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ORIGINAL ARTICLE

Analytical investigation of squeezing unsteady nanofluid flow between parallel plates by LSM and CM

O. Pourmehran, M. Rahimi-Gorji, M. Gorji-Bandpy, D.D. Ganji *

Babol Noshirvani University of Technology, Department of Mechanical Engineering, Babol, Iran

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Abstract An analytical investigation is applied for unsteady flow of a nanofluid squeezing between two parallel plates. Collocation Method (CM), Least Square Method (LSM) and fourth-order Runge-Kutta numerical method (NM) are used to solve the present problem. The results were compared with those obtained from Collocation Method (CM), Least Square Method and the established Numerical Method (Fourth order Runge-Kutta) scheme. It demonstrated LSM and CM presented accurate results. Water (H_2O) was the base fluid that contained different kinds of nanoparticles that is, Copper, Silver, Alumina and Titanium Oxide. The effective thermal conductivity and viscosity of the nanofluid are calculated using the Maxwell–Garnetts (MG) and Brinkman models, respectively. The analytical investigation is carried out for various governing parameters such as the squeeze number, nanoparticle volume fraction and Eckert number. As a main outcome from the present study, it is observed that the results of LSM are more accurate than CM and they are in excellent agreement with numerical ones, so LSM can be used for finding analytical solutions of coupled equations in nanofluid problems easily. The results demonstrate when two plates are moving together, the Nusselt number increases b of nanoparticle volume fraction and Eckert number while it decreases with growth of the squeeze number.

1. Introduction

Nanofluids are widely encountered in many industrial and technology applications, for example, melts of polymers, biological solutions, paints, asphalts and glues, etc. Nanofluids appear to have the potential to significantly increase heat transfer rates in a variety of areas. Heat and mass transfer for unsteady squeezing viscous flow between two parallel plates is one of the most important research topics due to its wide range of scientific and engineering applications, such as

* Corresponding author at: Babol University of Technology, Babol, P.O. Box 484, Iran. Tel./fax: +98 111 3234205.
E-mail addresses: oveis87@yahoo.com (O. Pourmehran), m69_rahimi@yahoo.com (M. Rahimi-Gorji), gorji@nit.ac.ir (M. Gorji-Bandpy), ddg_davood@yahoo.com (D.D. Ganji).
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hydrodynamical machines, polymer processing, lubrication system, chemical processing equipment, formation and dispersion of fog, damage of crops due to freezing, food processing and cooling towers. The first research on the squeezing flow in lubrication system was reported by Stefan [1]. Mahmood et al. [2] investigated the heat transfer characteristics in the squeezed flow over a porous surface. Mustafa et al. [3] studied heat and mass transfer characteristics in a viscous fluid which is squeezed between parallel plates.

They found that the magnitude of local Nusselt number is an increasing function of Pr and Ec. Magnetohydrodynamic squeezing flow of a viscous fluid between parallel disks was analyzed by Domairry and Aziz [4].

There are some simple and accurate approximation techniques for solving differential equations called the Weighted Residuals Methods (WRMs). Collocation (CM), Galerkin (GM) and Least Square (LSM) are examples of the WRMs. Stern and Rasmussen [5] used Collocation Method for solving a third order linear differential equation. Vaferi et al. [6] studied the feasibility of applying of Orthogonal Collocation Method to solve diffusivity equation in the radial transient flow system. Hendi and Albugami [7] used the Collocation and Galerkin methods for solving Fredholm–Volterra integral equation. Recently Least Square Method is introduced by Aziz and Bouaziz [8] and is applied for prediction of the performance of a longitudinal fin [9]. They found that least squares method is simple compared with other analytical methods. Shaoqin and Huoyuan [10] developed and analyzed least squares approximations for the incompressible magnetohydrodynamic (MHD) equations. Recently Hatami et al. [11] and Sheikholeslami et al. [12] applied LSM and CM on fin performance and nanofluid in porous channel respectively. Ellahi [13] used homotopy analysis method (HAM) analytical solution of MHD non-Newtonian nanofluid in a pipe. In another study, Rashad et al. [14] investigated the natural convection of non-Newtonian nanofluid around a vertical permeable cone. Hajmohammadi and Nourazar [15] considered the fluid flow and heat transfer repercussions for introducing a thin (micro) gas layer into a cylindrical Couette flow assembly dealing with a power law liquid (lubricant). They concluded that the thin gas layer stabilizes or destabilizes the flow, depending on the magnitude of the power index number characterizing the liquid.

Enhancement of heat transfer performance in many industrial fields such as power, manufacturing and transportation, is an essential topic from an energy saving perspective. The low thermal conductivity of conventional heat transfer fluids such as water and oils is a primary limitation in enhancing the performance and the compactness of such systems. Solids typically have a higher thermal conductivity than liquids. For example, copper has a thermal conductivity 700 times greater than water and 3000 times greater than engine oil. An innovative technique to enhance heat transfer is using solid particles in the base fluid (i.e. nanofluids) in the range of sizes up to 100 nm [16]. Khanafer et al. [17] firstly conducted a numerical investigation on the heat transfer enhancement due to adding nano-particles in a differentially heated enclosure. They found that the suspended nanoparticles substantially increase the heat transfer rate at any given Grashof number. Hajmohammadi and Nourazar [18] studied a conjugate forced convection heat transfer from a good conducting plate with temperature-dependent thermal conductivity by using DTM. They concluded that for a good conducting plate with a finite thickness the distribution of the conjugate heat flux at the upper surface is significantly affected by the plate thickness. Also they solved two problems, the conjugate heat transfer problem [19] and a characteristic value problem occurring in linear stability analysis [20] by ADM and DTM.

Khanafer et al. [17] studied magnetohydrodynamic flow in a nanofluid filled inclined enclosure with sinusoidal wall. They reported that for all values Hartmann number, at $Ra = 10^4$ and 10^5 maximum values of E is obtained at $\gamma = 60°$ and $\gamma = 0°$, respectively. Kalidas Das et al. [21] investigated the unsteady boundary layer flow of a nanofluid over a heated stretching sheet with thermal radiation. They found that the heat transfer rate at the surface increases in the presence of Brownian motion but reverse effect occurs for thermophoresis. Rizwan Ul Haq et al. [22] analyze the flow of three-dimensional water-based nanofluid over an exponentially stretching sheet. Their results illustrate the effects of various parameters including the temperature exponent, stretching parameter and volume fraction of three different types of nanoparticles, such as copper (Cu), alumina ($Al_2\,O_4$) and titanium dioxide (TiO_2) with water as a base fluid. Noreen Sher Akbar et al. [23] studied MHD peristaltic flow of a Carreau nanofluid in an asymmetric channel. They understood that the pressure rise increases with increase in Hartmann Number and thermophoresis parameter. Kalidas Das [24] studied the problem of unsteady MHD free convection flow of nanofluids via a porous medium bounded by a moving vertical semi-infinite permeable flat plate with constant heat source and convective boundary condition in a rotating frame of reference theoretically. Sheikholeslami et al. [25] studied the flow and heat transfer of a nanofluid over a stretching cylinder in the presence of magnetic field. They found that choosing copper (for small values of magnetic parameter) and alumina (for large values of magnetic parameter) leads to the highest cooling performance for this problem. Effect of static radial magnetic field on natural convection heat transfer in a horizontal cylindrical annulus enclosure filled with nanofluid was investigated numerically using the Lattice Boltzmann method by Ashorynejad et al. [26]. They showed that the average Nusselt number increases as nanoparticle volume fraction and Rayleigh number increase, while it decreases as Hartmann number increases. Sheikholeslami et al. [27] performed a numerical analysis for natural convection heat transfer of cu-water nanofluid in a cold outer circular enclosure containing a hot inner sinusoidal circular cylinder in the presence of horizontal magnetic field. They concluded that in the absence of magnetic field, enhancement ratio decreases as Rayleigh number increases while an opposite trend is observed in the presence of magnetic field. Sheikholeslami et al. [28] investigated the nanofluid flow and heat transfer characteristics between two horizontal plates in a rotating system. Their results showed that for suction and injection, the heat transfer rate at the surface increases with increase of the nanoparticle volume fraction, Reynolds number. There have been published several recent numerical studies on the modeling of natural convection heat transfer in nanofluids [29–35]. Sheikholeslami et al. [36] have studied analytically with Adomian Decomposition Method (ADM) scheme on present problem and their results had very good agreement with the older researches. Therefore, Collocation Method and Least Square Method were used to find efficient, reliable and precise solutions.

In this study, Collocation Method and Least Square Method are applied to find the semi-analytical solutions of nonlinear differential equations governing the problem of unsteady squeezing nanofluid flow and heat transfer. The effects of the squeeze number, the nanofluid volume fraction and Eckert number on Nusselt number and skin friction coefficient are investigated.

2. Description of the problem

The unsteady flow and heat transfer in a two-dimensional squeezing nanofluid between two infinite parallel plates are considered in this study. The two plates are placed at $z = \pm l(1-\alpha t)^{\frac{1}{2}} = \pm h(t)$. For $\alpha > 0$, two plates are squeezed until they touch $t = \frac{1}{\alpha}$. For $\alpha < 0$, the two plates are separated. The viscous dissipation effect, is retained. This behavior occurs at high Eckert number ($\gg 1$). The Eckert number expresses the relationship between a flow's kinetic energy and enthalpy. The fluid is a water based nanofluid containing copper, silver, alumina and titanium oxide. The nanofluid is a two component mixture with the following assumptions: incompressible; no-chemical reaction; negligible radiative heat transfer; nano-solid-particles and the base fluid are in thermal equilibrium and no slip occurs between them. Flow is considered laminar and stable. All body forces are neglected. All properties are considered fixed and the boundary condition assumed as isothermal. The thermophysical properties of the nanofluid are given in Table 1. The governing equations are as follows:

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0 \tag{1}$$

$$\rho_{nf}\left(\frac{\partial u}{\partial t} + u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y}\right) = -\frac{\partial p}{\partial x} + \mu_{nf}\left(\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2}\right) \tag{2}$$

$$\rho_{nf}\left(\frac{\partial v}{\partial t} + u\frac{\partial v}{\partial x} + v\frac{\partial v}{\partial y}\right) = -\frac{\partial p}{\partial y} + \mu_{nf}\left(\frac{\partial^2 v}{\partial x^2} + \frac{\partial^2 v}{\partial y^2}\right) \tag{3}$$

$$\frac{\partial T}{\partial t} + u\frac{\partial T}{\partial x} + v\frac{\partial T}{\partial y} = \frac{k_{nf}}{(\rho C_p)_{nf}}\left(\frac{\partial^2 T}{\partial x^2} + \frac{\partial^2 T}{\partial y^2}\right) + \frac{\mu_{nf}}{(\rho C_p)_{nf}}\left(4\left(\frac{\partial u}{\partial x}\right)^2 + \left(\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y}\right)^2\right) \tag{4}$$

where u and v are the velocities in the x and y directions, respectively. Effective density (ρ_{nf}), the effective dynamic viscosity (μ_{nf}), effective heat capacity $(\rho C_p)_{nf}$ and the effective thermal conductivity k_{nf} of the nanofluid are defined as [4]:

$$\rho_{nf} = (1-\varphi)\rho_f + \varphi\rho_s,$$

$$(\rho C_p)_{nf} = (1-\varphi)(\rho C_p)_f + \varphi(\rho C_p)_s$$

$$\mu_{nf} = \frac{\mu_f}{(1-\varphi)^{2.5}}, \quad (Brinkman) \tag{5}$$

$$\frac{k_{nf}}{k_f} = \frac{k_s + 2k_f - 2\varphi(k_f - k_s)}{k_s + 2k_f + 2\varphi(k_f - k_s)} \quad (Maxwell\text{–}Garnetts).$$

The relevant boundary conditions for the problem are

$$v = v_w = \frac{dh}{dt}, \ T = T_H \quad at\ y = h(t),$$

$$v = \frac{\partial u}{\partial y} = \frac{\partial T}{\partial y} = 0 \quad at\ y = 0 \tag{6}$$

We introduce these parameters:

$$\eta = \frac{y}{[l(1-\alpha t)^{\frac{1}{2}}]}, \quad u = \frac{\alpha x}{[2(1-\alpha t)]}f'(\eta),$$

$$v = -\frac{\alpha l}{\left[2(1-\alpha t)^{\frac{1}{2}}\right]}f(\eta), \quad \theta = \frac{T}{T_H}, \tag{7}$$

$$A_1 = (1-\varphi) + \varphi\frac{\rho_s}{\rho_f}.$$

The above variables are substituted into Eqs. (2) and (3). Then the pressure gradient is eliminated from the resulting equations:

$$f^{iv} - SA_1(1-\varphi)^{2.5}(\eta f''' + 3f'' + f'f'' - ff'') = 0 \tag{8}$$

Using Eq. (7), Eqs. (3) and (4) reduce to the following differential equations:

$$\theta'' + PrS\left(\frac{A_2}{A_3}\right)(f\theta' - \eta\theta') + \frac{PrEc}{A_3(1-\varphi)^{2.5}}\left(f''^2 + 4\delta^2 f'^2\right) = 0, \tag{9}$$

where A_2 and A_3 are dimensionless constants given by

$$A_2 = (1-\varphi) + \varphi\frac{(\rho C_p)_s}{(\rho C_p)_f},$$

$$A_3 = \frac{k_{nf}}{k_f} = \frac{k_s + 2k_f - 2\varphi(k_f - k_s)}{k_s + 2k_f + 2\varphi(k_f - k_s)} \tag{10}$$

With the following boundary conditions,

$$\begin{aligned} &f(0) = 0, \ f''(0) = 0 \\ &f(1) = 1, \quad f'(1) = 0 \\ &\theta'(0) = 0, \quad \theta(1) = 1 \end{aligned} \tag{11}$$

In Eq. (9), S is the squeeze number, Pr is the Prandtl number and Ec is the Eckert number, which are defined as follows:

$$S = \frac{\alpha l^2}{2v_f}, \quad Pr = \frac{\mu_f(\rho C_p)_f}{\rho_f k_f}, \quad Ec = \frac{\rho_f}{(\rho C_p)_f}\left(\frac{\alpha x}{2(1-\alpha t)}\right)^2, \quad \delta = \frac{l}{x} \tag{12}$$

Physical quantities of interest are the skin friction coefficient and Nusselt number which are defined as follows:

$$C_f = \frac{\mu_{nf}\left(\frac{\partial u}{\partial y}\right)_{y=h(t)}}{\rho_{nf}v_w^2}, \quad Nu = \frac{-lk_{nf}\left(\frac{\partial T}{\partial y}\right)_{y=h(t)}}{kT_H} \tag{13}$$

In terms of Eq. (7), it can be obtained

Table 1 Thermophysical properties of water and nanoparticles.

	ρ (kg/m^3)	C_p (j/kjk)	k (W/m k)
Pure water	997.1	4179	0.613
Copper (Cu)	8933	385	401
Silver (Ag)	10,500	235	429
Alumina (Al_2O_3)	3970	765	40
Titanium oxide (TiO_2)	4250	686.2	8.9538

$$C_f^* = \frac{l^2}{x^2(1-\alpha t)Re_x C_f} = A_1(1-\varphi)^{2.5} f''(1) \tag{14}$$

$$Nu^* = \sqrt{1-\alpha t}Nu = -A_3\theta'(1).$$

3. Applied methods

Before presenting the results, it is necessary to provide some background knowledge about the mathematical methods employed. Therefore, in this section, some basic relationships and theories concerning Collocation method (CM) and fourth order Runge-Kutta Numerical Method are presented.

There are some simple and accurate approximation techniques for solving differential equations called the Weighted Residuals Methods (WRMs). Collocation (CM), Galerkin (GM) and Least Square (LSM) are examples of the WRMs. Collocation Method (CM) was firstly introduced by Ozisik [37] for solving differential equations in heat transfer problems. Stern and Rasmussen [5] used collocation method for solving a third order linear differential equation. Vaferi et al. [38] studied the feasibility of applying of Orthogonal Collocation method to solve diffusivity equation in the radial transient flow system. Many advantages of CM compared to other analytical make it more valuable and motivate researchers to use it for solving problems. Some of these advantages are listed below [39]:

(a) WRMs solve the equations directly and no simplifications are needed.
(b) They do not need any perturbation, linearization or small parameter versus Homotopy Perturbation Method (HPM) and Parameter Perturbation Method (PPM).
(c) They are simple and powerful compared to numerical methods and achieve final results faster than numerical procedures while their results are acceptable and have excellent agreement with numerical outcomes, furthermore their accuracy can be increased by increasing the statements of the trial functions.
(d) They do not need to determine the auxiliary parameter and auxiliary function versus Homotopy Analysis Method (HAM).

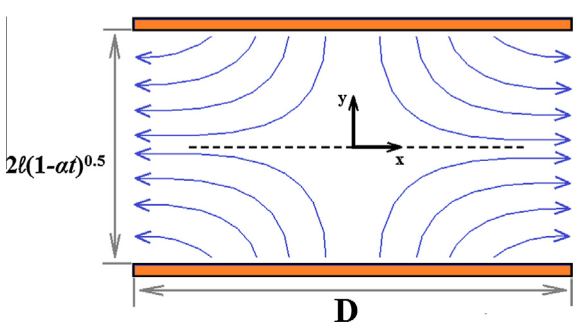


Figure 1 Geometry of physical model.

3.1. Collocation method

For conception of the main idea of this method, suppose a differential operator D is acted on a function u to produce a function p [11]:

$$D(u(x)) = p(x) \tag{15}$$

We wish to approximate u by a function $\tilde{u}$, which is a linear combination of basic functions chosen from a linearly independent set. That is,

$$\tilde{u} = \tilde{u} = \sum_{i=1}^{n} c_i \varphi_i \tag{16}$$

Now, when substituted into the differential operator, D, the result of the operations is not $p(x)$. Hence an error or residual will exist:

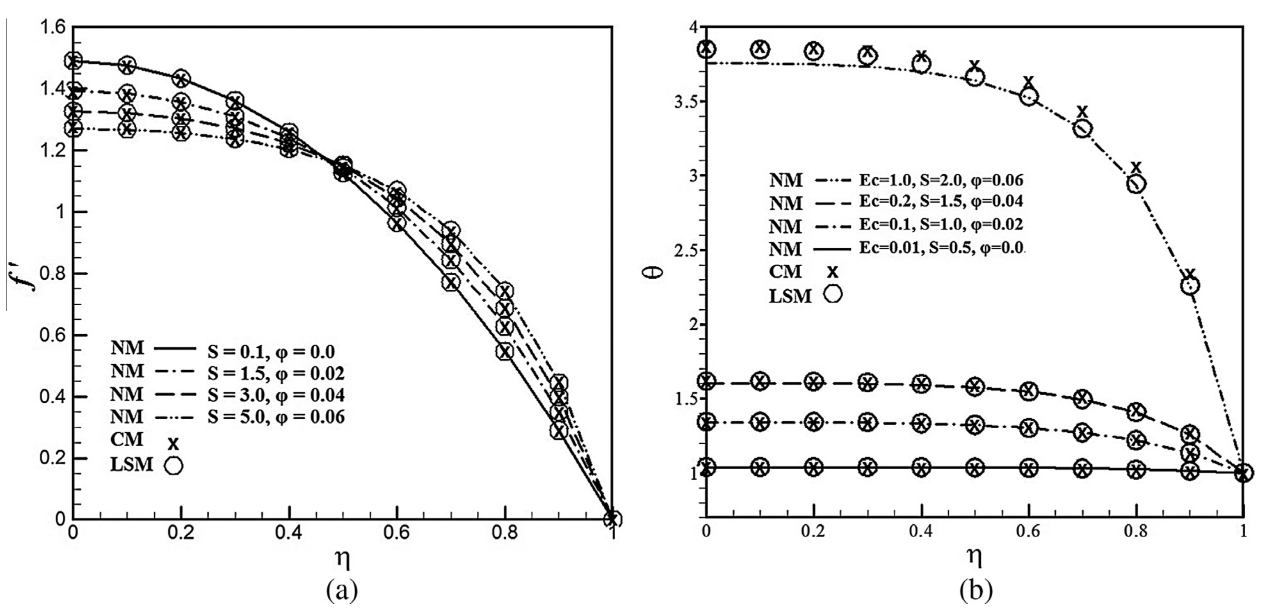


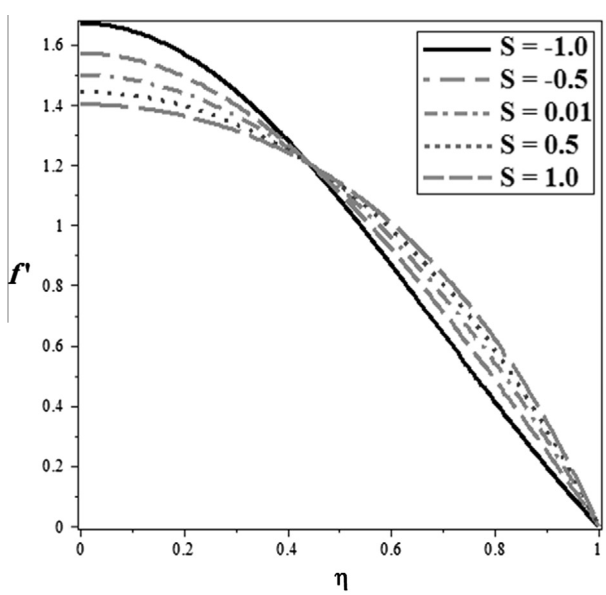
Figure 2 Comparison between results obtained via numerical solution, CM and LSM *at* $\delta = 0.1$ and $Pr = 6.2$ (Titanium Oxide–Water).

Table 2 Comparison of Nusselt number ($Nu^* = -\theta'(1)$) between the present results and analytical results obtained by Mustafa et al. [3] and Sheikholeslami et al. [28] for Copper (Cu) when $S = 0.5$ and $\delta = 0.1$ and $\varphi = 0.0$.

Pr	Ec	Mustafa et al.	Sheikholeslami et al.	Present work		NM
				LSM	CM	
0.5	1	1.522368	1.522367495	1.520649143	1.526577091	1.518859607
1	1	3.026324	3.026323559	3.023438178	3.03732009	3.019545607
2	1	5.98053	5.980530397	5.976762396	6.012625387	5.967887511
5	1	14.43941	14.43941323	14.44158461	14.59172334	14.41394678
1	0.5	1.513162	1.513161806	1.511719088	1.518660044	1.509772834
1	1.2	3.631588	3.631588268	3.628125812	3.644784107	3.623454726
1	2	6.052647	6.052647107	6.046876352	6.074640177	6.039091204
1	5	15.13162	15.13161783	15.11719088	15.18660044	15.09772808

Table 3 Comparison between the results of NM and LSM and CM solution for $f(\eta)$ and $\theta(\eta)$ when $S = 1$, $Pr = 6.2$, $Ec = 0.01$, $\phi = 0.02$ (Cu-Water) and $\delta = 0.01$.

η	$f(\eta)$			$\theta(\eta)$		
	NM	LSM	CM	NM	LSM	CM
0	0	0	0	1.032064	1.032857	1.032904953
0.1	0.141359	0.141345	0.141355	1.032061	1.032843	1.032894552
0.2	0.280666	0.280643	0.28066	1.03203	1.032749	1.032819532
0.3	0.415781	0.415753	0.415772	1.03189	1.032483	1.032602703
0.4	0.544379	0.544353	0.544368	1.031506	1.031934	1.032134625
0.5	0.663857	0.663838	0.663844	1.030663	1.030933	1.031239748
0.6	0.771229	0.771218	0.771215	1.029039	1.029205	1.029621244
0.7	0.863016	0.863012	0.863002	1.026151	1.026284	1.026777451
0.8	0.93512	0.93512	0.935109	1.021259	1.021394	1.021882826
0.9	0.982695	0.982696	0.982691	1.013186	1.013282	1.01362631
1	1	1	1	1	1	1



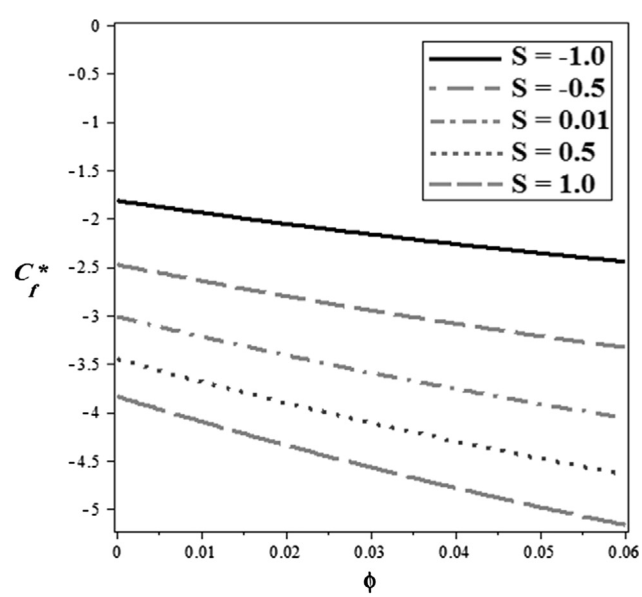


Figure 3 Effect of the squeeze number on the (a) velocity profile at $Pr = 6.2$, $Ec = 0.5$, $\varphi = 0.06$, $\delta = 0.1$ and (b) skin friction coefficient at $Ec = 1$ (Silver–Water).

$$E(x) = R(x) = D(\tilde{u}(x)) - p(x) \neq 0 \tag{17}$$

The notion in the Collocation is to force the residual to zero in some average sense over the domain. That is [34],

$$\int R(x)W_i(x)dx = 0 \quad i = 0, 1, 2, \ldots, n \tag{18}$$

where the number of weight functions W_i is exactly equal to the number of unknown constants in $\tilde{u}$. The result is a set of n algebraic equations for the unknown constants c_i. For collocation method, the weighting functions are taken from the family of Dirac δ functions in the domain. That is $W_i(x) = \delta(x - x_i)$. The Dirac δ function has the property of [11,40,13]

$$\delta(x - x_i) = \begin{cases} 1 & \text{if } x = x_i \\ 0 & \text{otherwise} \end{cases} \tag{19}$$

And residual function in Eq. (9) must be forced to be zero at specific points.

3.2. Least square method

If the continuous summation of all the squared residuals is minimized, the rationale behind the name can be seen. In other words, a minimum of [37]

$$S = \int R(x)R(x)dx = \int R^2(x)dx \tag{20}$$

In order to achieve a minimum of this scalar function, the derivatives of S with respect to all the unknown parameters must be zero. That is,

$$\frac{\partial S}{\partial c_i} = 2\int R(x)\frac{\partial R}{\partial c_i}dx = 0 \tag{21}$$

Comparing with Eq. (22), the weight functions are seen to be [39]

$$W_i = k\frac{\partial R}{\partial c_i}, \quad k = 2 \tag{22}$$

However, the "k" coefficient can be dropped, since it cancels out in the equation. Therefore the weight functions for the Least Squares Method are just the derivatives of the residual with respect to the unknown constants

$$W_i = \frac{\partial R}{\partial c_i} \tag{23}$$

3.3. Fourth order Runge-Kutta method (NM)

It is obvious that the type of the current problem is boundary value problem (BVP) and the appropriate method needs to be chosen. The available sub-methods in the Maple 17.0 are a combination of the base schemes; trapezoid or midpoint method. There are two major considerations when choosing a method for a problem. The trapezoid method is generally efficient for typical problems, but the midpoint method is so capable of handling harmless end-point singularities that the trapezoid method cannot. The midpoint method, also known as the fourth-order Runge–Kutta–Fehlberg method, improves the Euler method by adding a midpoint in the step which increases the accuracy by one order. Thus, the midpoint method is used as a suitable numerical technique in present study [41–46].

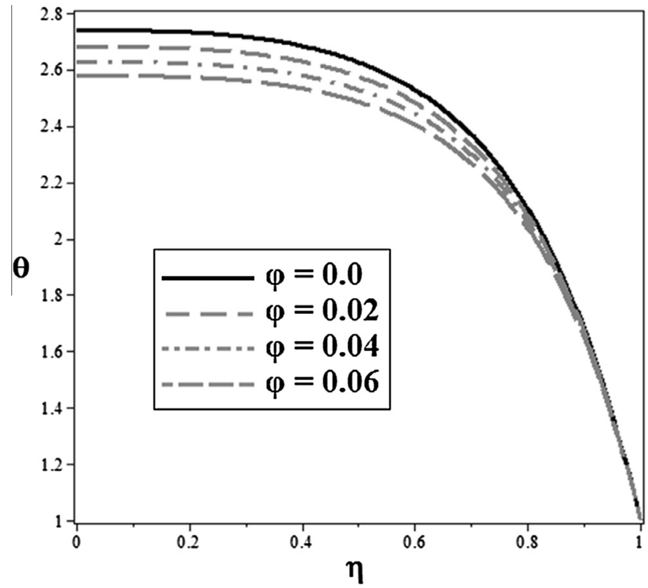


Figure 4 The temperature profile by effect of volume fraction of nanofluid when $Pr = 0.6$, $\delta = 0.1$, $Ec = 0.5$ and $S = 1$ (Silver–Water).

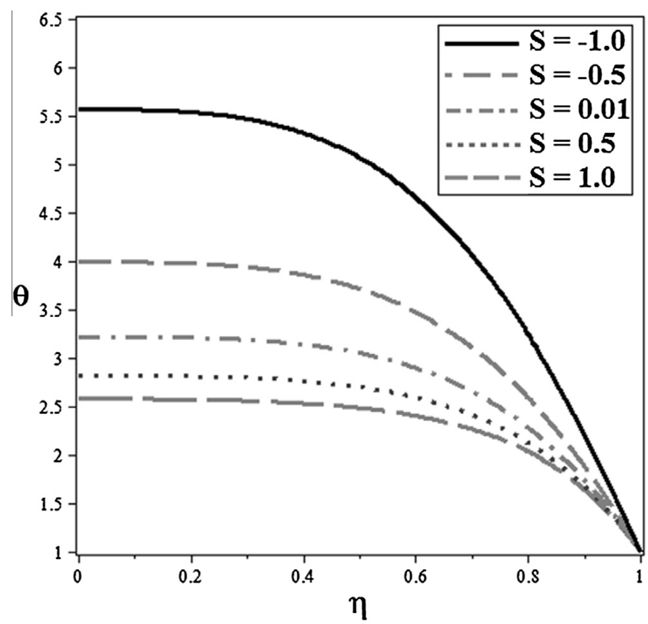


Figure 5 The temperature profile by effect of squeeze number when $Pr = 0.6$, $\delta = 0.1$, $Ec = 0.5$ and $\varphi = 0.0$ (Silver–Water).

4. Results and discussion

In present study, pure water (H_2O) is the base fluid that containing different kinds of nanoparticles namely copper, silver, alumina and titanium oxide and governing equations for nanofluid are solved by LSM, CM and NM. For solving Eqs. (8)

and (9) by WRMs, because trial functions must satisfy the boundary conditions in Eq. (11), so these functions ($f(\eta)$ and $\theta(\eta)$) should contain statements which satisfy boundary condition. In this study, four statements are considered for velocity and temperature profiles and as explained in above WRMs advantages, accuracy of results can be increased by increasing the number of statements, so

$$f(\eta) = \eta^3 + C_1(\eta^3 - \eta) + C_2(\eta^3 - \eta^5) + C_3(\eta^3 - \eta^7) \tag{24}$$

$$\theta(\eta) = 1 + +C_4(1 - \eta^3) + +C_5(1 - \eta^5) + +C_6(1 - \eta^7) \tag{25}$$

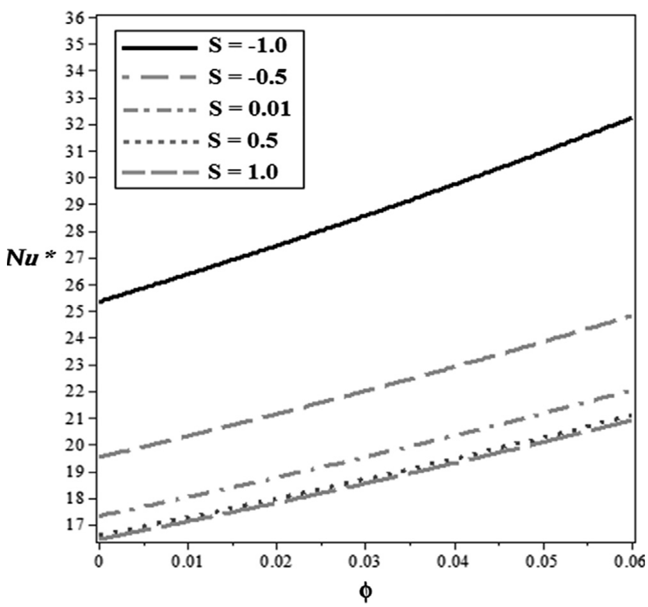


Figure 6 Variety of Nusselt number by effect of the squeeze number Vs. nanoparticle volume fraction.

For example, Using Collocation method for silver nanoparticle when $Ec = 0.1$, $S = 1$, $\varphi = 0.02$, $Pr = 6.2$ and $\delta = 0.01$ leads to

$$f(\eta) = 1.415122767\ \eta - 0.334731955\ \eta^3 - 0.07590439091\ \eta^5 - 0.004486421265\ \eta^7 \tag{26}$$

$$\theta(\eta) = 1.032802398 - 0.01018717042\ \eta^3 - 0.008179374723\ \eta^5 - 0.01443585266\ \eta^7 \tag{27}$$

Also LSM results for this special case will be

$$f(\eta) = 1.415020845\ \eta - 0.334210605\ \eta^3 - 0.07664132372\ \eta^5 - 0.004168915860\ \eta^7 \tag{28}$$

$$\theta(\eta) = 1.032754259 - 0.01318660040\ \eta^3 - 0.004362611968\ \eta^5 - 0.01520504741\ \eta^7 \tag{29}$$

For showing the efficiency of analytical applied methods (CM and LSM) a special case is considered and results are depicted in Fig. 2-a and b. As seen in these figures, both analytical methods have good agreement with numerical method. For better perception, Tables 2 and 3 are presented for velocity, temperature and nanoparticle concentration respectively. These tables confirm that LSM has lower errors compared to CM, so it is more accurate and reliable than CM.

In this study, LSM and CM are used to solve the problem of unsteady squeezing nanofluid flow (Fig. 1). The effects of active parameters such as the squeeze number, the nanofluid volume fraction and Eckert number on flow and heat transfer characteristics are investigated. The present code is validated by comparing the obtained results with other works reported in the literature [3,28] (Table 2). Different values of active parameters are shown in Fig. 2 and Table 3. All these

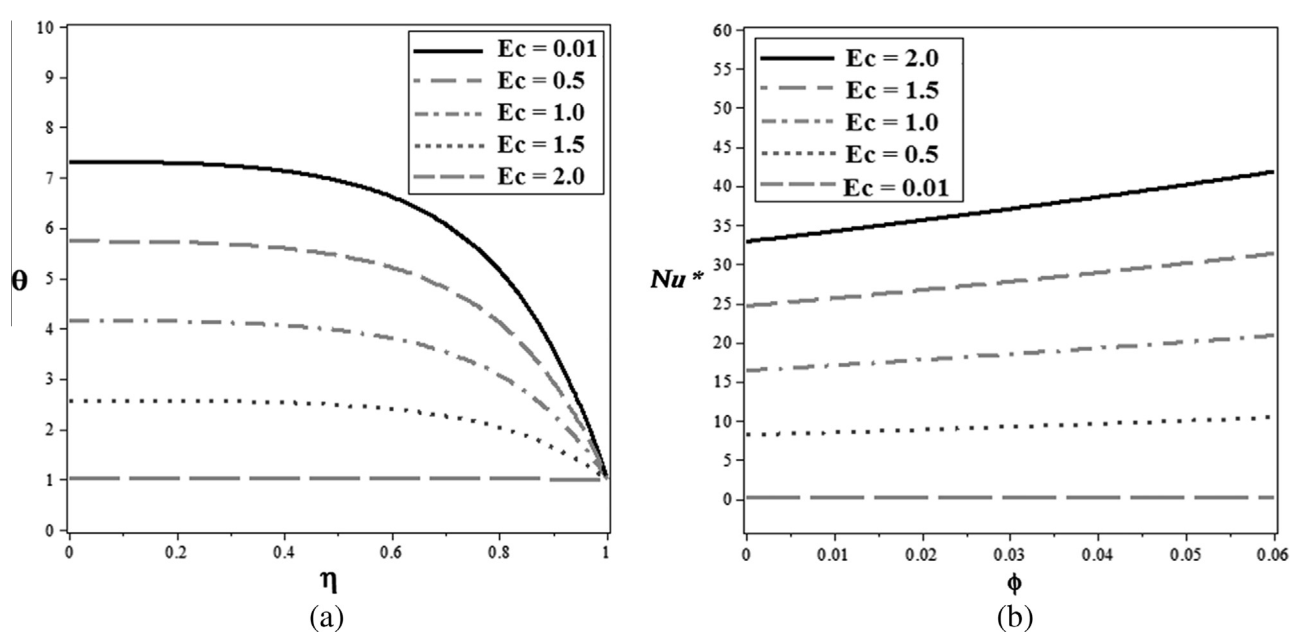


Figure 7 Effect of the Eckert number on the (a) temperature profile at $\varphi = 0.06$, $Pr = 6.2$, $\delta = 0.1$, $S = 1$ and (b) Nusselt number (Silver–water).

comparisons illustrate that LSM and CM offer highly accurate solution for the present problem.

Fig. 3 shows the effect of squeeze number on the velocity profile and skin friction coefficient. It is important to note that the squeeze number (S) describes the movement of the plates ($S > 0$ corresponds to the plates moving apart, while $S < 0$ corresponds to the plates moving together (the so-called squeezing flow)). The positive and negative squeeze numbers have different effects on the velocity profile. For the case of squeezing flow, the velocity increases due to an increase in the absolute value of squeeze number when $\eta < 0.5$, while it decreases for $\eta > 0.5$. Also it can be seen that the absolute value of squeeze number has a reverse relationship with the absolute values of skin friction coefficient for squeezing flow case. Also Fig. 3 shows that opposite trend is observed for the case in which the plates are moving apart. Besides, it can be found that velocity components of nanofluid increase as a result of an increase in the energy transport in the fluid as the volume fraction increases. Thus, the absolute value of skin friction coefficient increases with increase of volume fraction of nanofluid.

Effects of the nanofluid volume fraction and the squeeze number on the temperature profile are shown in Figs. 4 and 5, respectively. Fig. 6 shows the effect of squeeze number and nanoparticle volume fraction on the Nusselt number. Increasing the volume fraction of nanofluid leads to decrease in the thermal boundary layer thickness. Hence the Nusselt number increases as the volume fraction of nanofluid increases. When two plates move together, thermal boundary layer thickness increases as the absolute magnitude of the squeeze number enhances. This increase in thermal boundary layer thickness reduces the Nusselt number. Also it can be found that opposite behavior is observed when two plates move apart.

Fig. 7 shows the effect of Eckert number on the temperature profile and Nusselt number. The viscous dissipation effect significantly increases the temperature of the fluid between two plates. Hence the Nusselt number increases with increase of Eckert number (see Figs. 8 and 9).

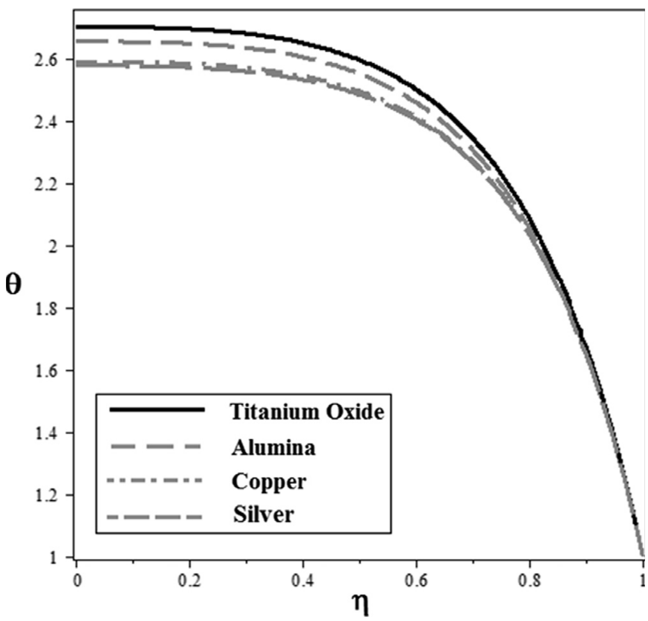


Figure 8 Temperature profile for different types of nanoparticles when $Ec = 0.5$, Pr-6.2, $\delta = 0.1$, $S = 1$ and $\varphi = 0.06$.

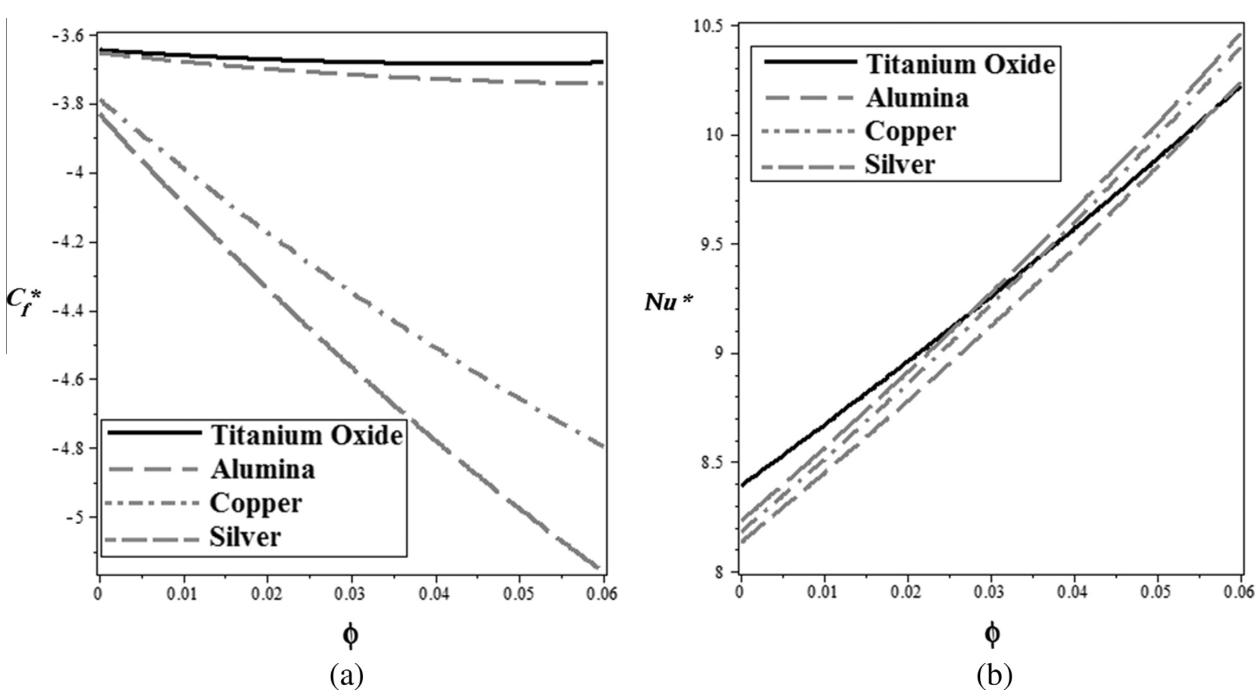


Figure 9 Effect of different types of nanoparticles on the skin friction coefficient and Nusselt number when (a) $S = 1$ and (b) $Ec = 0.5$, $Pr = 6.2$, $\delta = 0.1$ and $S = 1$.

5. Conclusion

In this paper, two analytical approaches called Least Square Method (LSM) and Collocation Method (CM) along a numerical method have been successfully applied to find the most accurate solution of the heat transfer of unsteady flow of a nanofluid squeezing between two parallel plates. As a main outcome from the present study, it is observed that the results of LSM are more accurate than CM and they are in excellent agreement with numerical ones, so LSM can be used for finding analytical solutions of coupled equations in nanofluid problems easily. The effects of the squeeze number, the nanofluid volume fraction and Eckert number on Nusselt number and skin friction coefficient are studied. The results show that the type of nanofluid is an important key factor for heat transfer enhancement. Selecting silver as nanoparticle leads to obtain the highest values of Nusselt number. Also, it can be found that when the two plates move toward together, the Nusselt number has a direct relationship with nanoparticle volume fraction and Eckert number while it has a reverse relationship with the squeeze number.

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