Physics Letters B 737 (2014) 120–123

Contents lists available at ScienceDirect

Physics Letters B

www.elsevier.com/locate/physletb

Leading large-x logarithms of the quark–gluon contributions to inclusive Higgs-boson and lepton-pair production

N.A. Lo Presti [a], A.A. Almasy [b,1], A. Vogt [c,*]

[a] Institut de Physique Théorique, CEA-Saclay, F-91191, Gif-sur-Yvette cedex, France
[b] Deutsches Elektronensynchrotron DESY, Platanenallee 6, D-15738 Zeuthen, Germany
[c] Department of Mathematical Sciences, University of Liverpool, Liverpool L69 3BX, United Kingdom

ARTICLE INFO

Article history:
Received 9 July 2014
Accepted 18 August 2014
Available online 22 August 2014
Editor: A. Ringwald

ABSTRACT

We present all-order expressions for the leading double-logarithmic threshold contributions to the quark–gluon coefficient functions for inclusive Higgs-boson production in the heavy top-quark limit and for Drell–Yan lepton-pair production. These results have been derived using the structure of the unfactorized cross sections in dimensional regularization and the large-x resummation of the gluon–quark and quark–gluon splitting functions. The resummed coefficient functions, which are identical up to colour factor replacements, are similar to their counterparts in deep-inelastic scattering but slightly more complicated.

The discovery of a particle with a mass of about 125 GeV [1] and properties consistent with those of the standard-model Higgs boson [2] at the LHC has led to increased interest in precision predictions for Higgs production and decay. The main channel for the total production cross section is gluon–gluon fusion via a top quark loop, known at all $M_{\rm H}/M_{\rm top}$ to next-to-leading order (NLO) of perturbative QCD [3,4]. The convergence of the perturbation series is particularly slow in this case, hence calculations are required at, and beyond, the next-to-next-to-leading order (NNLO).

These calculations can be carried out, at a sufficient accuracy [5], for an effective Hgg interaction in the heavy-top limit [6],

$$\mathcal{L}_{\rm eff} = -\frac{1}{4} C_H H G^a_{\mu\nu} G^{a,\mu\nu}, \tag{1}$$

where $G^a_{\mu\nu}$ denotes the gluon field strength tensor. The prefactor C_H includes all QCD corrections to the top quark loop; it is of first order in the strong coupling constant α_s and fully known up to N^{3}LO (α_s^4) [7], see also Ref. [8]. The NNLO contributions to the total cross sections were computed in this effective theory in Refs. [9–11]; a high-accuracy threshold resummation and a first approximation for N^{3}LO corrections were subsequently obtained in Refs. [12,13].

Recently a major step has been taken towards deriving the complete N^{3}LO corrections: the calculation of the soft-gluon and virtual contributions at this order [14]. This result directly leads to a further improvement in the threshold limit [15–17] by fixing the remaining parameter required for a full N^{3}LO + next-to-next-to-next-to-leading logarithmic (N^{3}LL) accuracy [18] of the soft-gluon exponentiation. The same soft + virtual N^{3}LO and resummation accuracy has also been reached for Drell–Yan lepton-pair production $pp \to \ell^+\ell^- +$ *anything*, calculated at NNLO in Refs. [19,20], due to its close similarity with inclusive Higgs-boson production [15,17].

Generally fixed- or all-order results for logarithmically enhanced endpoint contributions, e.g., in the large-x or threshold limit, can provide checks of elaborate Feynman-diagram calculations and estimates of corrections that cannot (yet) be calculated directly. Quite a few studies of the threshold limit have addressed the dominant channels in Higgs and lepton-pair production, i.e., gluon–gluon fusion and quark–antiquark annihilation, respectively. Here we present first all-order results for the sub-dominant quark–gluon contributions to both processes. In particular, we derive the leading large-x logarithms of the coefficient functions $c_{P,\rm qg}$ for $P = \rm H$ and $P = \rm DY$.

Our derivation starts from the unfactorized partonic cross sections $\widehat{W}_{P,j\ell}$ in

$$\begin{aligned}\sigma_P &= \widetilde{\sigma}_{0,P} \widehat{W}_{P,j\ell} \otimes \widehat{f}_j \otimes \widehat{f}_\ell \\ &= \widetilde{\sigma}_{0,P} \widetilde{c}_{P,ik} \otimes Z_{ij} \otimes Z_{k\ell} \otimes \widehat{f}_j \otimes \widehat{f}_\ell,\end{aligned} \tag{2}$$

which lead to the mass-factorized expressions

$$\sigma_P = \sigma_{0,P} c_{P,ik} \otimes f_i \otimes f_k. \tag{3}$$

Here $\otimes$ abbreviates the Mellin convolutions, and summations over the light quarks and antiquarks and gluons are understood. All charge factors have been suppressed; see, e.g., Appendix A of

* Corresponding author.
1 Address until 31 August 2013.

http://dx.doi.org/10.1016/j.physletb.2014.08.044

Ref. [19] for the Drell–Yan process. We use dimensional regularization with $D = 4 - 2\epsilon$; a tilde marks the D-dimensional counterparts of quantities which are finite for $\epsilon = 0$. In particular, the coefficient functions in Eq. (2) can be written as

$$\widetilde{c}_{P,ik}\big(x, M^2\big) = \sum_{n=0}\sum_{\ell=0} a_s^n \epsilon^\ell c_{P,ik}^{(n,\ell)}(x) \quad \text{with } a_s \equiv \frac{\alpha_s(M^2)}{4\pi} \tag{4}$$

for the choice $\mu_r = \mu_f = M$ of the renormalization and mass-factorization scales, with $M = M_H$ or $M = M_{\ell^+\ell^-}$, which can be made without loss of information. All factorized expressions refer to the $\overline{\text{MS}}$ scheme; the additional terms defining its difference to MS are suppressed in Eq. (4) and below. The coefficient functions $c_{P,ik}$ in Eq. (3) are obtained from the above by setting $\epsilon = 0$.

The scale dependence of the factorized parton distributions f_i in Eq. (3) is governed by the splitting functions P_{ik}, which are related to the transition functions Z_{ik} in Eq. (2) by

$$P_{ik} \equiv -\gamma_{ik} = \frac{dZ_{ij}}{d\ln M^2} \otimes \big[Z^{-1}\big]_{jk} = \beta_D(a_s)\frac{dZ_{ij}}{da_s} \otimes \big[Z^{-1}\big]_{jk}, \tag{5}$$

where $\beta_D(a_s) = -\epsilon a_s - \beta_0 a_s^2 - \dots$ with $\beta_0 = \frac{11}{3}C_A - \frac{2}{3}n_f$ is the D-dimensional beta function. Eq. (5) can be solved for Z order by order in α_s.

The prefactors $\widetilde{\sigma}_{0,P}$ in Eq. (2) are defined such that the lowest-order contributions to the D-dimensional coefficient functions in Eq. (4) are normalized and independent of ϵ, i.e., given by

$$c_{\mathrm{H,gg}}^{(0,\ell)}(x) = c_{\mathrm{DY,q\bar{q}}}^{(0,\ell)}(x) = \delta(1-x)\delta_{0\ell}. \tag{6}$$

We further specify our notation for the coefficient functions and splitting functions by recalling the leading-logarithmic large-x contributions to the NLO quark–gluon coefficient functions:

$$\begin{aligned} c_{\mathrm{H,qg}}^{(1)\mathrm{LL}}(x) &= 2P_{\mathrm{gq}}^{(0)}(x)\ln(1-x) \\ &= 4C_F\big(2x^{-1} - 2 + x\big)\ln(1-x), \end{aligned} \tag{7}$$

$$\begin{aligned} c_{\mathrm{DY,qg}}^{(1)\mathrm{LL}}(x) &= 2P_{\mathrm{qg}}^{(0)}(x)\ln(1-x) \\ &= 4T_f\big(1 - 2x + 2x^2\big)\ln(1-x) \end{aligned} \tag{8}$$

with $C_F = \frac{4}{3}$, $T_f = \frac{1}{2}$ and $C_A = 3$ for QCD. Note that our convention in Eq. (7) differs from the quantities Δ_{ik} in Refs. [10,11] by a factor of x^{-1}. On the other hand, our normalization in Eq. (8) is the same as in Ref. [19]. The corresponding NNLO corrections read

$$c_{\mathrm{H,qg}}^{(2)\mathrm{LL}}(x) = \frac{1}{3}(13C_F + 35C_A)P_{\mathrm{gq}}^{(0)}(x)\ln^3(1-x), \tag{9}$$

$$c_{\mathrm{DY,qg}}^{(2)\mathrm{LL}}(x) = \frac{1}{3}(35C_F + 13C_A)P_{\mathrm{qg}}^{(0)}(x)\ln^3(1-x). \tag{10}$$

It is convenient to turn the convolutions above to products by Mellin transforming all quantities,

$$f(N) = \int_0^1 dx\big(x^{N-1}\{-1\}\big)f(x)_{\{+\}}, \tag{11}$$

where the parts in curly brackets refer to the case of $(1-x)^{-1}$ +-distributions. Here we mainly consider the leading powers of $(1-x)$ in the threshold limit, in particular $(1-x)^0$ corresponding to N^{-1} in the large-N limit for the quark–gluon quantities addressed in this letter. Keeping only the leading – and subleading, if $\ln^k N$ is replaced by $\ln^k N + k\gamma_e \ln^{k-1} N$ – contributions, the relations between the corresponding expressions in x-space and Mellin-N space read

$$\begin{aligned} \frac{\ln^n(1-x)}{(1-x)_+} &\overset{\mathrm{M}}{=} \frac{(-1)^{n+1}}{n+1}\ln^{n+1}N + \dots, \\ \ln^n(1-x) &\overset{\mathrm{M}}{=} \frac{(-1)^n}{N}\ln^n N + \dots. \end{aligned} \tag{12}$$

Here and below $\overset{\mathrm{M}}{=}$ denotes equality under the Mellin transformation (11).

The diagonal splitting function are not logarithmically enhanced at higher orders for the N^0 contributions [21] (nor at N^{-1}, see Refs. [22,23]). Hence only their leading-order contributions are relevant here (and at NLL), with

$$P_{\mathrm{qq}}^{(0)\mathrm{LL}}(N) = -4C_F\ln N, \qquad P_{\mathrm{gg}}^{(0)\mathrm{LL}}(N) = -4C_A\ln N. \tag{13}$$

The corresponding off-diagonal contributions can be readily read off from Eqs. (7) and (8),

$$P_{\mathrm{qg}}^{(0)\mathrm{LL}}(N) = 2T_f N^{-1}, \qquad P_{\mathrm{gq}}^{(0)\mathrm{LL}}(N) = 2C_F N^{-1}. \tag{14}$$

These functions do exhibit a double-logarithmic higher-order enhancement, derived in Ref. [24],

$$P_{\mathrm{qg}}^{\mathrm{LL}}(N, a_s) = a_s P_{\mathrm{qg}}^{(0)\mathrm{LL}}(N)\mathcal{B}_0(-\tilde{a}_s), \tag{15}$$

$$P_{\mathrm{gq}}^{\mathrm{LL}}(N, a_s) = a_s P_{\mathrm{gq}}^{(0)\mathrm{LL}}(N)\mathcal{B}_0(\tilde{a}_s) \tag{16}$$

in terms of the function

$$\mathcal{B}_0(x) = \sum_{n=0}^{\infty}\frac{B_n}{(n!)^2}x^n = 1 - \frac{x}{2} - \sum_{n=1}^{\infty}\frac{(-1)^n}{[(2n)!]^2}|B_{2n}|x^{2n}, \tag{17}$$

where B_n are the Bernoulli numbers in the standard normalization of Ref. [25], and

$$\tilde{a}_s \equiv 4a_s(C_F - C_A)\ln^2 N. \tag{18}$$

For the corresponding NLL and NNLL resummations of the splitting functions see Refs. [26,27].

We are now prepared to return to the unfactorized cross sections in Eq. (2). For brevity the following steps are written out only for Higgs-boson production. We have checked that the corresponding relations for the Drell–Yan case can be obtained, as expected from Eqs. (7)–(10) and (13)–(18), by interchanging gluon and (anti-)quark indices and colour factor replacements.

For the resummation of the quark–gluon coefficient function $c_{\mathrm{H,qg}} = c_{\mathrm{H,\bar{q}g}}$ we need to consider

$$\widehat{W}_{\mathrm{H,qg}} = \mathcal{O}\big(N^{-1}\big) = \widetilde{c}_{\mathrm{H,qg}}Z_{\mathrm{qq}}Z_{\mathrm{gg}} + \widetilde{c}_{\mathrm{H,gg}}Z_{\mathrm{gq}}Z_{\mathrm{gg}} + \mathcal{O}\big(N^{-3}\big) \tag{19}$$

and

$$\widehat{W}_{\mathrm{H,gg}} = \mathcal{O}\big(N^0\big) = \widetilde{c}_{\mathrm{H,gg}}Z_{\mathrm{gg}}Z_{\mathrm{gg}} + \mathcal{O}\big(N^{-2}\big) \tag{20}$$

which provides $\widetilde{c}_{\mathrm{H,gg}}$ for the right-hand-side of Eq. (19). Other coefficient functions such as $\widetilde{c}_{\mathrm{H,q\bar{q}}}$ are not relevant for the leading logarithms in Eq. (19) even at higher orders in N^{-1}.

At the leading (and next-to-leading) power in N^{-1} the a_s^n contributions to the diagonal and off-diagonal transition functions are given by [24]

$$Z_{ii}^{(n)\mathrm{LL}} = \frac{1}{n!}\epsilon^{-n}\big(\gamma_{ii}^{(0)}\big)^n, \tag{21}$$

$$\begin{aligned} Z_{ik}^{(n)\mathrm{LL}} = \frac{1}{n!}\sum_{m=0}^{n-1}\epsilon^{-n+m}\sum_{\ell=0}^{n-m-1}\frac{(m+\ell)!}{\ell!}\big(\gamma_{ii}^{(0)}\big)^{n-m-\ell-1} \\ \times \gamma_{ik}^{(m)}\big(\gamma_{kk}^{(0)}\big)^{\ell}. \end{aligned} \tag{22}$$

Here additional sign factors have been avoided by using the anomalous dimensions γ defined in Eq. (5). The D-dimensional coefficient function $\widetilde{c}_{\mathrm{H,gg}}$ can be determined from Eq. (20) with

$$\widehat{W}_{\mathrm{H,gg}}^{\mathrm{LL}} = \exp\big(a_s\widehat{W}_{\mathrm{H,gg}}^{(1)\mathrm{LL}}\big) \tag{23}$$

and

$$\widehat{W}^{(1)\mathrm{LL}}_{\mathrm{H,gg}} = 4C_F \frac{1}{\epsilon^2}\big(\exp(2\epsilon \ln N) - 1\big)$$
$$\stackrel{\mathrm{M}}{=} -4C_F \frac{1}{\epsilon}(1-x)_+^{-1-2\epsilon} + \text{virtual} \tag{24}$$

at order N^0. The difference of Eq. (24) to the corresponding structure function in deep-inelastic scattering (DIS) is the replacement $\epsilon \to 2\epsilon$ in the exponentials due to the different phase space. An extension of Eqs. (21)–(24) to higher logarithmic accuracy is no problem, but not required here.

The right-hand-side of Eq. (19) is thus known at LL accuracy at all powers of α_s and ϵ except for the quark–gluon coefficient function. Hence an all-order result for $\widehat{W}_{\mathrm{H,qg}}$ on the left-hand-side corresponding to Eqs. (23) and (24) leads to a LL resummation of $c_{\mathrm{H,qg}}$; determining this result is the crucial step of our calculations.

Taking into account $(1-x)^{-k\epsilon}$ factors due to real and virtual corrections, cf. the discussion of the phase-space master integrals in Ref. [10], the general form of the a_s^n contribution to $\widehat{W}_{\mathrm{H,qg}}$ is

$$\widehat{W}^{(n)}_{\mathrm{H,qg}} = \frac{1}{\epsilon^{2n-1}} \sum_{\ell=2}^{2n} (1-x)^{-\ell\epsilon}\big(\bar{A}^{(n,\ell)}_{\mathrm{H,qg}} + \epsilon\, \bar{B}^{(n,\ell)}_{\mathrm{H,qg}} + \ldots\big)$$
$$+ \mathcal{O}\big((1-x)^{1-k\epsilon}\big)$$
$$\stackrel{\mathrm{M}}{=} \frac{1}{N\epsilon^{2n-1}} \sum_{\ell=2}^{2n} e^{\ell\epsilon \ln N}\big(A^{(n,\ell)}_{\mathrm{H,qg}} + \epsilon\, B^{(n,\ell)}_{\mathrm{H,qg}} + \ldots\big)$$
$$+ \mathcal{O}\big(N^{-2} e^{k\epsilon \ln N}\big). \tag{25}$$

The parameters $A^{(n,\ell)}_{\mathrm{H,qg}}$ combine to the coefficients of the LL contributions $a_s^n \epsilon^{-2n+m} \ln^{m-1} N$ in Eq. (19), which, of course, vanish for $1 \le m \le n-1$ due to Eqs. (21) and (22). Correspondingly, the quantities $B^{(n,\ell)}_{\mathrm{H,qg}}$ determine the NLL contributions at all powers of α_s and ϵ.

The presence of $2n-1$ terms in the sums (25) represents a crucial difference to $\widehat{W}^{(n)}_{\mathrm{H,gg}}$ in the N^0 soft-gluon limit, where only the n even values of ℓ occur [13], and inclusive DIS and semi-inclusive e^+e^- annihilation (SIA), where the corresponding sums run from $\ell = 1$ to $\ell = n$ [26,28]. In those cases, a N^{n}LO calculation leads to a N^{n}LL resummation with a large number of relations to spare. Here, instead, all $2n-1$ terms with negative powers of ϵ are required to fix the LL coefficients $A^{(n,\ell)}_{\mathrm{H,qg}}$, i.e., the terms to ϵ^{-2} fixed by lower-order contributions together with the ϵ^{-1} term provided by the splitting-function resummation (16), see Fig. 1. Consequently, due to the extra factor of ϵ, the NLL coefficients $B^{(n,\ell)}_{\mathrm{H,qg}}$ in Eq. (25) cannot be determined without additional information.

We have determined the coefficients $A^{(n,\ell)}_{\mathrm{H,qg}}$ in Eq. (25) to a sufficiently high order in α_s and find

$$A^{(n,2)}_{\mathrm{H,qg}} = 2C_F \frac{(-1)^n}{(n-1)!}(4C_A)^{n-1},$$
$$A^{(n,3)}_{\mathrm{H,qg}} = 2C_F \frac{(-1)^n}{(n-2)!} 2(C_F - C_A)(4C_A)^{n-2},$$
$$\ldots$$
$$A^{(n,2n)}_{\mathrm{H,qg}} = 2C_F \frac{-1}{n!} \sum_{k=0}^{n-1} (4C_A)^k (4C_F)^{n-1-k}, \tag{26}$$

which can be cast in a closed, if not very transparent, form in terms of binomial coefficients:

α_s \ ε	-9	-8	-7	-6	-5	-4	-3	-2	-1	0	1	...
1									R	D	D	D
2							0	M	R	D	D	D
3					0	0	M	M	R	D	D	D
4			0	0	0	M	M	M	R	D	D	D
5	0	0	0	0	M	M	M	M	R	D	D	D

Fig. 1. The origin of the LL coefficients of $a_s^n \epsilon^k$ in Eqs. (19) and (25) for $n \le 5$. '0' indicates double-pole combinations of n and k which are present in the latter but not the former equation. Entries marked by 'M' are fixed by lower-order quantities through the mass factorization formula. The ϵ^{-1} terms ('R') are required at each order to determine the $2n-1$ coefficients $A^{(n,\ell)}_{\mathrm{H,qg}}$, they involve the splitting functions provided by fixed-order calculations at $n \le 3$ and the resummations (15) and (16). Finally entries marked by 'D' are determined, at each order, from the above coefficients via Eq. (25). Checks of this procedure are provided by the $a_s^2\epsilon^0$ terms of Refs. [9–11,19,20], see Eqs. (9) and (10), and the $a_s^2\epsilon^1$ contributions to Higgs production calculated in Ref. [29].

$$A^{(n,\ell)}_{\mathrm{H,qg}} = \frac{4^n}{2n!} \sum_{m=1}^{\lfloor \ell/2 \rfloor} (-1)^{n+m+1} \binom{n}{\ell - m} \sum_{k=0}^{m-1} \binom{\rho + k}{k}$$
$$\times (C_F - C_A)^{\rho} C_F^{k+1} C_A^{n-k-\rho-1} \tag{27}$$

with $\rho = \ell - 2m$ and $\lfloor a \rfloor$ the largest integer not greater than a. The simplicity of especially the special cases (26) provides some additional insurance against calculational errors. It is interesting to note that not only $A^{(n,3)}_{\mathrm{H,qg}}$, but all odd-$\ell$ coefficients vanish for $C_F = C_A$.

With these results the LL mass-factorization of $\widehat{W}_{\mathrm{H,qg}}$ can be performed order by order; it leads to a table of coefficients which has been given to $n = 12$ in Ref. [30]. Finally this table can be used to find and verify the all-order resummation formula for the quark–gluon coefficient functions,

$$c^{\mathrm{LL}}_{\mathrm{H,qg}}(N, a_s) = \frac{1}{2N \ln N} \frac{C_F}{C_F - C_A}\big\{\exp\big(8C_A a_s \ln^2 N\big)\mathcal{B}_0(\tilde{a}_s)$$
$$- \exp\big((2C_A + 6C_F) a_s \ln^2 N\big)\big\}, \tag{28}$$

which involves the same ingredients as its counterpart for DIS [24] but is slightly more complicated. The corresponding coefficient function for the Drell–Yan process can be obtained from (28) by $C_F \to T_f$ in the numerator of the prefactor and $C_A \leftrightarrow C_F$ everywhere else, including the argument of the function $\mathcal{B}_0$. Expansion of Eq. (28) and Mellin inversion yields the explicit third- and fourth-order predictions

$$c^{(3)\mathrm{LL}}_{\mathrm{H,qg}}(x, a_s) = \ln^5(1-x)\left(18C_F^3 + \frac{100}{3}C_F^2 C_A + \frac{230}{3}C_F C_A^2\right), \tag{29}$$

$$c^{(4)\mathrm{LL}}_{\mathrm{H,qg}}(x, a_s) = \ln^7(1-x)\left(\frac{3646}{135}C_F^4 + \frac{2834}{45}C_F^3 C_A\right.$$
$$\left. + \frac{3166}{135}C_F^2 C_A^2 + \frac{24434}{135}C_F C_A^3\right) \tag{30}$$

and their obvious analogues for lepton-pair production.

To summarize, we have derived the leading-logarithmic large-x resummation of the quark–gluon coefficient functions for inclusive Higgs-boson and lepton-pair production; our main results are Eq. (28) and its closely related counterpart for the Drell–Yan process. Our calculations have been confined to the leading term in the expansion in powers of $(1-x)$; yet we definitely expect the structure with $P_{ik}^{(0)}(x)$ in Eqs. (7)–(10) to occur at all orders. An extension of our results to the next-to-leading double logarithms, $\alpha_s^n \ln^{2n-2}(1-x)$, would require additional all-order insight into the corresponding coefficients in the crucial decomposition of the unfactorized partonic cross section (25). One may hope that an extension of Ref. [14] to the complete N^{3}LO corrections will soon provide useful information also for the large-x resummation of the quark–gluon channel.

Acknowledgements

We thank S. Marzani for his interest in these hitherto unpublished results which led to the writing of the present article. This research has been supported by the European Research Council under Advanced Investigator Grant ERC-AdG-228301; the German Research Foundation (DFG) through Sonderforschungsbereich Transregio 9, Computergestützte Theoretische Teilchenphysik; the UK Science & Technology Facilities Council (STFC) under grant number ST/G00062X/1, and the Research Executive Agency (REA) of the European Union under the Grant Agreement number PITN-GA-2010-264564 (LHCPhenoNet). Our calculations were performed using the symbolic manipulation system FORM [31].

References

[1] G. Aad, et al., ATLAS Collab., Phys. Lett. B 716 (2012) 1, arXiv:1207.7214 [hep-ex];
S. Chatrchyan, et al., CMS Collab., Phys. Lett. B 716 (2012) 30, arXiv:1207.7235 [hep-ex].
[2] CMS Collab., CMS-PAS-HIG-13-005, http://cds.cern.ch/record/1542387;
ATLAS Collab., ATLAS-CONF-2013-034, http://cds.cern.ch/record/1528170.
[3] D. Graudenz, M. Spira, P.M. Zerwas, Phys. Rev. Lett. 70 (1993) 1372;
M. Spira, A. Djouadi, D. Graudenz, P.M. Zerwas, Nucl. Phys. B 453 (1995) 17, arXiv:hep-ph/9504378.
[4] R. Harlander, P. Kant, JHEP 12 (2005) 015, arXiv:hep-ph/0509189.
[5] A. Pak, M. Rogal, M. Steinhauser, JHEP 02 (2010) 025, arXiv:0911.4662 [hep-ph];
R.V. Harlander, H. Mantler, S. Marzani, K.J. Ozeren, Eur. Phys. J. C 66 (2010) 359, arXiv:0912.2104 [hep-ph].
[6] J.R. Ellis, M.K. Gaillard, D.V. Nanopoulos, Nucl. Phys. B 106 (1976) 292;
M.A. Shifman, A.I. Vainshtein, M.B. Voloshin, V.I. Zakharov, Sov. J. Nucl. Phys. 30 (1979) 711.
[7] K.G. Chetyrkin, B.A. Kniehl, M. Steinhauser, Nucl. Phys. B 510 (1998) 61, arXiv:hep-ph/9708255.
[8] Y. Schröder, M. Steinhauser, JHEP 01 (2006) 051, arXiv:hep-ph/0512058;
K.G. Chetyrkin, J.H. Kühn, C. Sturm, Nucl. Phys. B 744 (2006) 121, arXiv:hep-ph/0512060.
[9] R.V. Harlander, W.B. Kilgore, Phys. Rev. Lett. 88 (2002) 201801, arXiv:hep-ph/0201206.
[10] C. Anastasiou, K. Melnikov, Nucl. Phys. B 646 (2002) 220, arXiv:hep-ph/0207004.
[11] V. Ravindran, J. Smith, W.L. van Neerven, Nucl. Phys. B 665 (2003) 325, arXiv:hep-ph/0302135.
[12] S. Catani, D. de Florian, M. Grazzini, P. Nason, JHEP 07 (2003) 028, arXiv:hep-ph/0306211.
[13] S. Moch, A. Vogt, Phys. Lett. B 631 (2005) 48, arXiv:hep-ph/0508265.
[14] C. Anastasiou, C. Duhr, F. Dulat, E. Furlan, T. Gehrmann, F. Herzog, B. Mistlberger, arXiv:1403.4616.
[15] T. Ahmed, M. Mahakhud, N. Rana, V. Ravindran, arXiv:1404.0366 [hep-ph].
[16] M. Bonvini, S. Marzani, arXiv:1405.3654 [hep-ph].
[17] S. Catani, L. Cieri, D. de Florian, G. Ferrera, M. Grazzini, arXiv:1405.4827 [hep-ph].
[18] S. Moch, J.A.M. Vermaseren, A. Vogt, Nucl. Phys. B 726 (2005) 317, arXiv:hep-ph/0506288v2.
[19] R. Hamberg, W. van Neerven, T. Matsuura, Nucl. Phys. B 359 (1991) 343;
R. Hamberg, W. van Neerven, T. Matsuura, Nucl. Phys. B 644 (2002) 403 (Erratum).
[20] R.V. Harlander, W.B. Kilgore, Phys. Rev. D 64 (2001) 013015, arXiv:hep-ph/0102241.
[21] G.P. Korchemsky, Mod. Phys. Lett. A 4 (1989) 1257.
[22] S. Moch, J.A.M. Vermaseren, A. Vogt, Nucl. Phys. B 688 (2004) 101, arXiv:hep-ph/0403192;
A. Vogt, S. Moch, J.A.M. Vermaseren, Nucl. Phys. B 691 (2004) 129, arXiv:hep-ph/0404111.
[23] Y.L. Dokshitzer, G. Marchesini, G.P. Salam, Phys. Lett. B 634 (2006) 504, arXiv:hep-ph/0511302.
[24] A. Vogt, Phys. Lett. B 691 (2010) 77, arXiv:1005.1606 [hep-ph].
[25] M. Abramowitz, I.A. Stegun (Eds.), Handbook of Mathematical Functions, Dover, New York, 1965.
[26] A.A. Almasy, G. Soar, A. Vogt, JHEP 03 (2011) 030, arXiv:1012.3352 [hep-ph].
[27] A.A. Almasy, N.A. Lo Presti, A. Vogt, to appear.
[28] N.A. Lo Presti, A. Vogt, A.A. Almasy, PoS RADCOR 2001 (2011) 023, arXiv:1202.5224 [hep-ph].
[29] M. Höschele, J. Hoff, A. Pak, M. Steinhauser, T. Ueda, Phys. Lett. B 721 (2013) 244, arXiv:1211.6559 [hep-ph].
[30] N.A. Lo Presti, PhD thesis, University of Liverpool, 2012.
[31] J.A.M. Vermaseren, New features of FORM, arXiv:math-ph/0010025;
M. Tentyukov, J.A.M. Vermaseren, Comput. Phys. Commun. 181 (2010) 1419, arXiv:hep-ph/0702279;
J. Kuipers, T. Ueda, J.A.M. Vermaseren, J. Vollinga, Comput. Phys. Commun. 184 (2013) 1453, arXiv:1203.6543 [cs.SC].