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# A self-consistent method to analyze the effects of the positive $Q$ -value neutron transfers on fusion

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## ABSTRACT

Considering the present limitation of the need for external parameters to describe the nucleus–nucleus potential and the couplings in the coupled-channels calculations, this work introduces an improved method without adjustable parameter to overcome the limitation and then sort out the positive  $Q$ -value neutron transfers (PQNT) effects based on the CCFULL calculations. The corresponding analysis for  $\text{Ca} + \text{Ca}$ ,  $\text{S}$ ,  $\text{Ca} + \text{Sn}$ , and  $\text{S}$ ,  $\text{Ca} + \text{Zr}$  provides a reliable proof and a quantitative evaluation for the residual enhancement (RE) related to PQNT. In addition, the RE for  $^{32}\text{S}$ ,  $^{40}\text{Ca} + ^{94}\text{Zr}$  shows an unexpected larger enhancement than  $^{32}\text{S}$ ,  $^{40}\text{Ca} + ^{96}\text{Zr}$  despite the similar multi-neutron transfer  $Q$ -values. This method should rather strictly test the fusion models and be helpful for excavating the underlying physics.

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## 1. Introduction

The involvement of the couplings between the inertial degrees of freedom, such as the inelastic collective excitations as well as nucleon transfers, and the relative motion degree of freedom dramatically change the tunneling processes, leads to strong sub-barrier fusion enhancement. Consequently near- and sub-barrier heavy-ion fusion offers a good platform to study the two basic mechanisms of tunneling and coupling [1,2] in quantum world. So far, couplings to the inelastic excitations can be described well in near-barrier fusion by using the coupled-channels (CC) theory [2]. While coupling to nucleon transfers is even troublesome and has not yet been well understood from both microscopic and macroscopic viewpoints.

For the effects of the transfer reaction on fusion, neutron transfers might be more important than proton transfers, because of the nonexistence of the Coulomb barrier. The possible effects of neutron transfer with positive  $Q$ -value on near-barrier fusion were firstly discovered by Beckerman et al. [3] in the experimental comparison of the fusion excitation functions of  $^{58,64}\text{Ni} + ^{58,64}\text{Ni}$ , and whereafter widely explained [4–7] by considering the gained kinetic energy for the (virtual) intermediate states induced by the

positive  $Q$ -value neutron transfers (PQNT) as an important doorway to fusion [8].

Afterwards, many studies [1,9–11] have been devoted to PQNT effects. Some systems show strong fusion enhancement related to PQNT experimentally [3,12]. Recently, Jiang et al. [13] showed a universal correlation between sub-barrier fusion enhancement and the strength of the total neutron-transfer cross sections by using a systematic analysis. However, some other systems don't show such a correlation [9–11]. For building up the correlation between PQNT and fusion, direct transfer measurements have been performed [14–16]. Experimental data have shown that the most dominant transfer process is to populate the excited states when the  $Q$ -value is positive [16], which is consistent with the optimum  $Q$ -value argument. Theoretically, different models have been proposed [17–19] to study the relevant reaction mechanisms. But it is still not well clarified for the dynamics of the neutron-transfer-mediated sub-barrier fusion up to now, in spite of the abundant experimental data and long-standing theoretical studies.

Fusion enhancement reflects the whole effects of couplings to all relevant channels. For understanding the PQNT effects theoretically, the key is to exclude the impact of the collective excitation states with a convincing calculation. But the current CC calculations cannot satisfy this point due to the need of external parameters to describe the nucleus–nucleus potential and the couplings. There is no denying that different theoretical models as well as parameter-dependent analyses artificially bring an additional obstacle for understanding the PQNT effects. Fortunately, a theoretical

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step has been taken recently to constraint these parameters by using the microscopic quantum calculations [20]. Before achieving a future comprehensive dynamic calculation, it is very meaningful to search for some physical quantity, which can reflect the PQNT effects reliably with a less model-dependent method.

## 2. Analysis method

In this article, residual enhancement (RE) is defined, to quantify the PQNT effects by excluding the inelastic coupling effects calculated by the CC theory [2], as the ratio of the experimental fusion data ( $\sigma_{\text{Exp}}$ ) to the CC calculation result ( $\sigma_{\text{CC}}$ ), that is  $\text{RE} = \sigma_{\text{Exp}}/\sigma_{\text{CC}}$ . Then, a systematic analysis of RE with existing experimental fusion data for some typical systems, measured by the same group, is performed in the following.

For the systems without expected strong PQNT effects, the experimental fusion data can be reproduced by the code CC-FULL [21] considering the suitable couplings, with the Akyüz–Winther (AW) proximity potential [22] parameterized into the Woods–Saxon (WS) form with three parameters  $V_0$ ,  $r_0$ , and  $a$ . Considering the impediment to understanding of the reaction mechanism caused by different potentials (type, diffuseness parameter and barrier energy) used in different works, the same type of potential is used here.

In order to obtain a consistent conclusion, comparison of RE for different systems with similar nuclear structures, should be done only in the case of the same kind calculation scheme for the inelastic couplings. To this end, some near-spherical systems without expected PQNT effects were used as benchmarks to calibrate the vibrational couplings that should be considered in the CC calculations. Then, it is extrapolated to study the adjacent systems with similar nuclear structures without any parameter adjustment. For avoiding the complication due to the presence of many complex channels, such as deep-inelastic collision and quasifission, while maintaining at the same time sizeable effects of the couplings to the vibrational states, only the medium-mass systems are selected.

The relevant information on the low-lying vibrational states of the reactants involved in the following CC calculations is shown in Table 1 [23,24], where  $\lambda^\pi$  is spin and parity,  $E_\lambda$  excitation energy,  $\beta_\lambda$  deformation parameter, and ‘NP’ represents the number of phonons. All the mutual excitations are considered in the calculations. The effects of higher excitation states on sub-barrier fusion are expected to be small due to the adiabatic nature of the fusion process [25] and are ignored here. All the values of  $\beta_\lambda$  for the nuclei are in agreement with those obtained from the reduced transition probability  $B(E\lambda)$ , except for  $^{40}\text{Ca}$ . Here, a smaller  $\beta_3$ -value of 0.27 for  $^{40}\text{Ca}$  is obtained as in Ref. [26] by best fitting the experimental fusion data of  $^{40}\text{Ca} + ^{40}\text{Ca}$  [27].

## 3. Results

### 3.1. Ca + Ca

The typical fusion of various combinations of calcium isotopes has been studied widely [28,29] for probing the PQNT effects. Fig. 1 shows the variation of RE with the reduced energy for  $^{40,48}\text{Ca} + ^{40,48}\text{Ca}$  [27,30,31]. Among them,  $^{40}\text{Ca} + ^{40}\text{Ca}$  and  $^{48}\text{Ca} + ^{48}\text{Ca}$  have no PQNT channels and were used as a benchmark for the vibrational coupling effects in the CC calculations, by trying to make RE to be nearly unity at sub-barrier energy region. This provides constraints on the considered couplings to the vibrational states.

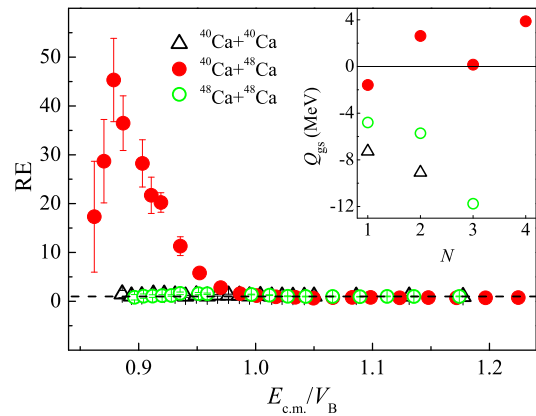
While for the asymmetric  $^{40}\text{Ca} + ^{48}\text{Ca}$  system, the RE deviates from unity with decreasing energy and shows a maximum of several tens at sub-barrier energy region. This isotopic dependence of

**Table 1**

The parameters used [23,24] and the coupling schemes considered in the CC calculations.

Nucleus	$\lambda^\pi$	$E_\lambda$ (MeV)	$\beta_\lambda$	NP
$^{32}\text{S}$	$2^+$	2.230	0.312	2
$^{40}\text{Ca}$	$3^-$	3.737	0.27 <sup>a</sup>	1
$^{48}\text{Ca}$	$2^+$	3.832	0.11	1
$^{90}\text{Zr}$	$2^+$	2.186	0.09	2
	$3^-$	2.748	0.22	2
$^{94}\text{Zr}$	$2^+$	0.919	0.09	2
	$3^-$	2.058	0.20	2
$^{96}\text{Zr}$	$2^+$	1.751	0.08	2
	$3^-$	1.897	0.27	2
$^{112}\text{Sn}$	$2^+$	1.257	0.123	2
	$3^-$	2.355	0.203	2
$^{116}\text{Sn}$	$2^+$	1.294	0.112	2
	$3^-$	2.266	0.18	2
$^{120}\text{Sn}$	$2^+$	1.171	0.108	2
	$3^-$	2.400	0.155	2
$^{124}\text{Sn}$	$2^+$	1.132	0.095	2
	$3^-$	2.614	0.130	2

<sup>a</sup> Obtained from the best fitting of the experimental fusion data of  $^{40}\text{Ca} + ^{40}\text{Ca}$  [27].



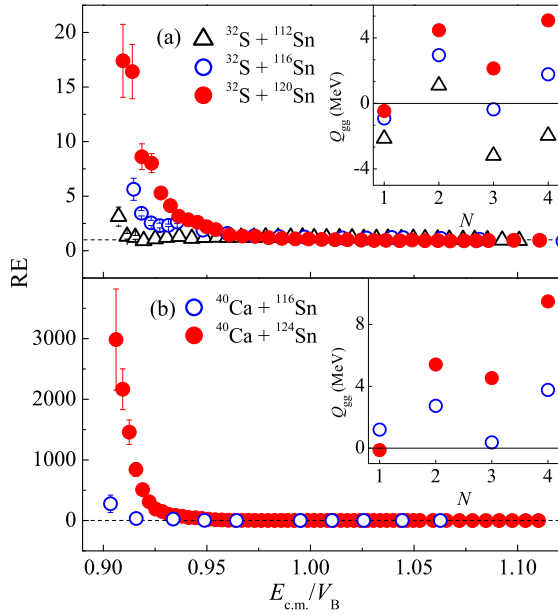
**Fig. 1.** (Color online.) RE as a function of  $E_{\text{c.m.}}/V_B$  for  $^{40,48}\text{Ca} + ^{40,48}\text{Ca}$  and the inserted  $Q_{\text{gg}}$ -values as a function of the number of pickup neutrons ( $N$ ). The original experimental fusion data are taken from Refs. [27,30,31].

RE is a strong sign for the PQNT effect, considering that the vibrational couplings which should be considered in the CC calculations have been calibrated by using the fusion data of  $^{40}\text{Ca} + ^{40}\text{Ca}$  and  $^{48}\text{Ca} + ^{48}\text{Ca}$ .

### 3.2. S, Ca + Sn

Similar PQNT effects on fusion also occur for some other medium-mass systems. Among them, the stable Sn isotopes have very similar collective excitation properties and are ideal for such an analysis. Here the analysis for five systems involving Sn is shown in Fig. 2. The RE for  $^{32}\text{S} + ^{112}\text{Sn}$ , without expected strong PQNT effect, shown in Fig. 2(a) is always close to unity except the lowest one. While for  $^{32}\text{S} + ^{116,120}\text{Sn}$ , RE obviously deviates from unity with decreasing energy. This again shows an isotopic dependence of fusion enhancement related to PQNT. The RE analysis result is consistent with the original argument of the increasing influence of coupling to transfer channels with increasing target mass number (corresponding to the increasing  $Q_{\text{gg}}$ -value) [32]. Furthermore, it seems that RE has a positive-going correlation with  $Q_{\text{gg}}$ -values, as shown in the insert.

For  $^{40}\text{Ca} + ^{116,124}\text{Sn}$ , the experimental fusion [33] also shows significant PQNT effects. The corresponding RE for the two systems is shown in Fig. 2(b). It can be seen that  $^{40}\text{Ca} + ^{124}\text{Sn}$  shows bigger



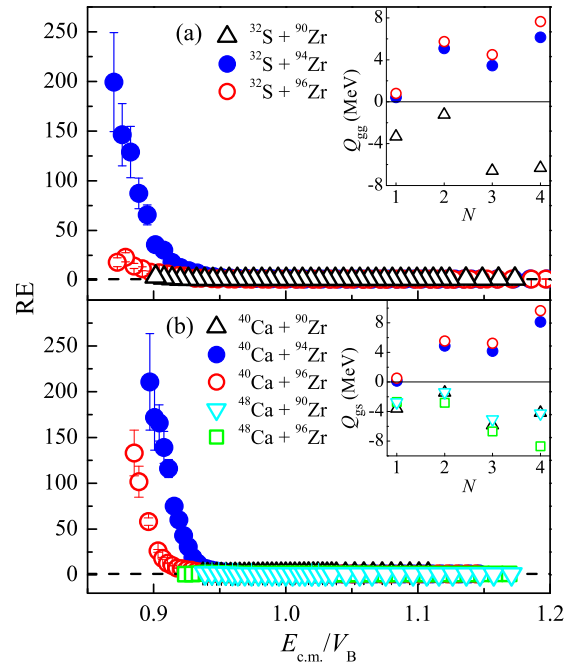
**Fig. 2.** (Color online.) Same as Fig. 1 but for  $^{32}\text{S} + ^{112,116,120}\text{Sn}$  (a) and  $^{40}\text{Ca} + ^{116,124}\text{Sn}$  (b). The original experimental fusion data are taken from Refs. [32,33].

RE than  $^{40}\text{Ca} + ^{116}\text{Sn}$  at the same reduced energy, which is consistent with the semi-classical consideration of the neutron transfer  $Q_{gg}$ -values [19]. Here a positive-going correlation of RE with  $Q_{gg}$ -values is also shown. Furthermore, the heavier  $^{58}\text{Ni} + ^{124}\text{Sn}$ , with more PQNT channels and higher  $Q_{gg}$ -values, shows a stronger PQNT effect than  $^{64}\text{Ni} + ^{124}\text{Sn}$  as well [34], although of the occurrence of the deep-inelastic collision [35]. In brief, RE for the Sn-involved systems analyzed here shows a consistent positive-going correlation with the PQNT  $Q_{gg}$ -values.

### 3.3. S, Ca + Zr

Moreover, the near-barrier nuclear reactions of the typical S, Ca + Zr have been studied extensively as well. For Ca + Zr, the larger experimental neutron pickup cross sections for  $^{40}\text{Ca} + ^{96}\text{Zr}$  than  $^{40}\text{Ca} + ^{90}\text{Zr}$  at near-barrier energies [36] suggests such a correlation between PQNT and fusion enhancement. Also Stefanini et al. [37] confirmed the strong PQNT effects in the fusion of  $^{40}\text{Ca} + ^{96}\text{Zr}$ , rather than the effect of the strong multi-phonon  $3^-$  excitation in  $^{96}\text{Zr}$  [18], by a pure experimental comparison of the  $^{40,48}\text{Ca} + ^{96}\text{Zr}$  fusion data.

For the relatively close systems of  $^{32}\text{S}, ^{40}\text{Ca} + ^{90,94,96}\text{Zr}$ , the experimental fusion [12,26,37,38] really shows a strong relation of the sub-barrier fusion enhancement to PQNT. The corresponding RE for  $^{32}\text{S} + ^{90,94,96}\text{Zr}$  is shown in Fig. 3(a). It can be seen that fusion of  $^{32}\text{S} + ^{90}\text{Zr}$  is reproduced well, while RE shows deviation from unity with decreasing energy for  $^{32}\text{S} + ^{94,96}\text{Zr}$ . Notably, bigger RE shows for  $^{32}\text{S} + ^{94}\text{Zr}$  than  $^{32}\text{S} + ^{96}\text{Zr}$ , although of the similar neutron transfer  $Q_{gg}$ -values. This situation, that  $^{32}\text{S} + ^{94}\text{Zr}$  shows a much larger fusion enhancement than  $^{32}\text{S} + ^{96}\text{Zr}$  despite the similar transfer  $Q_{gg}$ -values, is unanticipated according to the CC calculations as well as semi-classical consideration. This is difficult to explain within the existing models as already discussed [12]. Even a CC calculation including the triple-phonon quadrupole state ( $2^+$ ) $^3$  in  $^{94}\text{Zr}$  additionally only brings negligible sub-barrier fusion enhancement and does not change the conclusion. This suggests that more should be considered for a full clarification of the relevant dynamic reaction processes. At least, the role of the neutron orbital populations, such as shell closure of  $^{90}\text{Zr}$  and sub-shell



**Fig. 3.** (Color online.) Same as Fig. 1 but for  $^{32}\text{S} + ^{90,94,96}\text{Zr}$  (a) and  $^{40}\text{Ca} + ^{90,94,96}\text{Zr}$ ,  $^{48}\text{Ca} + ^{90,96}\text{Zr}$  (b). The original experimental fusion data are taken from Refs. [12, 26,37,38].

closure of  $^{96}\text{Zr}$ , should be considered theoretically. Experimentally, measurement for the transfer reactions of the  $^{94,96}\text{Zr}$ -involved systems should give a meaningful clue. The RE for  $^{40}\text{Ca} + ^{94,96}\text{Zr}$  illustrated in Fig. 3(b) also shows a similar correlation as that for  $^{32}\text{S} + ^{94,96}\text{Zr}$ .

Here, the good reproduction for the fusion of  $^{48}\text{Ca} + ^{96}\text{Zr}$ , as shown in Fig. 3(b), further supports the suitable consideration for the inelastic coupling effect of  $^{96}\text{Zr}$  in the present calculations. This analysis result suggests that a similar reference system for confirming the collective coupling effect of  $^{94}\text{Zr}$  is highly desired to judge whether this is caused by the imperfect CC calculation or by some underlying physics, although it should have been considered well by the present CC calculation.

## 4. Summary

A self-consistent technique to investigate the PQNT effects on sub-barrier fusion is introduced without adjustable parameters. This analysis provides a further reliable proof for the PQNT effects on sub-barrier heavy-ion fusion. Moreover, an unanticipated much larger fusion enhancement for  $^{32}\text{S}, ^{40}\text{Ca} + ^{94}\text{Zr}$  than  $^{32}\text{S}, ^{40}\text{Ca} + ^{96}\text{Zr}$  is pointed out despite the similar transfer  $Q_{gg}$ -values, which cannot be understood from the semi-classical consideration. Examining and explaining this point should be critical for unveiling the underlying physical mechanism, and ultimately developing a reliable model with predictive power and quantified uncertainties for heavy-ion fusion. The proposed technique offers an intuitionistic estimation for the PQNT effects. This improves much the current CC calculations and will be proved useful in other similar studies.

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