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Incremental approach to the nonlinear analysis of reinforcement concrete with cracks at plane stress state

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Abstract

Incremental approach to the nonlinear analysis of reinforcement concrete with cracks at plane stress state is presented. Calculation method is based upon the N. Karpenko deformation model where the essential features of physical nonlinearity of reinforcement concrete such as acquired orthotropy of concrete, formation and propagation of cracks in tension concrete, yield of steel reinforcement are given serious consideration. Constitutional relations are set as relations between finite increments of strains and finite increments of stress at steps of load with regard to stage of material resistance and history of stress state. The proposed incremental modal with the finite element method were used for numerical analysis of experimental deep beams from NIIMosstroy investigation. As a result of calculation the modes of formation and propagation of cracks as well as deflation curves fully corresponding to the experiment data were obtained. Incremental approach has made it possible to substantially reduce the number of iterations at steps of load and is recommended for computer modelling of complicated space systems of buildings and structures.

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1. Introduction

The essential problem of physically nonlinear numerical implementation of models and methods of calculation of reinforced concrete structures is the convergence of iterative processes. As the experience of the finite element analysis [1] shows, after the formation of cracks is required from 50 to 100 iterations at each step of loading to ensure the accuracy of 5% solution of the task in displacements.

This paper deals with the construction of the incremental model of cracked concrete based on general mechanics of reinforced concrete [2], which has been more than experimental testing and accepted as a basis for comparing the results of the transition to the incremental constitutive relations.

The nonlinear anisotropic model of a deep beam accounts for the non-linear nature of the deformation of the concrete and reinforcement and crack formation, which leads to acquired anisotropy. According to this model, the relationship between strains and stresses is represented as:

$$\begin{Bmatrix} \varepsilon_x \\ \varepsilon_y \\ \gamma_{xy} \end{Bmatrix} = \begin{bmatrix} C_{11} & C_{12} & C_{13} \\ C_{12} & C_{22} & C_{23} \\ C_{13} & C_{23} & C_{33} \end{bmatrix} \times \begin{Bmatrix} \sigma_x \\ \sigma_y \\ \tau_{xy} \end{Bmatrix}, \quad (1)$$

where $[C]$ is the symmetric matrix of stiffness.

In forming the matrix $[C]$ are taken into account anisotropy, the physical nonlinearity of concrete and reinforcement, communication linking the crack, the effect of coupling reinforcement with concrete between the cracks in it chart stretching and other factors of nonlinearity. Constitutive relations (1) and the entries of compliance $[C]$ shall be established on the basis of the analysis of stress-strain state of minor elements of the deep beam. Here are four stages of work of the material:

- linear uncracked (elastic stage of concrete and reinforcement);
- nonlinear without cracks (with the influence of plastic deformations in the concrete);
- cracked when the elastic reinforcement in the cracks;
- cracked when the steel beyond the elastic limit.

Cracks in the concrete are formed of the major sites of action of tensile stress when their value reaches the limit of tensile strength of concrete under plane stress state (violation of strength criterion). The constitutive relation between the strain and stress vectors are nonlinear and depend strongly on the efforts achieved in concrete and reinforcement.

2. Procedure for solving the problem

Application of stepwise, iterative procedure for solving the problem provides a complete curve of the equilibrium state of design and trace the history of its operation under increasing load until fracture, but it requires a very large amount of computation. This shortcoming is particularly noticeable in solving large-tasks. Step method with a small increase in the load parameter allows to obtain an approximate curve of equilibrium states, but this approach, using the secant stiffness characteristics, resulting in significant errors.

Consider the decision of physically nonlinear task of concrete in finite increments. In this case, the system of constitutive relations in the cross-sections of the drive module is converted to the system of constitutive relations in the conditional tangent modulus method. The initial system of constitutive relationships with secant stiffness characteristics in step j is as follows:

$$\begin{Bmatrix} \varepsilon_{xj} \\ \varepsilon_{yj} \\ \gamma_{xyj} \end{Bmatrix} = \begin{bmatrix} C_{11j} & C_{12j} & C_{13j} \\ C_{12j} & C_{22j} & C_{23j} \\ C_{13j} & C_{23j} & C_{33j} \end{bmatrix} \times \begin{Bmatrix} \sigma_{xj} \\ \sigma_{yj} \\ \tau_{xyj} \end{Bmatrix} \quad (2)$$

a similar system in step $j+1$:

$$\begin{Bmatrix} \mathcal{E}_{xj+1} \\ \mathcal{E}_{yj+1} \\ \mathcal{E}_{xyj+1} \end{Bmatrix} = \begin{bmatrix} C_{11j+1} & C_{12j+1} & C_{13j+1} \\ C_{12j+1} & C_{22j+1} & C_{23j+1} \\ C_{13j+1} & C_{23j+1} & C_{33j+1} \end{bmatrix} \times \begin{Bmatrix} \sigma_{xj+1} \\ \sigma_{yj+1} \\ \tau_{xyj+1} \end{Bmatrix}. \quad (3)$$

Subtract from the system (3) system (2):

$$(\mathcal{E}_{xj+1} - \mathcal{E}_{xj}) = (C_{11j+1}\sigma_{xj+1} - C_{11j}\sigma_{xj}) + (C_{12j+1}\sigma_{yj+1} - C_{12j}\sigma_{yj}) + (C_{13j+1}\tau_{xyj+1} - C_{13j}\tau_{xyj}); \quad (4)$$

$$(\mathcal{E}_{yj+1} - \mathcal{E}_{yj}) = (C_{12j+1}\sigma_{xj+1} - C_{12j}\sigma_{xj}) + (C_{22j+1}\sigma_{yj+1} - C_{22j}\sigma_{yj}) + (C_{23j+1}\tau_{xyj+1} - C_{23j}\tau_{xyj}); \quad (5)$$

$$(\mathcal{E}_{xyj+1} - \mathcal{E}_{xyj}) = (C_{13j+1}\tau_{xyj+1} - C_{13j}\tau_{xyj}) + (C_{23j+1}\tau_{xyj+1} - C_{23j}\tau_{xyj}) + (C_{33j+1}\tau_{xyj+1} - C_{33j}\tau_{xyj}). \quad (6)$$

Let

$$\Delta\sigma_{xj+1} = \sigma_{xj+1} - \sigma_{xj}; \quad \Delta\sigma_{yj+1} = \sigma_{yj+1} - \sigma_{yj}; \quad \Delta\sigma_{xyj+1} = \tau_{xyj+1} - \tau_{xyj}; \quad (7)$$

$$\Delta\mathcal{E}_{xj+1} = \mathcal{E}_{xj+1} - \mathcal{E}_{xj}; \quad \Delta\mathcal{E}_{yj+1} = \mathcal{E}_{yj+1} - \mathcal{E}_{yj}; \quad \Delta\mathcal{E}_{xyj+1} = \mathcal{E}_{xyj+1} - \mathcal{E}_{xyj}. \quad (8)$$

Divide and multiply while the first differences on the right side of (4)-(6) to $\Delta\sigma$ and denote:

$$C_{11j+1}^k = \frac{C_{11j+1}\sigma_{xj+1} - C_{11j}\sigma_{xj}}{\Delta\sigma_{xj+1}}; \quad C_{22j+1}^k = \frac{C_{22j+1}\sigma_{yj+1} - C_{22j}\sigma_{yj}}{\Delta\sigma_{yj+1}}; \quad (9)$$

$$C_{12j+1}^k = \frac{1}{2} \left(\frac{C_{12j+1}\sigma_{yj+1} - C_{12j}\sigma_{yj}}{\Delta\sigma_{yj+1}} + \frac{C_{12j+1}\sigma_{xj+1} - C_{12j}\sigma_{xj}}{\Delta\sigma_{xj+1}} \right); \quad C_{13j+1}^k = \frac{1}{2} \left(\frac{C_{13j+1}\tau_{xyj+1} - C_{13j}\tau_{xyj}}{\Delta\tau_{xyj+1}} + \frac{C_{13j+1}\sigma_{xj+1} - C_{13j}\sigma_{xj}}{\Delta\sigma_{xj+1}} \right); \quad (10)$$

$$C_{23j+1}^k = \frac{1}{2} \left(\frac{C_{23j+1}\tau_{xyj+1} - C_{23j}\tau_{xyj}}{\Delta\tau_{xyj+1}} + \frac{C_{23j+1}\sigma_{yj+1} - C_{23j}\sigma_{yj}}{\Delta\sigma_{yj+1}} \right); \quad C_{33j+1}^k = \frac{C_{33j+1}\tau_{xyj+1} - C_{33j}\tau_{xyj}}{\Delta\tau_{xyj+1}}. \quad (11)$$

Taking into account the expressions (7)-(11) the system (4)-(6) is converted to the form:

$$\begin{Bmatrix} \Delta\mathcal{E}_{xj+1} \\ \Delta\mathcal{E}_{yj+1} \\ \Delta\mathcal{E}_{xyj+1} \end{Bmatrix} = \begin{bmatrix} C_{11j+1}^k & C_{12j+1}^k & C_{13j+1}^k \\ C_{12j+1}^k & C_{22j+1}^k & C_{23j+1}^k \\ C_{13j+1}^k & C_{23j+1}^k & C_{33j+1}^k \end{bmatrix} \times \begin{Bmatrix} \Delta\sigma_{xj+1} \\ \Delta\sigma_{yj+1} \\ \Delta\tau_{xyj+1} \end{Bmatrix}. \quad (12)$$

In solving the problem step method with equal increments of load dependencies (9)-(11) can take approximately:

$$\Delta\sigma_{xj+1} \approx \Delta\sigma_{xj} = \sigma_{xj} - \sigma_{xj-1}; \quad (13)$$

similarly

$$\Delta\sigma_{yj+1} \approx \Delta\sigma_{yj}; \quad \Delta\tau_{xyj+1} \approx \Delta\tau_{xyj}; \quad \sigma_{xj+1} \approx \sigma_{xj} + \Delta\sigma_{xj}; \quad \sigma_{yj+1} \approx \sigma_{yj} + \Delta\sigma_{yj}; \quad \tau_{xyj+1} \approx \tau_{xyj} + \Delta\tau_{xyj}. \quad (14)$$

The meaning of the above formal transformations is that by using the expressions (14) in steps of loading accumulated errors are small and at each step can be dispensed one iteration.

The proposed low iterative approach to physically nonlinear analysis of reinforced concrete deep beams in the mean value is realized in the form of an algorithm and a computer program for calculating. The adequacy of the

proposed approach is evaluated by comparing the calculated and experimental data characterizing the operation of the structure in the process of loading. We investigated stepwise, iterative procedures, built on the basis of cross-sections of the stiffness matrix and stepping procedure using the tangent matrix.

Calculation of reinforced concrete deep beam of the experiment of NIIMosstroy was made. The deep beam height of 258 cm, a thickness of 22 cm and 5.7 m span was tested for short-term load. Prism strength of concrete at test was 20 MPa. Reinforcement of structure was made welded mesh from rods A400 grade 18 mm diameter with step 100 mm. For the calculation the symmetric part of the beam was divided into 50 rectangular finite elements.

3. Conclusions

The experiment was carried out in stages loading samples delayed at every stage for 10-15 minutes. In calculating the loading step was determined taking into account the gradual increase in load and was received in the amount of 5% of the estimated breaking force. The results of calculation of the deep beam by the proposed method were compared with experimental data, linear-elastic analysis and nonlinear analysis performed by an iterative method using the secant stiffness characteristics.

The first cracks were appeared in a lowest layer of finite elements with a total load of 400 kN. Experimental fracture load was 385 kN. The direction of the principal axes was 86° , which also corresponds to the experimental data.

Before cracking behavior of structure been mostly linear-elastic nature. In the future, the behavior becomes pronounced nonlinear. The theoretical curve of deflection calculated by the proposed method is fully consistent with the divergence experienced at certain points to 10%. The deviation of the theoretical curve obtained using an iterative method with secant stiffness characteristics, was 7%.

The carried out checks showed the high efficiency of low iterative process. The achieved results allow us to recommend a low iterative method for numerical simulation of complex concrete structures.

References

- [1] A. N. Petro, Deformation model of nonlinear creep of reinforcement concrete and its application to analysis of elements at plane stress state. Petrozavodsk, 2002 (in Russian).
- [2] N. I. Karpenko, General models of mechanics of reinforced concrete. Moscow, Stroyizdat, 1996, pp. 60-91 (in Russian).