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Hydrodynamic coefficients of the transverse force on a circular cylinder oscillating sinusoidally in still water

Zhida Yuan\textsuperscript{a} and Zhenhua Huang\textsuperscript{b}\textsuperscript{*}

\textsuperscript{a} Neptune Offshore Engineering Development Co., LTD, Tianjin China
\textsuperscript{b} Department of Ocean and Resources Engineering, School of Ocean and Earth Science and Technology, University of Hawaii at Manoa, Honolulu, HI 96822 USA

Abstract

The alternating nature of vortex shedding and the induced transverse or lift forces are of significant importance in predicting structural responses, such as vortex-induced vibration and the associated fatigue. This paper reports a set of experimental results of the transverse force acting on a circular cylinder oscillating sinusoidally in still water. The measured transverse forces were analyzed using FFT (Fast Fourier Transform), and the coefficients of the mean and first four dominant frequency components, which are important for fatigue analysis, are examined for frequency parameter ($\beta$) ranging from 417 to 2083 and Keulegan-Carpenter number ($KC$) ranging from 4.8 to 28. The results showed that dominant transverse force component had a frequency double the oscillation frequency for most combinations of $\beta$ and $KC$. We also found that for the transverse force coefficient it was possible to scale the force coefficient and the $KC$ number to better correlate the scaled force coefficient with the scaled $KC$ number. These scale factors reflect the influence of the frequency parameter on the measured transverse force coefficients.

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\textsuperscript{*} Corresponding author. Tel.: +1-808-956-8100; fax: +1-808-956-3498.

\textit{E-mail address:} zhenhua@hawaii.edu
1. Introduction

When a cylinder oscillating in still water or a fixed cylinder in regular waves, the total force will have an in-line component (in-line force) and a transverse component (transverse force or lift force). The transverse force is induced by the cyclic vortex shedding which gives an asymmetrical pressure gradient across the wake, and can be larger than the in-line force in some cases. The transverse force may have multiple frequencies (Isaacson and Maull, 1976; Obasaju, et al., 1988; Justesen, 1989; Justesen, 1991), and may trigger resonance at frequencies different from the wave frequency. Understanding of transverse force is important when designing offshore foundations for oil & gas platforms, wind turbines and substations, and performing fatigue analysis of these structures.

For certain values of Reynolds number and Keulagen-Carpenter number (KC number, defined as $UT/D$ with $D$ being the diameter of the cylinder, $T$ the oscillation/wave period and $U$ the oscillation/wave velocity), the vortex shedding may take place mainly on one side of the cylinder and the cylinder may also have a steady component in the transverse force in addition to the fluctuating component (Williamson, 1985; Obasaju, et al., 1988; Lam, et al., 2010). Therefore, a mean steady component should be included in the mathematical model describing the transverse force. In the literature, the transverse force $F_L(t)$ has been described using several transverse force coefficients. If the main concern is maximum transverse force, then a maximum transverse force coefficient may be more appropriate (Isaacson and Maull, 1976; Maull and Milliner, 1978; Justesen, 1991). Since the transverse force fluctuates around a mean with certain degree of irregularity, the Root-Mean-Square (RMS) transverse force coefficient is frequently used in the literature (Shankar, et al., 1988; Chaplin, 1988; Skomedal, et al., 1989; Vengatesan, et al., 2000). When performing fatigue analysis, it is desirable to know the harmonic components in the transverse force. For this purpose, it seems appropriate to use Eq. (1) to express the transverse force (Isaacson and Maull, 1976; Maull and Milliner, 1978; Obasaju, et al., 1988; Justesen, 1991; Wang, 1997).

$$F_L(t) = \bar{F}_L + \frac{1}{2} \rho D L U_m^2 \sum_{n=1}^{N} C_L^n \cos(2 \pi ft + \psi_n), \quad \bar{F}_L = \frac{C_{L\text{mean}}}{2} \rho D L U_m^2,$$

where $\bar{F}_L$ is the mean transverse force with $C_{L\text{mean}}$ being the mean transverse force coefficient; $D$ and $L$ are the diameter and length of the cylinder, respectively; $U_m$ is the maximum horizontal velocity component of the flow; $C_L^n$ is the transverse coefficient for the $n$-th harmonic force component and is a function of both the $KC$ number and Reynolds number; $f$ is the fundamental frequency of oscillatory flow, which is the frequency of cylinder oscillation in this study; $N$ the number of harmonic components used in the analysis; and $\psi_n$ is the phase angle associated with the $n$-th harmonic force component. A lift force coefficient for the dominant mode can also be defined. Note that the Strouhal number $S$ is related to the dominant frequency in Eq. (1).

The transverse force acting on an oscillating cylinder is directly related to vortex shedding from the cylinder. Investigations in the past decades have revealed that certain repeatable vortex shedding patterns can occur for certain flow amplitudes (Maull and Milliner, 1978; Williamson, 1985; Obasaju, et al., 1988; Lam, et al., 2010). Each of these repeatable patterns reflects the shedding of a particular number of vortices per half cycle; as the flow reverses in another half cycle, the shed vortices in the first half cycle are carried back over the cylinder, and one or more pairs with vortices of opposite sign can be formed. The pairs of vortices usually are carried away from the body through convection at a large angle to the direction of the main flow.

Transverse force on a circular cylinder have been studied by using a cylinder fixed in oscillatory flow (CFIOF) (Bearman, et al., 1984; Maull and Milliner, 1978; Obasaju, et al., 1988; Williamson, 1985; Chaplin, 1988; Skomedal, et al., 1989), a cylinder oscillating in still water (COISW) (Wang, et al., 1997), a vertical cylinder fixed in waves (VCFIW) (Stansby, et al., 1983; Vengatesan, et al., 2000; Yang and Rockwell, 2002), and a horizontal cylinder fixed in waves (HCFIW) (Shankar, et al., 1988). However, data about the hydrodynamic coefficients of harmonic transverse force are scarce and available data are found in the literature only for selected values of frequency parameter $\beta$, defined by $D^3 \nu / T$ with $\nu$ being the kinematic viscosity of the water. Results available in the literature about hydrodynamic coefficients of harmonic transverse force and the $\beta$ and $KC$ numbers used in these studies are summarized in Table 1.
This study reports a set of experimental results on transverse force on a circular cylinder oscillating sinusoidally in still water for frequency number ranging from 417 to 2083.

Table 1. Available data for hydrodynamic coefficients of harmonic transverse force

<table>
<thead>
<tr>
<th>Reference</th>
<th>Type</th>
<th>Method</th>
<th>$\beta$</th>
<th>$KC$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Maull and Milliner (1978)</td>
<td>CFIOF</td>
<td>Experimental</td>
<td>200,443</td>
<td>5.5-27.5</td>
</tr>
<tr>
<td>Justesen (1991)</td>
<td>CFIOF</td>
<td>Numerical</td>
<td>196</td>
<td>2.0-26.0</td>
</tr>
<tr>
<td>Obasaju, et al., (1988)</td>
<td>CFIOF</td>
<td>Experimental</td>
<td>483</td>
<td>5.0-52.0</td>
</tr>
<tr>
<td>Isaacson and Maull (1976)</td>
<td>VCFIW</td>
<td>Experimental</td>
<td>-</td>
<td>5.0-20</td>
</tr>
</tbody>
</table>

2. Experimental setup and data analysis

A specially-designed cylindrical force model was used to measure the in-line and transverse forces acting on a smooth circular cylinder of 50 mm in diameter. The model includes three parts: a test section of 50-mm long, an upper section and a lower dummy section. The test section was instrumented with a three-dimensional piezoelectric force transducer (Kistler). All these three sections were carefully assembled so that the center lines of these three parts were aligned. For details of the cylindrical force model, the reader is referred to Yuan and Huang (2010,2015), who used the same the cylindrical force model to study a different problem.

All experiments were conducted in a flume of 0.55-m wide, 0.6-m deep and 36-m long, located in the Hydraulics Modelling Laboratory in Nanyang Technological University, Singapore. The model was placed in the middle section of the flume to minimize any possible influence of the two ends of the flume. The force model was firmly attached to a stiff frame, which itself was firmly attached to an actuator. The actuator can be programmed and provide any specified oscillations. The motion of the force model was measured and monitored by a Ultralab sensor. The force model had been well calibrated through both static and dynamic tests (Yuan and Huang, 2010). The accuracy of our instrumented cylindrical force model was also verified by comparing our in-line force results for $\beta$=500, 1042 and 1563 with those reported in the literature with similar $\beta$ values (Obasaju, et al., 1988; Sarpkaya, 1986).

3. Results and discussion

3.1. The mean and first four harmonic force coefficients

The measured mean transverse force coefficients are shown in Fig. 1 for all test conditions. For a given $KC$ number, the mean transverse force coefficient $C_{L\text{mean}}$, in general, increases with decreasing frequency parameter $\beta$; for a given frequency parameter $\beta$, $C_{L\text{mean}}$, in general, decreases with increasing $KC$. In our experiments, $C_{L\text{mean}}$ can reach as large as 3.8, which is comparable to the dominant harmonic force coefficient, confirming again that the mean transverse force should be considered in the design of marine structures.

The measured harmonic force coefficients for all test conditions are shown in Fig. 2. Up to 6 harmonics were analyzed in our data analysis, but only the first 4 harmonic transverse force coefficients are shown in Fig. 3. In general, the third harmonic force coefficients are smaller than the other three. However, Wang, et al, (1997) found that the third harmonic component was the dominant mode in their experiments. The hydrodynamic coefficients are expected to drop sharply to zero when $KC \rightarrow 0$ (Skomedal, et al., 1989). When $KC<4$, it is possible that there is no flow separation, thus no transverse force (Williamson, 1985; Obasaju, et al., 1988). Since the low $KC$ number cases are not the focus of this study, the lower limit of the $KC$ number in Fig. 2 is above 4.
3.2. Primary and secondary frequencies

Referring to Fig. 2, it can be seen that except for $\beta = 2083$ where the primary (dominant) frequency is the same as the oscillation frequency $f$, the component of frequency $2f$ always dominate the transverse force. The peak value of the transverse coefficient for the second harmonic component moves toward lower $KC$ values when the value of $\beta$ changes from 2083 to 417. The second largest force coefficient is related to the so-called secondary frequency. For most cases, the secondary frequency is $4f$, except for very small or very large values of $\beta$: for $\beta = 2083$, the secondary frequency is $2f$; for $\beta = 417$, secondary frequency is $f$. For $\beta = 625$ and 600, the secondary frequency is $f$ when $KC$ is small, but $4f$ when $KC$ is large.

3.3. Trajectory of the total force vector

Fig. 3 shows four examples of the trajectory of force vector. These examples cover four cases: (a) for $\beta = 2083$ and $KC = 4.9$, the primary frequency is $f$ and the secondary frequency is $2f$; (b) for $\beta = 2083$ and $KC = 8.20$, the primary frequency is $2f$ and the secondary frequency is $f$; (c) for $\beta = 1563$ and $KC = 11.44$, the primary frequency is $2f$ and the secondary frequency is $4f$; (d) for $\beta = 833$ and $KC = 18.00$, the primary frequencies are $2f$ and $4f$. Fig. 3 also shows that the magnitude of the transverse force is comparable with the in-line force, except when $\beta$ number is large and $KC$ number is small.

3.4. Discussion

It can be seen from Fig. 2 that hydrodynamic coefficients of transverse force depend on both $KC$ and $\beta$ numbers. The measured force coefficients can be better correlated to the $KC$ number applying $\beta$-dependent scale factors on $C_L$ and $KC$. Formally, we can write

$$\alpha_n(\beta)C_L^n = F_n(\gamma_n(\beta)KC)$$

(2)

where $\beta$-dependent scale factors $\alpha_n(\beta)$ and $\gamma_n(\beta)$ are to be found through trial-and-error. The scaled force coefficients for the first, second and the fourth harmonic are shown in Fig. 4. For the fourth harmonic component, $\alpha_4(\beta) = \gamma_4(\beta) = 1$. For the first and second harmonic components, the two empirical $\beta$-dependent scale factors $\alpha_n(\beta)$ and $\gamma_n(\beta)$ used in Fig. 4 are

$$\gamma_n(\beta) = \gamma_n(\beta) = \left(\frac{\log(\beta)}{5}\right)^2$$

(3)

$$\alpha_n(\beta) = \frac{\exp(2.876 \times 10^{-1}\beta)}{\exp(1.946 \times 10^{-3}\beta) - 1.226 \times 10^{-3} \beta + 6.605}$$

(4)
\[ \alpha_2(\beta) = \frac{1.929 \times 10^{-2} \beta}{\ln(\beta)} \]  

These scale factors were obtained by trial-and-error. It can be seen that the degree of scatter is greatly reduced on the scaled abscissa and ordinate for \( c_{L}^{(1)}, c_{L}^{(2)} \) and \( c_{L}^{(4)} \). However, for the mean transverse force coefficient, no simple scale factors were found that can help achieve a better correlation.

Fig. 2 Harmonic force coefficients for 8 values of frequency parameter: \( \beta = 2083 \) (a), \( \beta = 1563 \) (b), \( \beta = 1250 \) (c), \( \beta = 1042 \) (d), \( \beta = 833 \) (e), \( \beta = 625 \) (f), \( \beta = 500 \) (g), and \( \beta = 417 \) (h).
Fig. 3. Trajectories of the total force vector: (a) for $\beta=2083$ and $KC=4.9$; (b) for $\beta=2083$ and $KC=8.20$; (c) for $\beta=1563$ and $KC=11.44$; (d) for $\beta=833$ and $KC=18.00$.

Fig. 4. Correlation of the scaled force coefficient with the scaled KC number. Top panel-- first harmonic force coefficient, middle panel--second harmonic force coefficient, bottom panel--fourth harmonic force coefficient. See Fig. 1 for the symbols used in this figure.
4. Conclusions

A set of hydraulic tests were performed to study the transverse forces acting on a cylinder oscillating with a single frequency in still water. The measured transverse force was analyzed using a Fourier analysis and up to four frequency components were analyzed to calculate the transverse force coefficient for each frequency component. The experimental results showed that the primary frequency in the transverse force has a frequency double the oscillation frequency, except for $\beta=2083$ where the primary frequency is the same as the oscillation frequency. The mean transverse force coefficient in general increases with decreasing $\beta$, and its value can be comparable to the transverse force coefficients. We also found that it was possible to use the proposed empirical scale factors, which are functions of frequency parameter $\beta$, to better correlate the scaled transverse force coefficients of the first, second and the fourth harmonic component with the scaled KC number.

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References


