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Procedia IUTAM 14 (2015) 394 – 402

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Stabilizing a leading-edge boundary layer subject to wall suction by increasing the Reynolds number

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Abstract

Laminar flow in the boundary layer at the leading edge of swept airplane wings typically becomes transitional and turbulent shortly downstream of the attachment line. Flow control techniques to maintain the flow laminar, such as suction into the wall, therefore must focus on this instability, which otherwise leads to turbulent flow and thus contaminates the flow over the entire wing chord. The present paper presents new results on how the linear leading-edge boundary layer (LEBL) instability of swept-cylinders flow, which models swept-wing flow, may be avoided. The classical Reynolds number definition is employed, which is based on the far-field velocity Q_∞ , the cylinder radius R^* , and the sweep angle Λ . It is demonstrated that the flow can be stabilized by increasing the Reynolds number at constant wall suction through an increase of R^* or Λ , but not of Q_∞ .

The stability analysis is carried out for the swept Hiemenz boundary layer (SHBL), a widely used flat-plate approximation of the swept-cylinder LEBL. As demonstrated recently¹, the SHBL with suction becomes similar to the two-dimensional asymptotic suction boundary layer (ASBL) when increasing the classical Reynolds number Re^{SH} to large values. In the limit of $Re^{SH} \rightarrow \infty$, the SHBL with suction becomes identical to the highly stable ASBL, and hence inherits its linear stability properties. The transformation of these recent findings concerning the linear stability of the SHBL with suction to the swept-cylinder LEBL unveils that stabilization of flow with constant suction can be observed by increasing Re^{SH} .

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Selection and peer-review under responsibility of ABCM (Brazilian Society of Mechanical Sciences and Engineering)

Keywords: boundary-layer instability, linear stability theory, leading-edge boundary layer, asymptotic suction boundary layer, flow control

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Nomenclature

x	cylinder-chordwise direction	Q_∞	free-stream velocity	θ	b.l. momentum thickness
η	cylinder-normal direction	R^*	cylinder radius	Q_I	flat-plate impingement vel.
z	streamwise/sweep direction	Λ	cylinder sweep angle	γ	flat-plate strain rate ($= Q_I/\theta$)
Re^{cyl}	cylinder Reynolds number ($= R^* Q_\infty \cos \Lambda / \nu$)	V_0	wall-normal suction velocity	κ	wall suction ($= V_0 / \sqrt{\nu \gamma}$)
Re^{SH}	swept Hiemenz Reynolds number ($= W_\infty / \sqrt{a \nu}$)	W_∞	sweep velocity ($= Q_\infty \sin \Lambda$)	φ	flat-plate sweep angle
		a	chordwise strain rate ($= 2Q_\infty \cos(\Lambda)/R^*$)	ν	kinematic viscosity
		Δ^{SH}	reference length ($= \sqrt{\nu/a}$)	$\bar{\theta}$	nondim. boundary-layer momentum thickness

1. Introduction*1.1. Motivation*

The leading-edge boundary layer flow on swept aircraft wings undergoes transition to turbulence usually immediately downstream of the attachment line. However, it is indispensable for achieving laminar flow over large portions of the chord and hence for significant wing drag reduction to avoid transition near the leading edge of swept wings, or swept cylinders. The aim of the present paper is to contribute to the understanding of the viscous linear instability in the very front part of the leading edge. New results on the influence of the leading-edge radius and sweep angle on the flow stability are presented. In light of the decade-long efforts to maintain the flow over wings laminar, these are potentially of relevance to wing design.

1.2. The LEBL, SHBL and their connection to the ASBL

The leading-edge geometry is defined by the cylinder radius R^* and the sweep angle Λ against a free-stream of velocity Q_∞ (cf. nomenclature). The irrotational far-field flow is determined by the three parameters $\{Q_\infty, R^*, \Lambda\}$, which lead to a chordwise strain rate $a = 2Q_\infty \cos(\Lambda)/R^*$ of the outer flow. Close to the attachment line, the resulting swept-cylinder LEBL may be approximated^{2,3} by the flat-plate swept Hiemenz boundary layer (SHBL). The latter flow is self-similar, and it is characterized⁴ by the sweep velocity W_∞ and a strain rate a , which must be matched to the outer strain rate of the cylinder flow.

The SHBL belongs to the class of Falkner-Skan (FS) boundary layers⁵ and requires the chordwise strain rate a to be nonzero. However, it was recently demonstrated^{6,1} by the introduction of a new boundary-layer formalism (figure 1) that the chordwise flow may vanish, $a = 0$, if suction is present. In that case, the SHBL with suction becomes identical to the two-dimensional asymptotic suction boundary layer (ASBL). The classical Reynolds number definition, however, diverges in this limit, $Re^{SH}|_{a \rightarrow 0} \rightarrow \infty$, even though the flow is well-behaved and possesses a nonzero boundary-layer thickness. In particular, the ASBL has a linear critical Reynolds number and favorable stability properties. This singularity makes a direct transformation (e.g. of stability results) of the SHBL with suction to the swept-cylinder LEBL impossible in this limit, when employing the classical FS formalism. By employing the new formalism, a quantitative transformation of the stability results to the LEBL is possible, as long as the boundary-layer thickness remains nonzero. In particular, it becomes possible in the flat-plate limit, i.e. when $R^* \rightarrow \infty$, or $\Lambda \rightarrow \pi/2$. Using the new transformation, linear stability results for swept-cylinder LEBL are obtained on the basis of stability results for the flat-plate SHBL when approaching the limit $Re^{SH}|_{a \rightarrow 0} \rightarrow \infty$. This will be demonstrated in section 2.

1.3. The linear stability of the SHBL and the ASBL

The linear stability analysis of the SHBL⁷ for a classical normal-mode ansatz^{8,9} results in a linear critical Reynolds number of $Re_{crit}^{SH} \approx 583.1$. The stability may be enhanced by applying homogeneous wall suction⁷, as verified in numerical simulations¹⁰ and experiments^{11,12,13}. Adding wall suction with constant dimensional velocity V_0 leads to the definition of a second nondimensional suction number⁷ $\kappa^{SH} = V_0 / \sqrt{\nu a}$. The ASBL has a linear critical Reynolds

number¹⁴ of $Re_{\text{class,crit}}^{AS} \approx 54\,370$ and Herron et al.¹⁵ reported a large portion of its stability diagram. The ASBL was shown to be subcritically unstable¹⁶ while its transition Reynolds number¹⁷ of $Re_{\text{class,tr}}^{AS} \approx 4\,800$ is far below $Re_{\text{class,crit}}^{AS}$.

1.4. Stabilization by suction

The nondimensional wall suction κ^{SH} required to stabilize a SHBL according to linear theory increases monotonically⁷ with the Reynolds number Re^{SH} . This suggests that increasing Re^{SH} also requires an increase of the dimensional suction velocity V_0 to maintain the flow linearly stable. This is the case for relatively low Reynolds numbers Re^{SH} . However, for large Reynolds numbers Re^{SH} , the classical stability condition of monotonically increasing κ^{SH} can be misleading: Both the classical Reynolds number Re^{SH} as well as κ^{SH} diverge when the chordwise strain rate a vanishes. However, the boundary-layer thickness may remain nonzero, if suction is present. Therefore, the case $Q_\infty \rightarrow \infty$, in which the boundary-layer thickness vanishes, must be investigated separately from the two cases of $R^* \rightarrow \infty$ and $\Lambda \rightarrow \pi/2$, in which the boundary-layer thickness remains finite.

It is the aim of the present paper to further elucidate the significance of the asymptotic solution⁷ $\kappa_{\text{crit}}^{SH} \rightarrow \infty$ for $Re^{SH} \rightarrow \infty$ when applied to swept-cylinder flow. It is demonstrated that this result only means that flows of increasing free-stream velocity Q_∞ require increasing physical suction velocities V_0 . To the contrary, it is shown that increasing $Re^{SH} \rightarrow \infty$ via the cylinder radius $R^* \rightarrow \infty$ or the sweep angle $\Lambda \rightarrow \pi/2$ may even lead to flow stabilization if the physical suction V_0 is held constant.

The remainder of this paper is structured as follows. First, the transformations between the LEBL, the SHBL and the ASBL are explained in section 2. The linear stability properties of the SHBL together with those of the ASBL are illustrated in section 3. Then, these linear stability results are transformed to the swept-cylinder geometry (LEBL) in section 4, before the stabilizing effect of an increasing Reynolds number Re^{SH} is discussed in section 5. Section 6 concludes this paper.

2. From swept-cylinder to flat-plate boundary-layer flow with finite and vanishing chordwise velocity

2.1. Transformation of the swept-cylinder LEBL to the flat-plate SHBL

The first step consists in the transformation of the swept-cylinder LEBL to the flat-plate SHBL. The three quantities $\{Q_\infty, R^*, \Lambda\}$ define the far-field flow around the swept cylinder, which gives rise to the chordwise strain rate a . The associated cylinder Reynolds number Re^{cyl} is defined with R^* as the reference length scale and the cylinder-normal reference velocity $Q_\infty \cdot \cos \Lambda$ (see nomenclature). The transformation to the self-similar flat-plate SHBL is carried out by requiring equal far-field strain rates a and sweep velocities W_∞ of both flows. The SHBL Reynolds number Re^{SH} is defined with the wall-parallel reference velocity W_∞ and the synthetic length scale $\Delta^{SH} = \sqrt{v/a}$. Then, Re^{cyl} is related to Re^{SH} according to $Re^{SH} = \sqrt{Re^{cyl}/2} \cdot \tan \Lambda$. When a vanishes (e.g. $R^* \rightarrow \infty$ or $\Lambda \rightarrow 90^\circ$), the classical ansatz¹⁸ becomes singular because the SHBL length scale Δ^{SH} diverges.

2.2. Unification of the flat-plate SHBL and ASBL flows

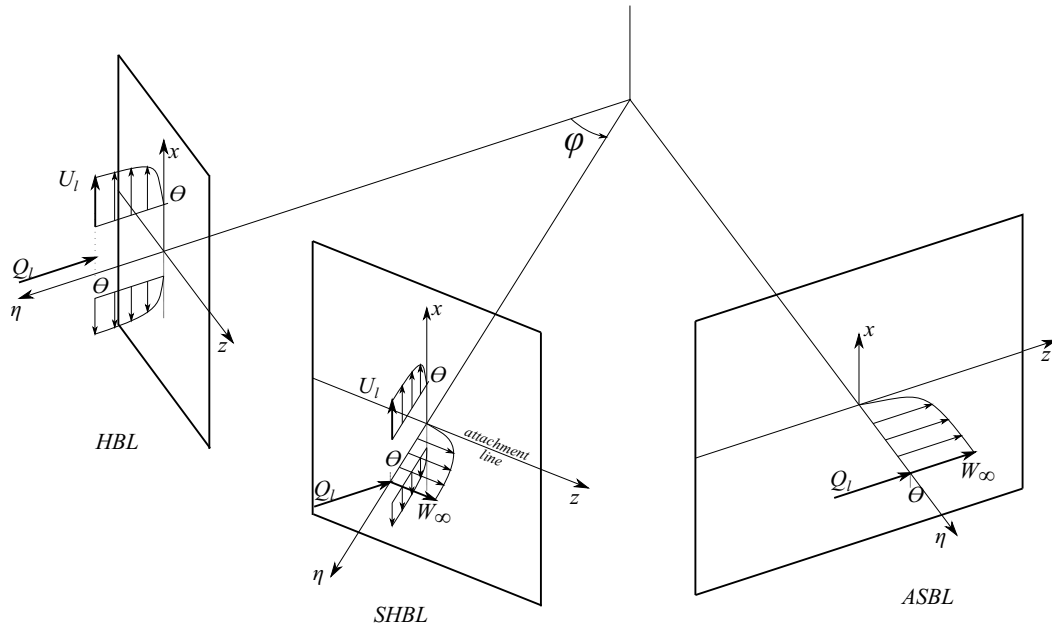


Fig. 1. The Hiemenz boundary layer ($\varphi = 0$), the SHBL ($0 < \varphi < 90^\circ$) and the ASBL ($\varphi = 90^\circ$). All three homogeneous flat-plate boundary layers are described by one formalism, which employs the sweep angle φ of the infinite flat plate. Adapted from (author?)¹ with permission.

The second step, i.e. the unification of the SHBL and the ASBL, is explained here only briefly^{6,1}. The SHBL singularity which occurs for vanishing chordwise flow is overcome by introducing the sweep angle φ of the infinite plate into the boundary-layer similarity ansatz (figure 1). This angle is a representation of the streamline orientation near the attachment line and is not to be confused with the swept-cylinder sweep angle Λ (see figure 2(c) below). The reference length scale Δ of the new solution is not constructed from viscosity and the chordwise strain rate a , unlike Δ^{SH} . Instead, it is based on the boundary-layer momentum thickness θ of the velocity profile in the z -direction. As a result, the similarity coordinate $\eta = y/\Delta$ remains bounded even if the chordwise strain rate a vanishes¹. In brief, the new reference velocity is $Q_l = W_\infty/\sin \varphi$ and the length scale is $\Delta = \sqrt{v/\gamma}$, with the strain rate $\gamma = a/\cos \varphi$. Thus, the new Reynolds number is $Re = Q_l/\sqrt{\gamma v}$ and the nondimensional suction is $\kappa = V_0/\sqrt{\gamma v}$. The transformation to the classical nondimensional numbers of the SHBL is given¹ by $Re^{SH} = \bar{\theta}_0 \tan \varphi$ and $\kappa^{SH} = \kappa/\sqrt{\cos \varphi}$, where $\bar{\theta}_0 \approx 0.4042$. As a result, the governing equations for the three velocity components U , V and W of homogeneous flat-plate boundary layers, in particular the SHBL ($0 < \varphi < \pi/2$) and the ASBL ($\varphi = \pi/2$), read

$$U(x, \eta) = x f' / Re, \quad V(\eta) = -f / Re, \quad W(\eta) = g \sin \varphi, \quad (1a,b,c)$$

$$f''' + f f'' - f'^2 + \cos^2 \varphi = 0, \quad g'' + f g' = 0, \quad (2a,b)$$

subject to the boundary conditions

$$f(0) = \kappa, \quad f'(0) = 0, \quad f'(\eta \rightarrow \infty) = \cos \varphi, \quad (3a,b,c)$$

$$g(0) = 0, \quad g(\eta \rightarrow \infty) = 1. \quad (3d,e)$$

In summary, the two preceding subsections 2.1 and 2.2 present two transformations, which allow us to relate the dimensional swept-cylinder parameters $\{Q_\infty, R^*, \Lambda\}$ and wall suction velocity V_0 of the LEBL, via the SHBL, to φ

and κ directly. These are the two nondimensional numbers that describe homogeneous flat-plate flow. The direct transformations read

$$Q_\infty(\varphi; R^*, \Lambda) = 2 \left(\bar{\theta}_0 \tan \varphi \right)^2 \frac{\nu \cos \Lambda}{\sin^2 \Lambda R^*}, \quad (4a)$$

$$R^*(\varphi; Q_\infty, \Lambda) = 2 \left(\bar{\theta}_0 \tan \varphi \right)^2 \frac{\nu \cos \Lambda}{\sin^2 \Lambda Q_\infty}, \quad (4b)$$

$$\Lambda(\varphi; Q_\infty, R^*) = \arccos \left(-\frac{(\bar{\theta}_0 \tan \varphi)^2 \nu}{Q_\infty R^*} + \sqrt{\left(\frac{(\bar{\theta}_0 \tan \varphi)^2 \nu}{Q_\infty R^*} \right)^2 + 1} \right), \quad (4c)$$

$$V_0(\varphi, \kappa; Q_\infty, \Lambda, R^*) = \kappa \frac{Q_\infty \sin \Lambda}{\bar{\theta}(\varphi, \kappa) \sin \varphi}. \quad (4d)$$

Examples of these explicit relations between φ and $\{Q_\infty, R^*, \Lambda\}$ are illustrated in figure 2. We emphasize that the relatively large values of $\varphi \approx 90^\circ$ result from the fact that the streamlines are almost parallel to the attachment line at the distance of one boundary-layer momentum thickness from the attachment line in the chordwise direction.

If φ is increased to 90° by increasing the cylinder radius R^* or the sweep angle Λ (figures 2(b) or (c)), the chordwise flow component vanishes, but the boundary-layer thickness θ remains finite. As a result, the SHBL flow becomes identical to the ASBL. However, this is not the case if φ is increased to 90° by letting $Q_\infty \rightarrow \infty$ as the boundary-layer momentum thickness vanishes in this limit.

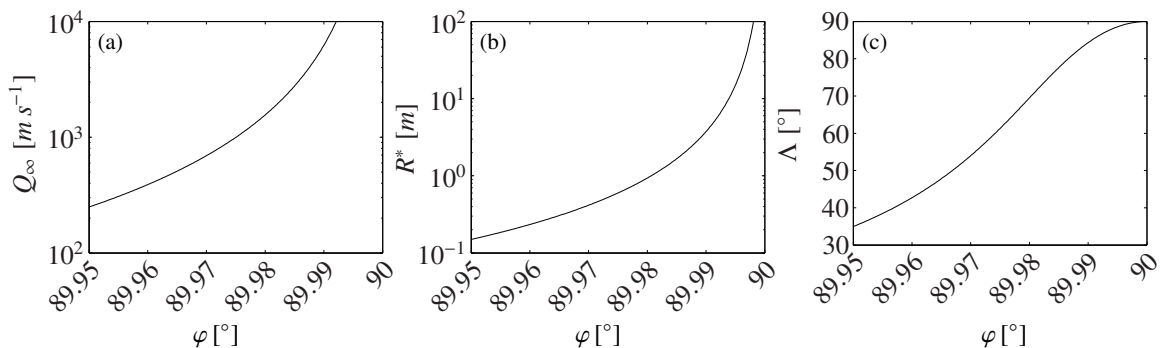


Fig. 2. (a) Free-stream velocity Q_∞ , (b) cylinder radius R^* , (c) sweep angle Λ as functions of φ (eq. (4a-c)) where the respective other quantities are $Q_\infty = 250 \text{ m/s}$, $\Lambda = 35^\circ$, $R^* = 0.15 \text{ m}$, $\nu = 3.5 \cdot 10^{-5} \text{ m}^2/\text{s}$.

3. Linear stability for finite and for vanishing chordwise flow

The transformation of stability results from the flat-plate SHBL configuration to the LEBL in the present publication is based on the solution of the stability equations in ref.¹. There, the temporal linear stability analyses were carried out both for finite ($a > 0$) as well as for vanishing chordwise flow ($a = 0$) for the first time. The neutral surface was obtained on the basis of the new formalism (1), by solving the stability equations for $\varphi \in [0, \pi/2]$ and various suction values $\kappa \geq 0$ for real wave numbers α . As a result, a smooth, compound neutral surface in the $\{\varphi, \kappa, \alpha\}$ parameter space was obtained¹. Neutral curves for various values of κ are illustrated in figure 3.

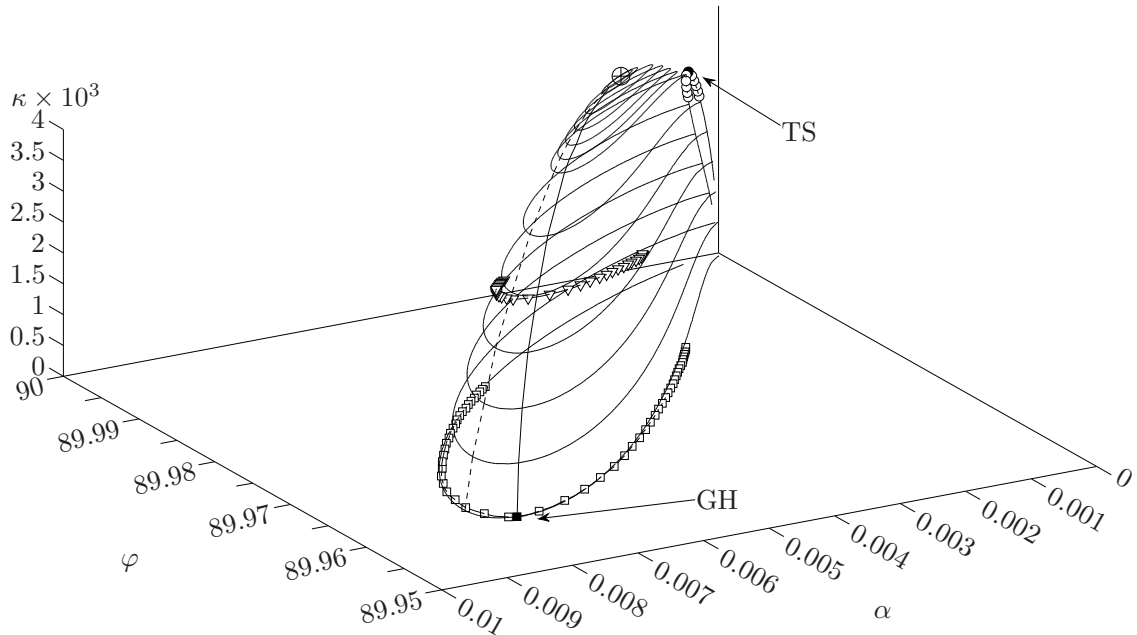


Fig. 3. Neutral curves of the compound linear stability investigation of the SHBL and the ASBL. The neutral curve for $\kappa = 0$ describes the classical Görtler-Hämmerlin (GH) instability with the linear critical Reynolds number of $Re_{crit}^{SH} \approx 583.1$ (filled square). This neutral curve as well as those obtained for $\kappa > 0$ are in good agreement with results by (author?)⁷ (open squares and triangles). For $\varphi = 90^\circ$ and positive suction $\kappa > 0$ all neutral curves remain bounded. The neutral curve of the ASBL reported by (author?)¹⁵ and the most unstable Tollmien-Schlichting (TS) mode are recovered (circles).

In particular, the neutral surface does not diverge in the limit of $\varphi = \pi/2$, which corresponds to the singularity $Re^{SH} \rightarrow \infty$ of the classical formalism. Furthermore, a maximum nondimensional wall suction κ_{max} exists for which the flow is linearly stable at all Reynolds numbers Re^{SH} .

These new stability results can be applied to the swept-cylinder LEBL: The most intuitive approach would consist in performing a two-step transformation: First, this neutral surface would be converted from the new scaling to the classical nondimensionalizations of the SHBL and the ASBL (cf. section 2.2). This would recover the classical results for the SHBL⁷ and the ASBL^{14,15}, as documented previously¹. Second, the results would be converted from the classical SHBL nondimensionalization to cylinder geometries, as presented in section 2.1. However, performing these two steps one after the other, i.e. converting φ to Re^{SH} and κ to κ^{SH} first, would again result in diverging solutions. In order to overcome the singularity $Re^{SH} \rightarrow \infty$ of the SHBL formalism, both steps must be carried out simultaneously, employing the direct transformation equations (4). This is done in the following section.

4. Transformation of the linear stability results for the SHBL with suction to the LEBL

4.1. Transformation of the neutral curve for several far-field flows

A projection of the neutral surface illustrated in figure 3 in the direction of α leads to the neutral curve $\kappa_{crit}(\varphi)$, shown in figure 3 by a solid curve. This needs to be converted into a dimensional suction velocity $V_{0,crit}(Q_\infty, R^*, \Lambda)$ as a function of the dimensional far-field quantities $\{Q_\infty, R^*, \Lambda\}$.

First, φ is converted to a set of parameters $\{Q_\infty, R^*, \Lambda\}$ according to equations (4a-c). Naturally, there are infinitely many parameter combinations which are described by the same angle φ (or Re^{SH}), since the stability results were obtained for a self-similar flat-plate boundary layer. Two of the three values $\{Q_\infty, R^*, \Lambda\}$ are chosen arbitrarily while

the third one is calculated. Second, $V_{0,\text{crit}}(Q_\infty, R^*, \Lambda)$ is calculated in a straightforward manner from equation (4d).

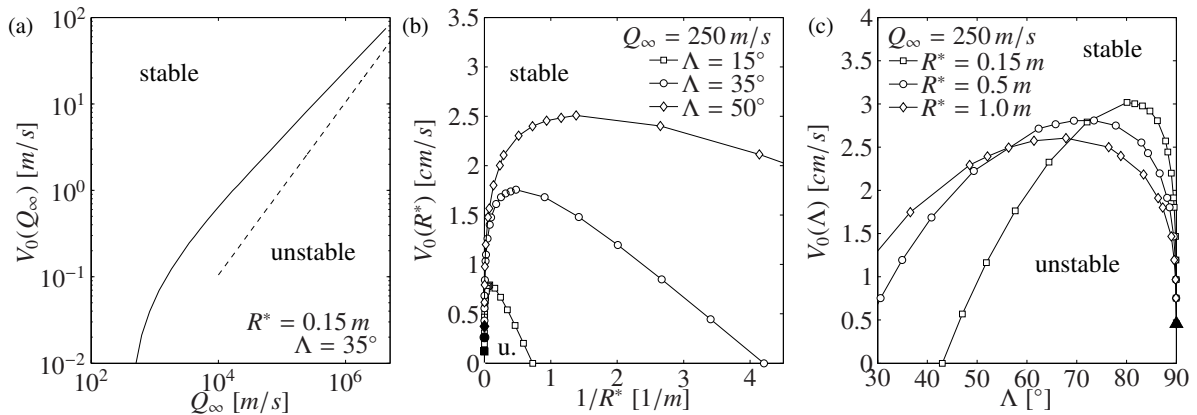


Fig. 4. Neutral curve in dimensional units V_0 as a function of (a) Q_∞ at $R^* = 0.15\text{ m}$, $\Lambda = 35^\circ$, (b) leading-edge radius R^* at $Q_\infty = 250\text{ m/s}$ for various Λ ($15^\circ, 35^\circ, 55^\circ$), (c) sweep angle Λ for various R^* ($0.15\text{ m}, 0.50\text{ m}, 1.0\text{ m}$) at $Q_\infty = 250\text{ m/s}$, $\nu = 3.5 \cdot 10^{-5}\text{ m}^2/\text{s}$. Increasing R^* or Λ can stabilize the flow, if the constant suction V_0 is stronger than the values denoted by the filled symbols (b) ■, ◆, ●, or (c) ▲. Increasing Q_∞ cannot stabilize the flow, the dashed curve in (a) shows the asymptotic behavior of the required suction V_0 as $Q_\infty \rightarrow \infty$.

Exemplary results in dimensional units are shown in figure 4. Panel (a) illustrates the neutral curve $V_{0,\text{crit}}(Q_\infty)$ as a function of the far-field velocity Q_∞ for a constant cylinder radius and sweep angle. The classical result⁷, which states that the suction required to stabilize a LEBL diverges with the far-field velocity, is confirmed. The asymptote is marked in the figure by a dashed line. Figure 4(b) shows three neutral curves $V_{0,\text{crit}}(R^*)$ as a function of the cylinder radius R^* for three different sweep angles Λ . Analogously, panel (c) illustrates three neutral curves $V_{0,\text{crit}}(\Lambda)$ as a function of the sweep angle Λ for three different radii R^* . Both panels (b) and (c) show that, for any given value of R^* and Λ , sufficiently strong finite suction velocities V_0 stabilize the LEBL. This holds even in the limit of infinite cylinder radius R^* or full sweep $\Lambda = 90^\circ$, where the baseflow becomes identical to the ASBL. There, the stability properties (e.g. wave number, phase velocity) of the LEBL become identical to those of the flat-plate ASBL.

4.2. Stability diagram of the swept-cylinder LEBL

The results from figure 4 are recovered and extended in figure 5. The neutral curve $\kappa_{\text{crit}}(\varphi)$ is illustrated by a black curve as it results from the stability equations, separating the regions of linear stability and instability. The φ and κ -axes are directly transformed to the classical Reynolds number definitions which describe the SHBL and the ASBL, respectively, labeled on the top and right boundaries of figure 5. The three sets of curves illustrated in panels (a)-(c) are isolines of V_0 for various $\{Q_\infty, R^*, \Lambda\}$. The curves of panel (a) illustrate the coordinate system $\{Q_\infty, V_0\}$, while those of panels (b) and (c) illustrate the systems $\{R^*, V_0\}$ and $\{\Lambda, V_0\}$, respectively. Along all these curves, V_0 is constant and either one of the three parameters $\{Q_\infty, R^*, \Lambda\}$ varies. From one curve to the next, V_0 is discretely altered (see caption).

From figure 4(a) it can be seen that increasing the far-field velocity Q_∞ makes all flow configurations enter the region of instability eventually. A flow with constant suction V_0 cannot be stabilized by increasing the far-field velocity Q_∞ . Likewise, panels (b)-(c) show that stable flows may become unstable by increasing R^* or Λ . The first linear instability occurs where the green (blue) curves intersect with the black neutral curve. In contrast to panel (a), however, some configurations illustrated in panels (b) and (c) eventually leave the region of unstable flow, when R^* or Λ is further increased. Thus, flow stabilization occurs even when V_0 is held constant. All flow configurations of panels (b)-(c) terminate at the right axis ($\varphi = 90^\circ$, positive suction $\kappa > 0$) when $R \rightarrow \infty$ or $\Lambda = 90^\circ$, respectively. Some of them end within the region of unstable flow, some of them in the stable domain. Whether or not the curves terminate inside the region of unstable flow is determined by the stability properties of the ASBL, which may or may not be stable, depending on the Reynolds number.

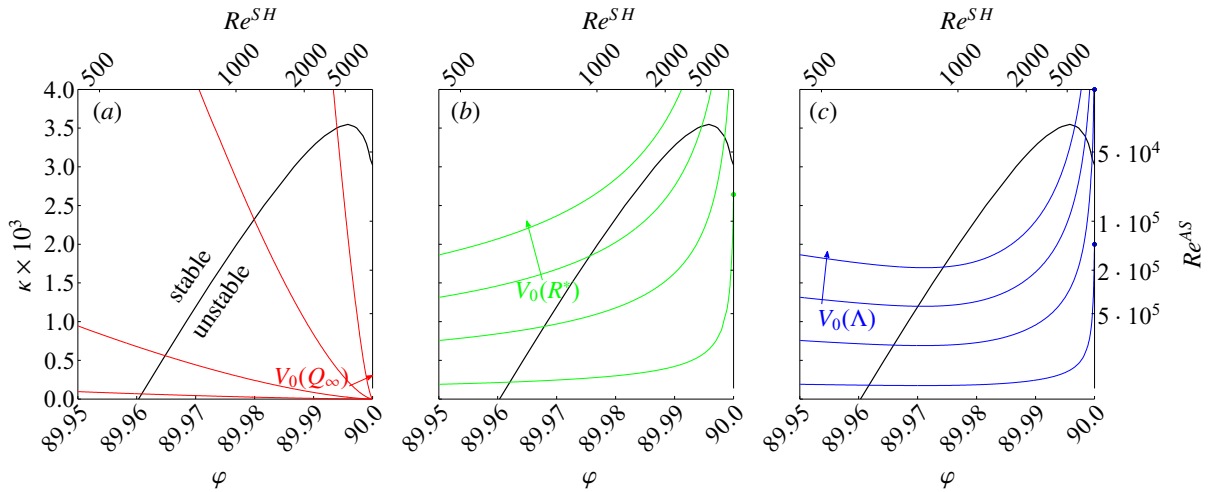


Fig. 5. (Color online) Neutral curve (black) and curves of constant suction velocity V_0 for (a) various far-field velocities Q_∞ , $V_0 = \{0.1, 1, 10, 100\} \text{ cm s}^{-1}$, (b) various cylinder radii R^* , $V_0 = \{0.2, 0.8, 1.4, 2.0\} \text{ cm s}^{-1}$, (c) various sweep angles Λ , $V_0 = \{0.2, 0.8, 1.4, 2.0\} \text{ cm s}^{-1}$. The constant reference parameters for the respective other panels are $Q_\infty = 250 \text{ m s}^{-1}$, $\Lambda = 35^\circ$, $R^* = 0.15 \text{ m}$, $\nu = 3.5 \cdot 10^{-5} \text{ m}^2 \text{ s}^{-1}$ (cf. fig. 2).

5. Flow stabilization by increasing the Reynolds number

A surprising result is the fact that the curves shown in figure 4(b) and (c) are non-monotonic, or, that the blue and green curves of figure 5(b-c) may intersect with the black neutral curve twice. This demonstrates that an unstable LEBL may not only be stabilized by increasing the wall suction or by decreasing the Reynolds number Re^{SH} . To the contrary, if the dimensional suction velocity V_0 exceeds a certain threshold, a LEBL with constant suction may be stabilized by increasing Λ or R^* and thus Re^{SH} . As an example, it may be observed from figure 4(c) that a swept cylinder of radius $R^* = 0.5 \text{ m}$ at $V_0 = 2.5 \text{ cm/s}$ wall suction first becomes linearly unstable when the sweep angle exceeds $\Lambda \approx 60^\circ$ but re-stabilizes again when $\Lambda \approx 80^\circ$.

Finally, it is pointed out that Re^{SH} increases monotonically both with R^* and with Λ . Thus, re-stabilization of any LEBL is observed for all wave numbers by increasing Λ or R^* if the physical wall suction V_0 is sufficiently large. Likewise, there are LEBL flows which are stable at all wave numbers for any value of Λ or R^* if V_0 is large enough. For the examples of leading-edge boundary-layer flow presented here, suction on the order of 1 cm/s is sufficient. On the other hand, Q_∞ must not be increased in order for the flow to be stabilized (see figure 5(a)).

6. Conclusions

We have shown that the leading-edge boundary layer (LEBL) with wall suction may be stabilized by increasing the Reynolds number. The stability properties were obtained from the swept Hiemenz boundary layer (SHBL) along a flat plate, a commonly used model flow with the classical Reynolds number definition Re^{SH} . The reason for this counterintuitive observation is the fact that, despite the singularity $Re^{SH} \rightarrow \infty$ in the limit of vanishing chordwise flow $a = 0$, the flow is physically sensible and identical to the very stable ASBL configuration¹ if nonzero wall suction is applied. Therefore, the LEBL with wall suction inherits the properties of the ASBL if the chordwise flow vanishes. Even before reaching the limit of vanishing chordwise flow, an increase of the cylinder sweep angle Λ or radius R^* (which lets Re^{SH} become large) may already stabilize the LEBL as it approaches the ASBL solution. An increase of the far-field velocity Q_∞ , however, does not lead to flow stabilization. In summary, a stabilization of the LEBL may be observed by an increase of the SHBL Reynolds number Re^{SH} , if wall suction is present.

We acknowledge funding of this work by ETH Research Grant ETH-19 12-2.

References

1. John, M. O., Obrist, D., and Kleiser, L., "A class of exact Navier-Stokes solutions for homogeneous flat-plate boundary layers and their linear stability," *Journal of Fluid Mechanics*, Vol. 752, Aug. 2014, pp. 462–484.
2. Pfenninger, W., "Flow Phenomena at the Leading Edge of Swept Wings," Tech. Rep. AGARDograph 97, North Atlantic Treaty Organization - Advisory Group for Aeronautical Research, Paris VII, 1965.
3. Gaster, M., "On the Flow Along Swept Leading Edges," *Aeronautical Quarterly*, Vol. 18, 1967, pp. 165–&.
4. Poll, D. I. A., "Some Aspects of the Flow Near a Swept Attachment Line With Particular Reference to Boundary Layer Transition," Tech. Rep. 7805, Cranfield Institute of Technology, College of Aeronautics, Cranfield, 1978.
5. Rosenhead, L., *Laminar Boundary Layers*, Clarendon, Oxford, 1963.
6. John, M. O., Obrist, D., and Kleiser, L., "An Exact Navier-Stokes Solution for Three-Dimensional, Spanwise-Homogeneous Boundary Layers," *Proceedings in Applied Mathematics and Mechanics*, Vol. 12, No. 1, 2012, pp. 477–478.
7. Hall, P., Malik, M. R., and Poll, D. I. A., "On the Stability of an Infinite Swept Attachment Line Boundary Layer," *Proceedings of the Royal Society of London Series A*, Vol. 395, No. 1809, 1984.
8. Görtler, H., "Dreidimensionale Instabilität der ebenen Staupunktströmung gegenüber wirbelartigen Störungen," *50 Jahre Grenzschichtforschung. Eine Festschrift in Originalbeiträgen.*, edited by H. Görtler and W. Tollmien, Vieweg, Braunschweig, 1955, pp. 304–314.
9. Hämmerlin, G., "Zur Instabilitätstheorie der ebenen Staupunktströmung," *50 Jahre Grenzschichtforschung. Eine Festschrift in Originalbeiträgen*, edited by H. Görtler and W. Tollmien, Vieweg, Braunschweig, 1955, pp. 315–327.
10. Spalart, P. R., "Direct Numerical Study of Leading-Edge Contamination," Tech. Rep. AGARD CP No. 438, NATO-AGARD, Neuilly-sur-Seine, 1988.
11. Danks, M. and Poll, D. I. A., "Non-Linear Instability and Transition in Flow Near a Swept Leading Edge," *IUTAM Symposium on Nonlinear Instability and Transition in Three-Dimensional Boundary Layers*, edited by P. W. Duck and P. Hall, Vol. 35, Kluwer Academic Publ, Dordrecht, Manchester, England, Jul 17-20, 1995, 1995, pp. 133–146.
12. Poll, D. I. A. and Danks, M., "Relaminarisation of the Swept Wing Attachment-Line by Surface Suction," *IUTAM Symposium on Laminar-Turbulent Transition*, Vol. 27, Sendai, Japan, September 5-9, 1994, 1995, pp. 137–144.
13. Arnal, D., Juillen, J., Reneaux, J., and Gasparian, G., "Effect of wall suction on leading edge contamination," *Aerospace Science and Technology*, Vol. 1, No. 8, Dec. 1997, pp. 505–517.
14. Hocking, L. M., "Non-Linear Instability of the Asymptotic Suction Velocity Profile," *Quarterly Journal of Mechanics and Applied Mathematics*, Vol. 28, No. Aug, 1975, pp. 341–353.
15. Herron, I. H., Von Kerczek, C. H., and Tozzi, J., "Instability Characteristics of the Asymptotic Suction Profile," *Journal of Applied Mechanics*, Vol. 52, No. 2, 1985, pp. 487.
16. Likhachev, O. A., "Self-oscillatory flow in asymptotic boundary layers," *Journal of Applied Mechanics and Technical Physics*, Vol. 17, No. 2, 1976, pp. 194–197.
17. Lifshits, A. M., "Stability of the asymptotic boundary layer with suction against finite-amplitude perturbations," *Fluid Dynamics*, Vol. 15, No. 3, 1980, pp. 438–442.
18. Hiemenz, K., "Die Grenzschicht an einem in den gleichförmigen Flüssigkeitsstrom eingetauchten geraden Kreiszyylinder," *Dinglers Polytechnisches Journal*, Vol. 326, No. 21-26, 1911, pp. 321–324,344–348,357–362,372–376,391–393,407–410.