+\delta)T}-1\right)\left(\frac{e^{-\theta T}}{e^{(\theta+\delta)T}-e^{-\theta T}}\right)^2<0. \qquad (13)$$

For optimum value of J, $\frac{dJ}{dR}=0$. This gives the optimum value of R, which is

$$\begin{aligned}
&-a\frac{2c_1}{a_1^2}(2\theta+\delta)\left(e^{(2\theta+\delta)T}-1\right)\left(\frac{e^{-\theta T}}{e^{(\theta+\delta)T}-e^{-\theta T}}\right)^2R+\frac{2c_1}{a_1^2}(2\theta+\delta)\left(e^{(2\theta+\delta)T}-1\right)\frac{\tau e^{-\theta T}\left(1-e^{-\theta T}\right)}{\theta(\theta+\delta)\left(e^{(\theta+\delta)T}-e^{-\theta T}\right)^2}\\
&+\left(\frac{e^{(\theta+\delta)T}}{e^{(2\theta+\delta)T}-e^{-\theta T}}\right)\Bigg[\Bigg(\left(p+\frac{2a_0c_1}{a_1^2}\right)\left(\frac{\theta}{\theta+\delta}\right)\left(1-e^{-((\theta+\delta)T)}\right)+\frac{2c_1\delta\tau}{a_1^2(\theta+\delta)^2}\left(1-e^{-(\theta+\delta)T}\right)-\left(\frac{2c_1a_0\theta}{a_1^2}-p\theta+c_0\theta+c_h\right)\left(\frac{1-e^{-(\theta+\delta)T}}{\theta+\delta}\right)\\
&-\left(\frac{e^{-\theta T}}{e^{(\theta+\delta)T}-e^{-\theta T}}\right)\Bigg)\Bigg]\\
&\times\left[-\left(p+\frac{2a_0c_1}{2_1^2}\right)\left(\frac{\theta+\delta}{\theta}\right)\left(e^{\theta T}-1\right)+\frac{2c_1\tau}{a_1^2\theta(\theta+\delta)}(2\theta+\delta)\left(e^{\theta T}-1\right)-\left(\frac{2c_1a_0\theta}{a_1^2}-p\theta+c_0\theta+c_h\right)\left(e^{\theta T}-1\right)\right]-\left(\frac{c_0}{\delta}\right)\left(1-e^{-\theta T}\right)\\
&=0\xrightarrow{yields}R=\Bigg[\frac{2c_1}{a_1^2}(2\theta+\delta)\left(e^{(2\theta+\delta)T}-1\right)\frac{\tau e^{-\theta T}\left(1-e^{-\theta T}\right)}{\theta(\theta+\tau)\left(e^{(\theta+\delta)T}-e^{-\theta T}\right)^2}+\left(\frac{e^{(\theta+\delta T)}}{e^{(\theta+\delta)T}-e^{-\theta T}}\right)\\
&\times\left[\left(p+\frac{2a_0c_1}{a_1^2}\right)\left(\frac{\theta}{\theta+\delta}\right)\left(1-e^{-(\theta+\delta)T}\right)+\frac{2c_1\delta_\tau}{a_1^2(\theta+\delta)^2}\left(1-e^{-(\theta+\delta)T}\right)-\left(\frac{2c_1a_0\theta}{a_1^2}-p\theta+c_0\theta+c_h\right)\left(\frac{1-e^{-(\theta+\delta)T}}{\theta+\delta}\right)\right]\left(\frac{e^{-\theta T}}{e^{(\theta+\delta)T}-e^{-\theta T}}\right)\\
&-\left[\left(p+\frac{2a_0c_1}{a_1^2}\right)\left(\frac{\theta+\delta}{\theta}\right)\left(e^{\theta T}-1\right)+\frac{2c_1\tau}{a_1^2\theta(\theta+\delta)}(2\theta+\delta)\left(e^{\theta T}-1\right)-\left(\frac{2c_1a_0\theta}{a_1^2}-p\theta+c_0\theta+c_h\right)\left(e^{\theta T}-1\right)\right]-\left(\frac{c_0}{\delta}\right)\left(1-e^{-\theta T}\right)\Bigg]\\
&\times\left[\frac{2c_1}{a_1^2}(2\theta+\delta)\left(e^{(2\theta+\delta)T}-1\right)\left(\frac{e^{-\theta T}}{e^{(\theta+\delta)T}-e^{-\theta T}}\right)^2\right]^{-1}
\end{aligned} \qquad (14)$$

Therefore, $J(R)$ is the concave function of R as $\frac{d^2J}{dR^2}=-\frac{2c_1}{a_1^2}(2\theta+\delta)(e^{(2\theta+\delta)T}-1)\left(\frac{e^{-\theta T}}{e^{(\theta+\delta)T}-e^{-\theta T}}\right)^2<0$ and is the global maximum. The proof is completed.

5. Numerical example

We consider the values of key parameters in appropriate units as follows: $a_0=50$ units, $a_1=10$ units, $T=3$ months, $\theta=0.05$, $\delta=0.06$, $c_h=\$\ 1.5$, $c_0=\$\ 10$, $c_1=\$\ 3$, and $p=\$\ 40$. Then, the required optimal solution is $R^*=379.82$ units, $E_0^*=126.51$ units and $J^*=\$851.005$. The optimal path of inventory (Fig. 1) declines sharply when adjusting demand $D(E)$. Similarly, the optimal path of sales team initiatives $E(t)$ (Fig. 2) decreases with time because the stock of inventory decreases with time. Another reason is that more initiatives are needed to boost the sales of the higher stock of the products as well as for the introduction of new products in the market. The demand and deterioration of the product over time t are shown in Fig. 3. The mathematical analysis and Fig. 4 indicate that $J(R)$ is a strictly concave function of R. Moreover, the optimal solution of the model is $R^*=209.89$ units, $R^*=209.89$, $E_0^*=126.51$ units and $J^*=\$\ 768.73$ when $a_0=0$, i.e., the demand of the customers is fully dependent on sales team initiatives.

6. Sensitivity analysis of the key parameters and managerial insights

We have conducted sensitivity analysis for the key parameters of the model. The optimal solutions for (−50%, −25%, +25%, +50%) changes of one parameter were obtained (see Table 1), keeping the

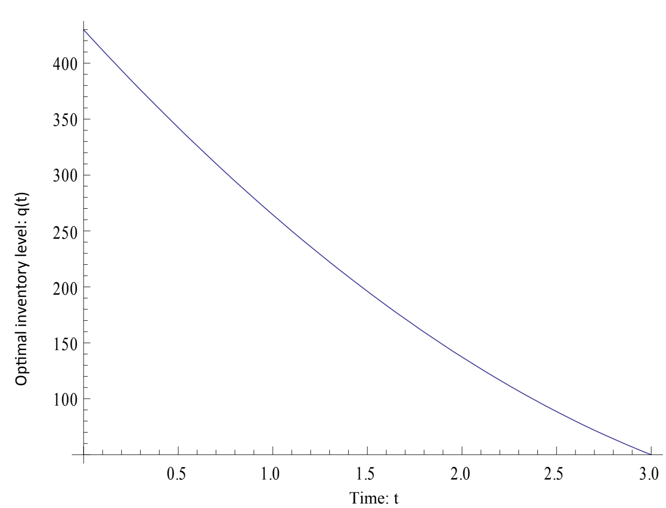

Fig. 1. Optimal level of inventory versus time.

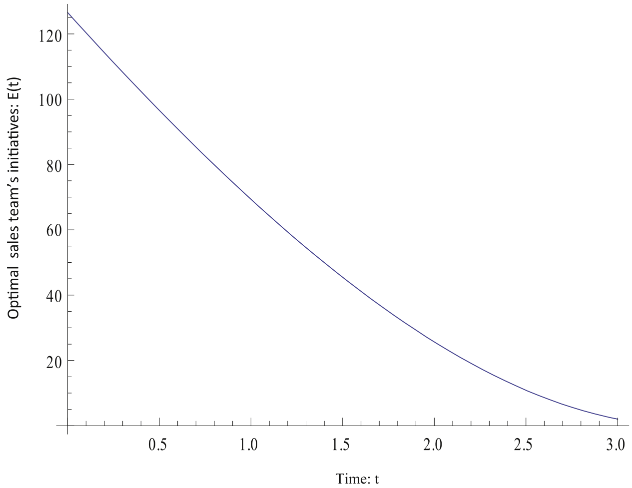

Fig. 2. Optimal initiatives of sales team versus time.

other parameters fixed. From the sensitivity analysis, the following natural phenomena are observed:

- The optimal values (R^*, E_0^*, J^*) are moderately sensitive with changes of the fixed demand rate (a_0). As the fixed demand rate (i.e., fixed customer demand rate) increases, the replenishment lot size (R^*) increases, which results in a higher profit. On the other hand, the initial level of the initiatives of the sales team (E_0^*) is unchanged because the scale parameter (a_1) is fixed. This is quite rational in real life problems.
- The optimal values (R^*, E_0^*, J^*) are highly sensitive with changes in the scale parameter (a_1) of the demand function. Higher values of a_1 result in more effort from the sales team to increase the demand for the items. As a result, the replenishment lot sizes and profits are higher due to more demand.
- When the deterioration rate (θ) increases, the replenishment lot size decreases to avoid more loss due to higher deterioration. The initial effort level (E_0^*) decreases with decreases in the lot size, and the profit (J^*) occasionally decreases due to higher deterioration.

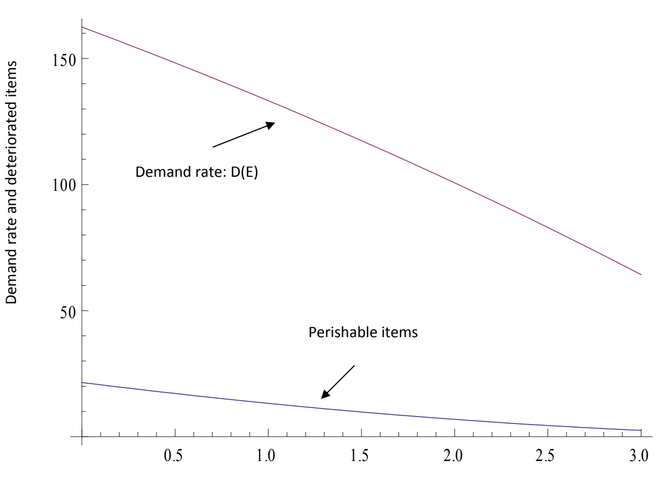

Fig. 3. Demand rate and perishable items versus time.

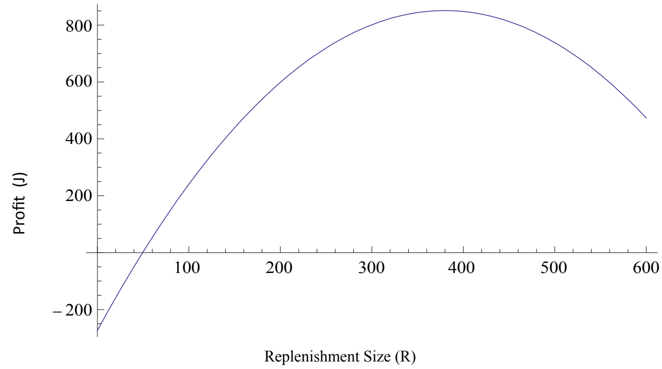

Fig. 4. Profit (J) versus replenishment size (R).

- The optimal values (R^*, E_0^*, J^*) are fairly sensitive to changes in the inventory holding cost(c_h). As the scale parameter a_1 of the initiatives of the sales team is fixed, the initial effort level of the sales team is insensitive to changes in c_h. However, the initial optimum replenishment lot size decreases with increases in c_h to avoid greater costs for holding more inventory. The profit decreases automatically with increases in inventory holding cost.
- The optimal replenishment lot size, initial effort level and profit decrease with increases in values of purchasing cost (c_0) per unit item. Here, the optimal solutions (R^*, E_0^*, J^*) are highly sensitive

Table 1
Sensitivity analysis of the key parameters.

Parameter	(Change in %)	R^*	E_0^*	J^*
a_0	−50%	294.85	126.51	809.87
	−25%	337.33	126.51	830.44
	+25%	422.30	126.51	871.57
	+50%	464.78	126.51	892.14
a_1	−50%	222.40	31.63	274.45
	−25%	289.99	71.16	514.68
	+25%	497.88	197.67	1283.42
	+50%	642.18	284.64	1811.92
θ	−50%	459.61	258.11	1621.45
	−25%	421.45	186.40	1233.03
	+25%	334.45	78.35	479.62
	+50%	285.08	41.81	123.45
c_h	−50%	449.56	126.51	1241.74
	−25%	414.69	126.51	1034.50
	+25%	344.94	126.51	691.26
	+50%	310.07	126.51	555.25
c_0	−50%	1284.02	1164.63	12,550.40
	−25%	831.92	514.70	5107.93
	+25%	Not feasible	Not feasible	Not feasible
	+50%	Not feasible	Not feasible	Not feasible
c_1	−50%	589.71	506.03	1619.74
	−25%	449.78	224.90	1107.25
	+25%	337.84	80.96	697.26
	+50%	309.85	56.22	594.76
p	−50%	Not feasible	Not feasible	Not feasible
	−25%	Not feasible	Not feasible	Not feasible
	+25%	919.26	650.20	6362.59
	+50%	1458.71	1580.71	16415.90
δ	−50%	393.14	81.54	697.84
	−25%	386.81	103.10	770.94
	+25%	372.15	151.57	937.62
	+50%	363.80	178.12	1030.41
T	−50%	737.43	1732.76	16,637.40
	−25%	746.81	747.87	6093.17
	+25%	Not feasible	Not feasible	Not feasible
	+50%	Not feasible	Not feasible	Not feasible

to changes in c_0. The model has no feasible solution while c_0 is higher. This occurs due to loss of the objective function.

- The optimal values (R^*, E_0^*, J^*) are highly sensitive with changes of the cost parameter (c_1) of the effort of the sales teams. A higher value of c_1 reduces the effort level of the objective function. Consequently, the demand rate decreases with increases in the effort level, and the replenishment lot size decreases to avoid greater inventory cost.
- The optimal values (R^*, E_0^*, J^*) are highly sensitive to the selling price (p) per unit item. The initial initiatives (E_0^*) and replenishment lot size increase due to higher revenue earned from selling the items. Consequently, demand and profit increase due to a higher selling price of the items. The lower sales prices (-50% & -25% changes in p) have no feasible solutions in this model.
- When the parameter δ increases, the value of the profit decreases to avoid greater inventory cost and the initial effort level increases to clear stock rapidly.
- When the cycle time (T) increases, the replenishment lot size decreases to reduce the cost for holding inventory. As the replenishment lot size is smaller, the initial initiative level decreases to adjust to the demand of the customers. Here, the profit reduces for a longer cycle time (T) due to inflation and time value of money tied up for the whole system. The model also has no feasible solutions for higher values of T, $+25\%$ & $+50\%$ changes in T.

In the above analysis, based on the values of cost and profit parameters, this model suggests how a manager of a firm could order the lot size and adjust the investment in the sales team to achieve the maximum profit.

7. Conclusion

In practice, pharmaceutical products may undergo deterioration over time. The deterioration includes decay, spoilage, evaporation, obsolescence and pilferage, which increase with on-hand inventory. Therefore, a larger stock of the products for a long period of time causes higher loss due to deterioration and inventory cost. Therefore, the optimal stock/replenishment order size should be sold within a short period of time. Consequently, more sales team initiatives are required to clear the stock within the stipulated period. The sales team initiatives are also important for launching new products in the market. Moreover, our model considers the inflation and time value of money for the cost and profit parameters, which so far has only been considered in a few articles. Our model helps a manager of a farm determine the optimal lot size and volume of sales team initiatives so that the total profit can be maximized.

The proposed model can be extended further for the time dependent perishable rate, which is relaxed in this model for the sake of simplicity. The continuous nature of sales team initiatives are considered in this model, which is also a limitation of this model. Another limitation of the model is a deterministic replenishment rate that is available instantaneously and infinitely in the market. This situation may arise when supply disruption does not occur, which is not possible all the time. Consequently, this model can be extended in the future by considering the discrete probabilistic replenishment rate as well as the demand rate.

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