Search for three-body force effects in neutron–deuteron scattering at 95 MeV

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Abstract
The neutron–deuteron (nd) elastic scattering differential cross section has been measured at 95 MeV incident neutron energy. The neutron–proton (np) differential cross section has also been measured for normalization purposes. An inclusion of three-nucleon forces gives a considerable improvement in the theoretical description of the nd data in the angular region of the cross-section minimum.

Keywords: Three-nucleon forces; Neutron–deuteron scattering; Angular distribution

1. Introduction
High-precision nucleon–nucleon (NN) potentials [1–4] have been developed recently with parameters adjusted by means of a large data base. Still, NN phenomenology alone cannot reproduce the binding energies of nuclei containing three nucleons or more. This indicates that three-nucleon (3N) forces may play some significant role and should be included in the description. However, 3N forces are difficult to investigate because they are expected to be weak in comparison with NN forces.

Recent developments in computational techniques allow exact solution of 3N bound and nd scattering
states through the Faddeev equations [5], with an \( NN \) potential as input for the calculations. It is also possible to include \( 3N \) forces in the Faddeev equations. Such computations of the \( nd \) differential cross section have been performed by Witała et al. [6], using for the \( 3N \) force a \( 2\pi \)-exchange model with parameters adjusted to the triton binding energy. For an incident neutron energy of 95 MeV, the estimates show that \( 3N \) forces should account for about 30\% of the cross section in the angular region of the cross-section minimum.

A basic way to understand \( 3N \) forces is that a 95 MeV neutron has a de Broglie wave length of 2.9 fm and is at the limit of resolving nucleons as extended objects. A fundamentally correct attitude would be to consider the process as interactions between quarks, the constituents of nucleons. However, quantum chromodynamics (QCD) is of limited use at these scales, where the coupling constant is too strong to allow perturbative calculations. Hybrid theories must be developed in between perturbative QCD and \( NN \) phenomenology. Effective field theories provide a systematic expansion with a rigorous connection to QCD, allowing in some cases to perform approximations in a very efficient way and to keep the error estimates under reasonable control [7]. An approach based on chiral perturbation theory (CHPT) has been developed recently by Epelbaum et al. [8]. The original idea of applying CHPT to few-nucleon systems was formulated by Weinberg [9,10]. The theory allows the description of interactions between pions, and between pions and nucleons (or other matter fields) and relies on a most general effective Lagrangian consistent with (approximate) chiral symmetry of QCD. Three-body forces appear naturally at next-to-next-to-leading order in the CHPT expansion. The \( nd \) differential cross section predicted by CHPT is close to the Faddeev calculations of \( NN \) interactions, except in the region of the minimum where three-body force effects raise the cross section, although not as much as predicted by Faddeev calculations including \( 3N \) forces.

There exist high-quality proton–deuteron \( (pd) \) scattering data at 65–250 MeV incident proton energy covering the full angular distribution [11–19]. Below about 130 MeV, the \( pd \) data fill the cross-section minimum as predicted by calculations including \( 3N \) forces. At higher energies, the measured cross section in the minimum is even higher than predicted. An important theoretical weakness at high energies comes from the fact that it is not well known to which extent relativistic effects contribute to the cross section. Moreover, Coulomb force effects are not included in the theoretical descriptions, and even if they are not expected to contribute strongly to the cross section in the minimum region, they must be investigated carefully before far-reaching conclusions can be drawn about the role of \( 3N \) forces in \( pd \) scattering. For these reasons, \( nd \) data sets around 100 MeV are of great value. Besides one set of \( nd \) data covering the full angular distribution at 65 MeV [20] with limited statistical precision in the minimum region, there exists data in the 80–170° range at 152 MeV [21] and recent data covering the full angular distribution at 250 MeV [22]. The \( nd \) data at 250 MeV agree reasonably well with \( pd \) data at the same energy [19] and are significantly underestimated by the calculations in the minimum region.

2. Experimental procedure

The data of the present experiment were obtained with the Medley experimental setup at The Svedberg Laboratory (TSL) in Uppsala. A 98 MeV proton beam of about 5 \( \mu \)A struck an 8 mm thick \( ^7Li \) target, producing full-energy peak neutrons of 94.8 MeV (2.7 MeV FWHM). The neutron beam intensity was about \( 5 \times 10^4 \) \( n/(cm^2s) \) at the target position. More details about the TSL neutron beam facility are presented in Ref. [23]. The Medley setup consists of eight detector telescopes. Each of them can be equipped with one or two silicon detectors and one CsI detector, being capable of light ion detection and identification in the energy range 3–130 MeV. Medley is described in detail in Ref. [24]. For the present experiment, the telescopes were all placed in the forward hemisphere at various angles and at distances from the target that optimize the conditions in the region of the \( nd \) cross-section minimum.

Polyethylene CD\(_2\) (280 \( \mu \)m thickness) and CH\(_2\) (1 mm or 195 \( \mu \)m thickness) target foils were used for the \( nd \) and \( np \) measurements respectively, as well as a C (150 \( \mu \)m thickness) target for carbon background subtraction and an empty target frame for instrumental background subtraction. The relative neutron fluences...
Fig. 1. Typical energy spectrum for a telescope placed at 35° obtained after deuteron selection, low-energy neutron rejection and background subtraction. The signal/background ratio before $^{12}\text{C}(n,d)$ subtraction is 1.3. The error bars are purely statistical. The $nd$ elastic events are integrated between the two arrows. The line is a Gaussian fit to the $nd$ elastic peak, used to define the integration limits. The peak at low energy is due to wrap-around effects, caused by low-energy neutrons from the preceding beam pulse.

were determined by two independent fission-based neutron monitors.

For each telescope, protons and deuterons were identified by a selection in a $\Delta E/E$ two-dimensional plot. Events due to low-energy neutrons were partly rejected by time-of-flight (TOF) techniques. The accepted events were projected as energy spectra. Instrumental background and carbon background were subtracted and the remaining peak was fitted with a Gaussian and integrated in the range mean $\pm 2.35\sigma$ to obtain the number of elastic events, as shown in Fig. 1.

A correction was needed to compensate for the unwanted inclusion of events due to low-energy neutrons. For a given neutron energy resolution, the fraction of contaminating events could be determined from an analysis of the neutron spectrum obtained with a magnetic spectrometer [25,26]. In the present experiment, the energy resolution was 3 to 15 MeV (FWHM) varying from telescope to telescope, and the corresponding correction factor was between 0.99 and 0.74, with an uncertainty of up to 2%. The data were also corrected for the CsI detection efficiency, which was estimated to be between 0.92 and 1.00 with an uncertainty of up to 1%. A few experimental points were affected by large energy losses inside the target. The loss of events was estimated using an MC program.

At the smallest neutron c.m. angle, i.e., where the deuteron energy is very low, the data were corrected by about a factor of two. The uncertainty in the correction is 10%, due to experimental uncertainties in the target orientation and thickness.

The uncertainty in the absolute neutron flux given by the neutron monitors was about 10%. However, high-quality $np$ data at 96 MeV obtained with the LISA magnetic spectrometer [27] exist in the 74–180° angular range, with a claimed uncertainty of 1.9% in the absolute scale. A precise absolute normalization could be achieved by normalizing the present $np$ data to the LISA data. The same factor was used to renormalize the $nd$ data. As a cross-check, a normalization using the Nijmegen partial-wave analysis PWA93 [28] as a reference $np$ cross section was also considered. The data normalized to PWA93 are about 4% higher on the absolute scale than those normalized to the LISA data. We have chosen to take this difference as the normalization uncertainty in our measurement.

Data points taken on both sides of the beam and whose neutron c.m. angles are separated by 2° or less from each other were combined in order to reduce the statistical uncertainty per point and cancel out systematic errors due to possible asymmetry effects. For the combined data, the uncertainty in the c.m. angle is 0.5°. Counting statistics account for about 4% relative uncertainty in the region of the $nd$ scattering minimum. The uncertainty in the absolute normalization does not affect all combined data points in the same way, because they are not always correlated to the same sets of data (that were normalized independently before the data were combined), and the normalization error is therefore included in the systematic uncertainty. The systematic uncertainty contains uncertainties in the setup geometry and detector response (typically about 2%), event identification (typically about 3%) and absolute normalization (4%), summing to about 5%.

3. Results and conclusion

The results are presented in Table 1 and in Fig. 2. The $np$ data are shown in the upper panel of the figure. They agree with the LISA data [27] with a reduced $\chi^2$ of 0.3, and are also in good agreement with PWA93 with a reduced $\chi^2$ of 1.2. When using PWA93 as a
Table 1

Measured $np$ and $nd$ elastic scattering differential cross sections at 95 MeV incident neutron energy. The absolute scale is determined by $np$ elastic scattering normalized to the LISA data [27]. The angular uncertainty is 0.5° (c.m.)

<table>
<thead>
<tr>
<th>$\theta_{2m}$ (degrees)</th>
<th>$\frac{d\sigma}{d\Omega}$ (mb sr)</th>
<th>$\pm \delta_{\text{stat}}$ (mb sr)</th>
<th>$\pm \delta_{\text{syst}}$ (mb sr)</th>
</tr>
</thead>
<tbody>
<tr>
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<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>27.5</td>
<td>7.66</td>
<td>0.38</td>
<td>0.41</td>
</tr>
<tr>
<td>50.6</td>
<td>5.53</td>
<td>0.11</td>
<td>0.30</td>
</tr>
<tr>
<td>58.9</td>
<td>4.69</td>
<td>0.08</td>
<td>0.21</td>
</tr>
<tr>
<td>69.7</td>
<td>4.63</td>
<td>0.07</td>
<td>0.25</td>
</tr>
<tr>
<td>87.2</td>
<td>4.13</td>
<td>0.04</td>
<td>0.24</td>
</tr>
<tr>
<td>98.6</td>
<td>4.16</td>
<td>0.06</td>
<td>0.19</td>
</tr>
<tr>
<td>107.0</td>
<td>4.89</td>
<td>0.05</td>
<td>0.28</td>
</tr>
<tr>
<td>127.6</td>
<td>6.41</td>
<td>0.05</td>
<td>0.37</td>
</tr>
<tr>
<td>139.1</td>
<td>7.83</td>
<td>0.10</td>
<td>0.43</td>
</tr>
<tr>
<td>150.0</td>
<td>9.25</td>
<td>0.11</td>
<td>0.54</td>
</tr>
<tr>
<td>$nd$ data</td>
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</tr>
<tr>
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<td>17.1</td>
<td>0.2</td>
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</tr>
<tr>
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<td>1.86</td>
<td>0.03</td>
<td>0.10</td>
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<tr>
<td>69.5</td>
<td>1.31</td>
<td>0.03</td>
<td>0.07</td>
</tr>
<tr>
<td>88.8</td>
<td>0.668</td>
<td>0.015</td>
<td>0.038</td>
</tr>
<tr>
<td>99.2</td>
<td>0.490</td>
<td>0.016</td>
<td>0.027</td>
</tr>
<tr>
<td>107.6</td>
<td>0.495</td>
<td>0.017</td>
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<tr>
<td>128.7</td>
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<tr>
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<tr>
<td>150.0</td>
<td>0.746</td>
<td>0.049</td>
<td>0.043</td>
</tr>
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</table>

reference, the reduced $\chi^2$ is 0.6 with respect to both the LISA data and PWA93. Since the goal of the measurement is to investigate an effect in the shape of the $nd$ angular distribution, it is quite comfortable that the $np$ angular distribution has the expected shape.

The $nd$ data are shown in the middle and lower panels of Fig. 2. They are compared with three different theoretical approaches: (i) Faddeev calculations of $NN$ interactions using the CD-Bonn potential without $3N$ forces, (ii) the same with $3N$ forces from a $2\pi$-exchange model [6], and (iii) CHPT at next-to-next-to-leading order [8]. All calculations are able to give a good overall description of the data. In order to investigate $3N$ forces, it is appropriate to compute the reduced $\chi^2$ in the angular region of the cross-section minimum, where the effects are expected to be significant. The reduced $\chi^2$ for the three points between 107° and 140° neutron c.m. angle are 7.9 for (i), 0.7 for (ii) and 2.0 for (iii). If the six points shown in the lower panel of Fig. 2 are included, the reduced $\chi^2$ are 5.4, 1.1 and 1.7, respectively. Using PWA93 instead of PWA93 instead of
Fig. 3. Ratio of the \( nd \) cross section to the \( np \) cross section at 95 MeV, as a function of the detected particle lab angle \( \theta_B \). The inner vertical error bars are due to statistics and the extended error bars represent the total uncertainty. The uncertainty in \( \theta_B \) is \( \pm 0.25^\circ \).

The theoretical curves are calculations with the CD-Bonn potential, the \( nd \) cross section being obtained from Faddeev calculations with and without \( 3N \) forces [3,6]. The same is plotted in the lower panel in the region of the minimum and with a linear vertical scale.

The LISA data for the normalization raises slightly the cross section, and the reduced \( \chi^2 \) for the three points in the minimum are then 11.0, 1.0 and 3.8 for (i), (ii) and (iii), respectively. Calculations with the CD-Bonn potential without three-body forces are unable to describe the data in the region of the minimum, while an inclusion of \( 3N \) forces in the Faddeev equations results in a remarkable agreement with the data. The approach based on CHPT gives also a considerable improvement.

Furthermore, the ratio of the \( nd \) cross section to the \( np \) cross section, which must be expressed as a function of the detected particle angle in the laboratory frame, allows normalization-free comparisons. This ratio is shown in Fig. 3. The conclusions are the same: a very good agreement with the theoretical curve based on calculations with the CD-Bonn potential with \( 3N \) forces, and large discrepancies in the minimum region when \( 3N \) forces are not included. The \( \chi^2 \) are comparable with the ones given above.

At the same energy in the c.m. system, \( pd \) data measured in inverse kinematics [12] are shown together with the present \( nd \) data in Fig. 2. The reduced \( \chi^2 \) between \( nd \) and \( pd \) is 2.6. The \( pd \) data are quite old (1954) and, although probably very good for their time, have larger uncertainties than the present \( nd \) data. This motivates a new \( pd \) experiment at this energy to study Coulomb effects in detail. Indeed, such an experiment has recently been performed at RCNP in Osaka, and the data are under analysis [30].

In conclusion, a combination of \( pd \) and \( nd \) elastic scattering data around 100 MeV permits to understand the role of Coulomb interactions and to perform a deep investigation of \( 3N \) force effects. We have measured the \( nd \) differential cross section at 95 MeV incident neutron energy and normalized the result to the \( np \) cross section. An inclusion of \( 3N \) forces is needed in order to describe the data in the minimum region, either by solving the Faddeev equations with an additional \( 3N \) force from a \( 2\pi \)-exchange model, or by performing CHPT calculations at next-to-next-to-leading order.

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