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Finitistic dimension conjecture and relative hereditary algebras

Jiaqun Wei [1]

Department of Mathematics, Nanjing Normal University, Nanjing 210097, PR China

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ABSTRACT

We introduce the notion of relative hereditary Artin algebras, as a generalization of algebras with representation dimension at most 3. We prove the following results. (1) The relative hereditariness of an Artin algebra is left–right symmetric and is inherited by endomorphism algebras of projective modules. (2) The finitistic dimensions of a relative hereditary algebra and its opposite algebra are finite.

As a consequence, the finitistic projective dimension conjecture, the finitistic injective dimension conjecture, the Gorenstein symmetry conjecture, the Wakamatsu-tilting conjecture and the generalized Nakayama conjecture hold for relative hereditary Artin algebras and endomorphism algebras of projective modules over them (in particular, over algebras with representation dimension at most 3).

We also show that the torsionless-finiteness of an Artin algebra is inherited by endomorphism algebras of projective modules, and consequently give a partial answer to the question if the representation dimension of the endomorphism algebra of any projective module over an Artin algebra A is bounded by the representation dimension of A.

1. Introduction

Throughout the paper, we work on Artin algebras and finitely generated left modules. If A is an Artin algebra, then we use A^o to denote the opposite algebra of A. We denote by A-mod the category of all A-modules.

E-mail address: weijiaqun@njnu.edu.cn.

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Let A be an Artin algebra. Recall that the *finitistic* (*projective*) *dimension* of A is defined to be the supremum of the projective dimensions of all finitely generated modules of finite projective dimension. Similarly, the finitistic injective dimension is defined, taking injective dimensions. Note that the finitistic injective dimension of A is just the finitistic (projective) dimension of A^o, due to the canonical duality in the realm of Artin algebras.

The famous finitistic dimension conjecture for Artin algebras asserts that the finitistic dimension of an Artin algebra is always finite. The conjecture is still open now. This conjecture is also related to many other homological conjectures (e.g., the Gorenstein symmetry conjecture, the Wakamatsu-tilting conjecture and the generalized Nakayama conjecture) and attracts many algebraists, see for instance [1,6,16,18], etc.

Only a few classes of algebras are known to have finite finitistic dimension, see for instance [4–6], etc. Among them, Igusa and Todorov [7] proved that the finitistic dimension conjecture holds for Artin algebras with representation dimension at most 3. Recently, Zhang and Zhang [17] proved that the finitistic dimension conjecture holds for endomorphism algebras of projective modules over Artin algebras with representation dimension at most 3.

Note the representation dimension of an Artin algebra is the same as the one of its opposite algebra, so the finitistic injective dimensions of Artin algebras with representation dimension at most 3 are also finite. It is natural to ask whether the finitistic injective dimensions of endomorphism algebras of projective modules over Artin algebras with representation dimension at most 3 are also finite. Notice that methods used in [17] cannot give us an answer.

We will give an affirmative answer to the above question in more general contexts. Namely, we introduce the notion of relative hereditary Artin algebras (i.e., 0-Igusa–Todorov algebras in [13]), which is a generalization of algebras with representation dimension at most 3, and then we study the finiteness of the finitistic dimensions of endomorphism algebras of projective modules over such algebras. We prove the following results.

Theorem 1.1. *The relative hereditariness of an Artin algebra is left–right symmetric and is inherited by endomorphism algebras of projective modules.*

As a corollary, we have

Theorem 1.2. *The finitistic dimensions of a relative hereditary algebra and its opposite algebra are finite.*

Consequently, we obtain that the finitistic projective dimension conjecture, the finitistic injective dimension conjecture, the Gorenstein symmetry conjecture, the Wakamatsu-tilting conjecture and the generalized Nakayama conjecture hold for relative hereditary Artin algebras and endomorphism algebras of projective modules over them (in particular, over algebras with representation dimension at most 3).

Concerning representation dimensions, it is an old question of Auslander [2, p. 177]: Is the representation dimension of the endomorphism algebra of any projective A-module bounded by the representation dimension of the Artin algebra A? We study the case when A is torsionless-finite. In this case, the representation dimension of A is at most 3 [9]. We show that the torsionless-finiteness of an Artin algebra is inherited by endomorphism algebras of projective modules too, and consequently give a partial answer to the above question.

2. Relative hereditary algebras

We first introduce the following definition.

Definition 2.1. Let A be an Artin algebra. We call A relative hereditary if there is a fixed ${}_AV$ such that, for any $M \in A$-mod, there is an exact sequence $0 \to V_1 \to V_0 \to M \to 0$ with $V_1, V_0 \in \operatorname{add}_A V$. Hereafter $\operatorname{add}_A V$ will denote the category of all direct summands of finite direct sums of ${}_AV$.

Relative hereditary algebras are special cases of Igusa–Todorov algebras studied in [13]. By the definition, one immediately obtains that torsionless-finite algebras are relative hereditary. Recall that a module is said to be *torsionless* provided it can be embedded into a projective module and an Artin algebra A is said to be *torsionless-finite* provided there are only finitely many isomorphism classes of indecomposable torsionless modules. The class of torsionless-finite algebras contain hereditary algebras, algebras with radical square zero, minimal representation infinite algebras, stably hereditary algebras, tame concealed algebras and right (or left) glued algebras, etc. (see for instance [9,14]).

It had been shown in [9] that torsionless-finite algebras have representation dimension at most 3. Recall that the *representation dimension* of an Artin algebra is the minimum of the global dimension of the endomorphism rings of generator–cogenerators. Note that an Artin algebra A has representation dimension at most $n+2$ if and only if there is a generator–cogenerator V such that, for any A-module X, there is an exact sequence $0 \to V_n \to \cdots \to V_0 \to X \to 0$ which stays exact under the functor $\mathrm{Hom}_A(V,-)$, where each $V_i \in \mathrm{add}_A V$, see for instance [3]. It follows that all algebras with representation dimension at most 3 are relative hereditary. Note that, among others, the following algebras have representation dimension at most 3: (1) special biserial algebra, (2) tilted algebra, (3) laura algebra, etc.

Remark 2.2. It is not known if every relative hereditary algebra has representation dimension at most 3. More specially, it is not known if the algebras in the following proposition have representation dimension at most 3 (these algebras were considered in [15]).

Proposition 2.3. *Let A be an Artin algebra with two ideals I, J such that $IJ = 0$. If both A/I and A/J are representation-finite, then A is relative hereditary.*

Proof. This is essentially a part of [13, Proposition 3.2]. We repeat the proof for reader's convenience. For any $N \in A$-mod, we denote $C_1 := JN$ and $C_2 := N/JN$. Then we have an exact sequence $0 \to C_1 \to N \to C_2 \to 0$. Note that, by assumptions, $IC_1 = IJN = 0$ and $JC_2 = J(N/JN) = 0$, so C_1 is also an A/I-module and C_2 is also an A/J-module. Let C_I (resp., C_J) be the direct sum of all non-isomorphic indecomposable A/I-modules (resp., A/J-modules). Then it is easy to see that ${}_AC_1 \in \mathrm{add}_A C_I$ and ${}_AC_2 \in \mathrm{add}_A C_J$. Now consider the following pullback diagram, where the right column is taken such that ${}_AP$ is the projective cover of ${}_AC_2$. Hereafter, the notion $\Omega_A M$ denotes the first syzygy of a module ${}_AM$.

$$\begin{tikzcd}
 & & 0 \arrow[d] & 0 \arrow[d] & \\
 & & \Omega_A C_2 \arrow[r, equal] \arrow[d] & \Omega_A C_2 \arrow[d] & \\
0 \arrow[r] & C_1 \arrow[r] \arrow[d, equal] & C_1 \oplus P \arrow[r] \arrow[d] & P \arrow[r] \arrow[d] & 0 \\
0 \arrow[r] & C_1 \arrow[r] & N \arrow[r] \arrow[d] & C_2 \arrow[r] \arrow[d] & 0 \\
 & & 0 & 0 &
\end{tikzcd}$$

Now fixed $U = C_I \oplus \Omega_A C_J \oplus A$ which is independent of N, we obtain an exact sequence $0 \to U_1 \to U_0 \to N \to 0$, where $U_1, U_0 \in \mathrm{add}_A U$, from the middle column in the diagram. It follows that A is relative hereditary. $\square$

A special case is as follows.

Corollary 2.4. *Let A be an Artin algebra with an ideal I such that $I^2 = 0$ and A/I is representation-finite. Then A is relative hereditary.*

In particular, the Hochschild extension algebra of a representation-finite algebra by a bimodule over it, and the trivially twisted extension algebra of two representation-finite algebras are relative hereditary [15]. We refer to [15] for more algebras satisfying the assumptions in Proposition 2.3 and Corollary 2.4, and hence for more examples of relative hereditary algebras.

Our interest in relative hereditary algebras comes from the recent study of the finitistic dimension conjecture via Igusa–Todorov functor introduced in [7,12]. The following lemma collects some important properties of the Igusa–Todorov functor, see [7].

Lemma 2.5. *For any Artin algebra A, there is a functor Ψ which is defined on the objects of A-*mod *and takes nonnegative integers as values, such that*:

(1) $\Psi(M) = \operatorname{pd}_A M$ *provided that* $\operatorname{pd}_A M < \infty$.
(2) $\Psi(X) \leqslant \Psi(Y)$ *whenever* $\operatorname{add}_A X \subseteq \operatorname{add}_A Y$. *The equation holds in case* $\operatorname{add}_A X = \operatorname{add}_A Y$.
(3) *If* $0 \to X \to Y \to Z \to 0$ *is an exact sequence in A-*mod *with* $\operatorname{pd}_A Z < \infty$, *then* $\operatorname{pd}_A Z \leqslant \Psi(X \oplus Y) + 1$.

Using the Igusa–Todorov functor, one can obtain the following result.

Proposition 2.6. *If A is a relative hereditary Artin algebra, then the finitistic dimension of A is finite.*

Proof. By assumption, there is a fixed ${}_AV$ such that, for any $M \in A$-mod, there is an exact sequence $0 \to V_1 \to V_0 \to M \to 0$ with $V_1, V_0 \in \operatorname{add}_A V$. Suppose now M is of finite projective dimension, then we have that $\operatorname{pd}_A M \leqslant \Psi(V_1 \oplus V_0) + 1 \leqslant \Psi(V) + 1 < \infty$. $\square$

One point of this paper is to show that the finitistic dimension of A^o is also finite in case the Artin algebra A is relative hereditary. Note that generally it is not known whether the finitistic dimension of A^o is finite provided that the finitistic dimension of A is finite. However, it is the case for an Artin algebra A with representation dimension at most 3, since the representation dimensions of A and A^o coincide.

We in fact prove the following result.

Proposition 2.7. *If A is a relative hereditary Artin algebra, then the opposite algebra A^o is also relative hereditary.*

Proof. Clearly we need to find a fixed A^o-module U such that for any $M^o \in A^o$-mod, there is an exact sequence $0 \to U_1 \to U_0 \to M^o \to 0$, where $U_1, U_0 \in \operatorname{add}_{A^o} U$. Note that $M^o \simeq \mathbf{D}M$ for some $M \in A$-mod, where $\mathbf{D}$ denotes the usual duality functor between A-mod and A^o-mod. Since A is a relative hereditary Artin algebra, there is a fixed ${}_AV$ such that, for any $M \in A$-mod, there is an exact sequence $0 \to V_1 \to V_0 \to M \to 0$ with $V_1, V_0 \in \operatorname{add}_A V$. Denote $V^o = \mathbf{D}V$ and by applying the functor $\mathbf{D}$ to the above sequence, we obtain an exact sequence $0 \to M^o \to V_0^o \to V_1^o \to 0$ with $V_0^o, V_1^o \in \operatorname{add}_A V^o$. Now consider the following pullback diagram, where P is the projective cover of V_0^o and $N = \Omega_{A^o}(V_0^o)$.

$$\begin{tikzcd}
 & 0 \arrow[d] & 0 \arrow[d] & & \\
 & N \arrow[r, equal] \arrow[d] & N \arrow[d] & & \\
0 \arrow[r] & Y \arrow[r] \arrow[d] & P \arrow[r] \arrow[d] & V_1^o \arrow[r] \arrow[d, equal] & 0 \\
0 \arrow[r] & M^o \arrow[r] \arrow[d] & V_0^o \arrow[r] \arrow[d] & V_1^o \arrow[r] & 0 \\
 & 0 & 0 & &
\end{tikzcd}$$

It is easy to see from the above exact commutative diagram that $Y \simeq \Omega_{A^o}(V_1^o) \oplus Q$ for some projective A^o-module Q. Therefore for any A^o-module M^o, we obtain an exact sequence $0 \to U_1 \to U_0 \to M^o \to 0$ with $U_1, U_0 \in \operatorname{add}_{A^o} U$ where $U = A^o \oplus \Omega_{A^o}(V^o)$ is the fixed module, which is independent of the module M^o. $\square$

Immediately from Propositions 2.6 and 2.7, we obtain the following theorem.

Theorem 2.8. *If A is a relative hereditary Artin algebra, then the finitistic dimensions of A and A^o are finite.*

Let A be an Artin algebra. An A-module ω is Wakamatsu-tilting if it satisfies that $A \simeq \operatorname{End} \omega_\Gamma$ where $\Gamma = \operatorname{End}_A \omega$ and that $\operatorname{Ext}_A^i(\omega, \omega) = 0 = \operatorname{Ext}_\Gamma^i(\omega, \omega)$ for all $i \geqslant 1$ [10,11]. Recall the following well-known conjectures (see for instance [6,8], etc.).

Gorenstein symmetry conjecture. $\operatorname{id}({}_A A) < \infty$ if and only if $\operatorname{id}({}_{A^o} A) < \infty$, where id denotes the injective dimension.

Wakamatsu-tilting conjecture. Let ${}_A\omega$ be a Wakamatsu-tilting module. (1) If $\operatorname{pd}_A \omega < \infty$, then ω is tilting. (2) If $\operatorname{id}_A \omega < \infty$, then ω is cotilting.

Generalized Nakayama conjecture. Each indecomposable injective A-module occurs as a direct summand in the minimal injective resolution of ${}_A A$.

Now we deduce the following corollary.

Corollary 2.9. *The Gorenstein symmetry conjecture, Wakamatsu-tilting conjecture and generalized Nakayama conjecture hold for relative hereditary algebras.*

Proof. Note that the Gorenstein symmetry conjecture and the generalized Nakayama conjecture are special cases of the second part of the Wakamatsu-tilting conjecture. Moreover, if the finitistic dimension conjecture holds for A and A^o, then the Wakamatsu-tilting conjecture holds by [8]. $\square$

3. Endomorphism algebras

Let A be an Artin algebra and $M \in A$-mod with $E = \operatorname{End}_A M$. Then M is also an E^o-module (i.e., right E-module). It is well known that $(M \otimes_E -, \operatorname{Hom}_A(M, -))$ is a pair of adjoint functors and that, for any E-module Y, there is a canonical homomorphism $\sigma_Y : Y \to \operatorname{Hom}_A(M, M \otimes_E Y)$ defined by $n \to [t \to t \otimes n]$. It is easy to see that σ_Y is an isomorphism provided that Y is a projective E-module.

The following lemma is elementary.

Lemma 3.1. *Let A be an Artin algebra and P be a projective A-module with $E = \operatorname{End}_A P$. Then σ_X is an isomorphism, i.e., $X \simeq \operatorname{Hom}_A(P, P \otimes_E X)$, for any $X \in E$-mod.*

Proof. Obviously, we have an exact sequence $E_1 \to E_0 \to X \to 0$ with $E_0, E_1 \in E$-mod projective. Applying the right exact functor $P \otimes_E -$, we obtain an induced exact sequence $P \otimes_E E_1 \to P \otimes_E E_0 \to P \otimes_E X \to 0$. Now applying the functor $\operatorname{Hom}_A(P, -)$, we further have an induced exact sequence $\operatorname{Hom}_A(P, P \otimes_E E_1) \to \operatorname{Hom}_A(P, P \otimes_E E_0) \to \operatorname{Hom}_A(P, P \otimes_E X) \to 0$, since ${}_A P$ is projective.

Moreover, there is the following exact commutative diagram

$$\begin{tikzcd}
E_1 \arrow[r] \arrow[d, "\sigma_{E_1}"] & E_0 \arrow[r] \arrow[d, "\sigma_{E_0}"] & X \arrow[r] \arrow[d, "\sigma_X"] & 0 \\
\operatorname{Hom}_A(M, M \otimes_E E_1) \arrow[r] & \operatorname{Hom}_A(M, M \otimes_E E_0) \arrow[r] & \operatorname{Hom}_A(M, M \otimes_E X) \arrow[r] & 0.
\end{tikzcd}$$

Since $E = \operatorname{End}_A M$ and $E_0, E_1 \in \operatorname{add}_E E$, the canonical homomorphisms σ_{E_0} and σ_{E_1} are isomorphisms. It follows that σ_X is also an isomorphism. $\square$

Now we can obtain a nice property of the class of relative hereditary algebras, which can help us to obtain new relative hereditary algebras from the old one.

Theorem 3.2. *Let A be a relative hereditary algebra and P be a projective A-module. Then $\mathrm{End}_A P$ is also relative hereditary.*

Proof. Let $E = \mathrm{End}_A P$ and take any $X \in E$-mod. Then $P \otimes_E X$ is an A-module. Since A is relative hereditary, there is an exact sequence $0 \to V_1 \to V_0 \to M \to 0$ with $V_1, V_0 \in \mathrm{add}_A V$ and ${}_AV$ fixed. Now applying the functor $\mathrm{Hom}_A(P, -)$, we further have an induced exact sequence $0 \to \mathrm{Hom}_A(P, V_1) \to \mathrm{Hom}_A(P, V_0) \to \mathrm{Hom}_A(P, P \otimes_E X) \to 0$, since ${}_AP$ is projective. By Lemma 3.1, $X \simeq \mathrm{Hom}_A(P, P \otimes_E X)$. Hence we obtain an exact sequence $0 \to U_1 \to U_0 \to X \to 0$, where $U_1, U_0 \in \mathrm{add}_{A^o} U$ and $U = \mathrm{Hom}_A(P, V)$ fixed independent of X. It follows that E is relative hereditary. $\square$

Combining results in Section 2, we obtain the following result.

Corollary 3.3. *Let A be an Artin algebra and P be a projective A-module. Then the finitistic dimension conjecture, the Gorenstein symmetry conjecture, Wakamatsu-tilting conjecture and generalized Nakayama conjecture hold for $\mathrm{End}_A P$ and $(\mathrm{End}_A P)^o$, if one of the following conditions holds*:

(1) *A has representation dimension at most 3*;
(2) *A has two ideals I, J such that $IJ = 0$ and both A/I and A/J are representation-finite.*

Remark 3.4. Let A be an Artin algebra with representation dimension at most 3 and P be a projective A-module. It is proved in [17] that the finitistic dimension of $\mathrm{End}_A P$ is finite. However, the method there used cannot be used to get the finiteness of the finitistic dimension of $(\mathrm{End}_A P)^o$.

In the rest, we concentrate ourself on torsionless-finite algebras, which is a special case of relative hereditary algebras.

Lemma 3.5. *Let A be an Artin algebra and P be a projective A-module with $E = \mathrm{End}_A P$. Then, for any torsionless E-module X, there is a torsionless A-module Y such that $X \simeq \mathrm{Hom}_A(P, Y)$.*

Proof. For any torsionless E-module X, we have an exact sequence $0 \to X \to E_0 \to X' \to 0$ with $E_0 \in E$-mod projective. Applying the functor $P \otimes_E -$, we obtain an induced exact sequence $0 \to Y \to P \otimes_E E_0 \to P \otimes_E X' \to 0$, for some $Y \in A$-mod. Note that $P \otimes_E E_0 \in \mathrm{add}_A P$ and ${}_AP$ is projective, so ${}_AY$ is torsionless. Now applying the functor $\mathrm{Hom}_A(P, -)$, we further have an induced exact sequence $0 \to \mathrm{Hom}_A(P, Y) \to \mathrm{Hom}_A(P, P \otimes_E E_0) \to \mathrm{Hom}_A(P, P \otimes_E X')$.

Moreover, there is the following commutative diagram

$$\begin{tikzcd}
0 \arrow[r] & X \arrow[r] \arrow[d, "\phi"] & E_0 \arrow[r] \arrow[d, "\sigma_{E_0}"] & X' \arrow[d, "\sigma_{X'}"] \\
0 \arrow[r] & \mathrm{Hom}_A(P, Y) \arrow[r] & \mathrm{Hom}_A(P, P \otimes_E E_0) \arrow[r] & \mathrm{Hom}_A(P, P \otimes_E X').
\end{tikzcd}$$

By Lemma 3.1, the canonical homomorphisms σ_{E_0} and $\sigma_{X'}$ are isomorphisms. It follows that ϕ is also an isomorphism and consequently, $X \simeq \mathrm{Hom}_A(P, Y)$ with ${}_AY$ torsionless. $\square$

Thanks to Lemma 3.5, we also obtain a nice property of the class of torsionless-finite algebras, which can help us to obtain new torsionless-finite algebras from the old one.

Proposition 3.6. *Let A be a torsionless-finite algebra and P be a projective A-module. Then $\mathrm{End}_A P$ is also torsionless-finite.*

Proof. Let $E = \mathrm{End}_A P$. By Lemma 3.5, every torsionless E-module is of the form $\mathrm{Hom}_A(P, Y)$ for some torsionless A-module Y. Under the correspondence, one easily see that there are only finitely many isomorphism classes of indecomposable torsionless E-modules, because there are only finitely many isomorphism classes of indecomposable torsionless A-modules. It follows that E is torsionless-finite. □

In particular, since torsionless-finite algebras have representation dimension at most 3, we have the following result.

Corollary 3.7. *Let A be a torsionless-finite algebra and P be a projective A-module. Then* $\mathrm{End}_A P$ *has representation dimension at most* 3.

Remark 3.8. The above result gives an affirmative answer for torsionless-finite algebras to the earlier mentioned question: Is the representation dimension of the endomorphism algebra of a projective module bounded by the representation dimension of the algebra?

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