The Serviceability Limit States in Reinforced Concrete Design

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Abstract

In order to satisfy the serviceability limit states, a concrete structure must be serviceable and perform its intended function throughout its working life. Excessive deflection should not impair the function of the structure or be aesthetically unacceptable. Cracks should not be unsightly or wide enough to lead to durability problems. Design for the serviceability limit states involves making reliable predictions of the instantaneous and time-dependent deformation of the structure. This is complicated by the non-linear behaviour of concrete caused mainly by cracking, tension stiffening, creep and shrinkage. This paper provides an overview of the behaviour of reinforced concrete beams and slabs at service loads and outlines a reliable method for the calculation of deflection.

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Keywords: Cracking, creep, deformation, reinforced concrete, serviceability, shrinkage, tension stiffening.
1. INTRODUCTION

The broad design objective for a concrete structure is that it should satisfy the needs for which it was contrived. In doing so, the structural designer must ensure that it is both safe and serviceable, so that the chances of it failing during its design lifetime are sufficiently small. The two primary structural design objectives are therefore strength and serviceability.

Modern design codes for structures, including AS3600-2009, have adopted the limit states method of design, whereby a structure must be designed to simultaneously satisfy a number of different limit states or design requirements, including adequate strength and serviceability. Minimum performance limits are specified for each of these limit states and any one may become critical and govern the design of a particular member. For each limit state, codes of practice specify both load combinations and methods of predicting the actual structural performance that together ensure an acceptably low probability of failure.

In order to satisfy the serviceability limit states, a concrete structure must be serviceable and perform its intended function throughout its working life. Excessive deflection should not impair the function of the structure or be aesthetically unacceptable. Cracks should not be unsightly or wide enough to lead to durability problems and vibration should not cause distress to the structure or discomfort to its occupants.

In this paper, the effects of creep and shrinkage on the deflection and cracking of reinforced concrete beams and slabs are discussed and quantified. An overview of recent research on the serviceability of reinforced concrete beams and slabs at the University of New South Wales is also provided.

2. EFFECTS OF CRACKING ON CROSS-SECTIONAL RESPONSE

Consider a reinforced concrete element subjected to uniform bending. The average instantaneous moment-curvature response is shown in Figure 1. At moments less than the cracking moment, $M_{cr}$, the element is uncracked and the moment-curvature relationship is essentially linear (OA in Figure 1) with a slope proportional to the second moment of area of the uncracked transformed section, $I_{uncr}$. When the extreme fibre tensile stress in the concrete reaches the flexural tensile strength, $f_{ctl}$, i.e. when the moment reaches the cracking moment $M_{cr}$, primary cracks form at reasonably regular centres. There is a sudden change in the local stiffness at and immediately adjacent to each crack. At a section containing a crack, the tensile concrete carries little or no stress, the flexural stiffness drops significantly and the moment-curvature relationship follows the dashed lines AA’C (when $M = M_{cr}$) in Figure 1, with the slope of line A’C proportional to the second moment of area of the cracked transformed section, $I_{cr}$.

In reality, the flexural rigidity of the fully-cracked cross-section ($EcI_{cr}$) underestimates stiffness after cracking because the tensile concrete between primary cracks carries stress due to bond between the tensile reinforcement and concrete. The average instantaneous moment-curvature response after cracking follows the solid line AB in Figure 1. At a typical in-service moment $M_t (\geq M_{cr})$, the flexural rigidity of the cracked region is $(EcI_{cr})$, as shown in Figure 1. As moment increases, there is a gradual breakdown in the steel-concrete bond and the average flexural stiffness of the entire member decreases. The difference between the actual and the zero tension response is known as tension stiffening (and is represented by a reduction in average instantaneous curvature, $\delta \kappa_{0.25r}$, as shown).

Branson’s equation for the effective second moment of area of a cracked region is specified in AS3600-2009 (Branson, 1965). A more realistic and rational equation, given in Equation 1, was recently proposed by Bischoff (2005) and can be derived from the average curvature approach in Eurocode 2 (2004):
Average $M$ versus $N_0$

$$I_{ef} = \frac{I_{cr}}{1 - \beta \left(1 - \frac{I_{cr}}{I_{uncr}} \right) \left(\frac{M_{cr}}{M_s}\right)^2} \leq 0.6I_{uncr}$$

3. Effects of Creep and Shrinkage on Cross-Sectional Response

3.1 Effects of Creep

The gradual development of creep strain in the compression zone of a reinforced concrete cross-section causes an increase of curvature and a consequent increase in the deflection of the member. For a plain concrete member, the increase in strain at every point on the section is proportional to the creep coefficient and so too, therefore, is the increase in curvature. For the uncracked, singly reinforced section (shown in Figure 2a), creep is restrained in the tensile zone by the reinforcement. Depending on the quantity of steel, the increase in curvature due to creep is proportional to a large fraction of the creep coefficient (usually between 0.7$\varphi(t)$ and 0.95$\varphi(t)$).

On the cracked, singly reinforced beam section (Figure 2b), the initial curvature is comparatively large and the cracked tensile concrete below the neutral axis can be assumed to carry no stress and therefore does not creep. Creep in the compression zone causes a lowering of the neutral axis and a consequent reduction in the cracking moment.
the compressive stress level. Creep is slowed down as the compressive stress reduces, and the increase in curvature is proportional to a small fraction of the creep coefficient (usually less than \( \frac{1}{4} \)). The relative increase in deflection caused by creep is therefore greater in an uncracked beam than in a cracked beam, although the total deflection in the cracked beam is significantly greater.

### 3.2 Effects of Shrinkage

Reinforcement embedded in the concrete provides restraint to shrinkage and, if the reinforcement is not symmetrically placed on a cross-section, a shrinkage-induced curvature develops with time. Consider the singly reinforced member shown in Figure 3a, and the small segment of length, \( \Delta z \). The shrinkage-induced stresses and strains on an uncracked and on a cracked cross-section are shown in Figures 3b and 3c, respectively.

As the concrete shrinks, it compresses the steel reinforcement, and the steel, in turn, imposes an equal and opposite tensile force, \( \Delta T \), on the concrete. This gradually increasing tensile force, acting at some eccentricity to the centroid of the concrete cross-section produces elastic plus creep strains and a resulting curvature on the section. The shrinkage-induced curvature \( (\kappa_{sh})_{cr} \) often leads to significant load independent deflection of the member. The magnitude of \( \Delta T \) (and hence the shrinkage-induced curvature) depends on the quantity and position of the reinforcement and on the size of the (uncracked) concrete part of the cross-section, and hence on the extent of cracking, and this in turn depends on the magnitude of the applied moment. Although shrinkage strain is independent of stress, it appears that shrinkage curvature is not independent of the external load. The shrinkage induced curvature on a previously cracked cross-section \( (\kappa_{sh})_{cr} \) is considerably greater than on an uncracked cross-section \( (\kappa_{sh})_{uncr} \), as can be seen in Figure 3.
4. MOMENT-CURVATURE RELATIONSHIPS

4.1 Effects of shrinkage prior to first loading

The average moment versus instantaneous curvature relationship (OAB in Figure 1 and reproduced in Figure 4) is significantly affected if shrinkage occurs prior to loading (and in practice this is most often the case). For example, for a singly reinforced element, a shrinkage induced curvature \( (\kappa_{sh})_{uncr} \) develops on the uncracked cross-section when the applied moment is still zero (i.e. \( M_s = 0 \)), shown in Figure 4 as point \( O' \). The curvature \( (\kappa_{sh})_{uncr} \) and the tensile stress caused by shrinkage in the extreme fibre of the uncracked cross-section \( \sigma_{cs} \), were illustrated in Figure 3.

The moment required to cause first cracking \( M_{cr,sh0} \) will be less than \( M_{cr} \) because of the initial tensile stress \( \sigma_{cs} \) in the concrete (as indicated in Figure 4) and the moment curvature relationship is now represented by curve \( O'A'B' \). The initial curvature due to early shrinkage on a fully-cracked cross-section \( (\kappa_{sh})_{cr} \), where the concrete is assumed to carry no tension, is significantly larger that that of the uncracked member \( (\kappa_{sh})_{uncr} \), as illustrated in Figure 3. Therefore, early shrinkage before loading causes the dashed line representing the fully-cracked response to move further to the right, shown as line \( O''C' \) in Figure 4.

![Figure 4: Average moment vs instantaneous curvature relationship after early shrinkage strain.](image)

Because the cracking moment is substantially reduced, it is likely that early shrinkage prior to loading affects the magnitude of tension stiffening under an applied moment \( M_s > M_{cr} \) but this is yet to be confirmed.

Empirical expressions for the shrinkage-induced curvature on cracked and uncracked cross-sections are given in Equations 2 and have been developed from a refined time analysis using a technique known as the age-adjusted effective modulus method (Gilbert & Ranzi, 2010):

\[
(\kappa_{sh})_{cr} = 1.2 \left( \frac{I_{cr}}{I_{ef}} \right)^{0.67} \left( 1 - 0.5 \frac{A_{st}}{A_{st}} \right) \left( \frac{E_{sh}}{d_o} \right) \quad (2a)
\]

and
\[ (\kappa_{hb})_{uncr} = (100p - 2500p^2)\left(\frac{d_o}{0.5D} - 1\right)\left(1 - \frac{A_{sc}}{A_{st}}\right)^{1.3} \varepsilon_{sh} \]  

(2b)

where \( A_{st} \) is the area of tensile reinforcement; \( A_{sc} \) is the area of compressive reinforcement (if any); \( p \) is the reinforcement ratio \((A_{st}/bd_o)\); \( \varepsilon_{sh} \) is the shrinkage strain; \( D \) is the overall depth of the member; and \( d_o \) is the depth to the layer of tensile reinforcement.

### 4.2 Effects of creep and shrinkage under sustained loads

For a cross-section subjected to constant sustained moment over the time period \( t_0 \) to \( t \), if no shrinkage has occurred prior to loading, the instantaneous moment versus curvature response of the cross-section is shown as curve OAB in Figure 5 (identical to curve OAB in both Figures 1 and 4). The instantaneous fully-cracked section response (calculated ignoring the tensile concrete) is shown as line OC in Figure 5. If the cross-section does not shrink with time (i.e. \( \varepsilon_{sh} \) remains at zero), creep causes an increase in curvature with time at all moment levels and the time-dependent \( M-N \) response shifts to curve OA’B’ in Figure 5a.

The creep-induced increase in curvature with time at an applied moment \( M \), may be expressed as \( \Delta \kappa_c(t) = \kappa_0 \varphi(t, t_0)/\alpha \), where \( \kappa_0 \) is the instantaneous curvature, \( \varphi(t, t_0) \) is the creep coefficient and \( \alpha \) is a factor that depends on the amount of cracking and the reinforcement quantity and location. For typical reinforcement ratios for beams and slabs, \( \alpha \) is in the range 1.0 – 1.3, prior to cracking, and in the range 4 – 6 when cracking is extensive. Empirical expressions for \( \alpha \) have been developed using the age-adjusted effective modulus method (Gilbert & Ranzi, 2010) and are given by Equations 3a for a cracked reinforced concrete section in bending \((I_{ef} < I_{uncr})\) and in Equation 3b for an uncracked section:

\[
\alpha = 0.48\rho^{-0.5} \left[ \frac{I_{cr}}{I_{ef}} \right]^{0.33} \left[ 1 + (125\rho + 0.1) \left( \frac{A_{sc}}{A_{st}} \right)^{1.2} \right] 
\]

(3a)

and

\[
\alpha = 1.0 + [45\rho - 900\rho^2] \left[ 1 + \frac{A_{sc}}{A_{st}} \right] 
\]

(3b)

When shrinkage before and after first loading is included, the curvature increases even further with time due to shrinkage and the time-dependent response of the cross-section is shown as curve O’A’B’ in Figure 5b. At \( M = 0 \), the curvature increases due to shrinkage of the uncracked cross-section and the point O moves horizontally to O’. Due to the restraint to shrinkage provided by the bonded reinforcement, tensile stress is induced with time and this has the effect of lowering the cracking moment from \( M_{cr} \) to \( M_{cr,sh} \). For any cross-section subjected to a sustained moment in the range \( M_{cr,sh} < M \leq M_{cr} \), cracking will occur with time and the increase in curvature will be exacerbated by the loss of stiffness caused by time-dependent cracking. In practice, critical sections of many lightly reinforced slabs are loaded in this range. The response of the cracked section (ignoring the tensile concrete) after creep and shrinkage is shown as line O’E’ in Figure 5b. The shrinkage induced curvature of the fully cracked cross-section when \( M = 0 \) is greater than that of the uncracked cross-section and the cracked section response is shifted horizontally from point O to point O’, as shown. The slope of the cracked section response in Figure 5b is softened by creep and the slope of the line O’E’ in Figure 5b is the same as the slope of line OC’ in Figure 5a.
Figure 5: Effects of creep and shrinkage on the average moment vs curvature relationship under sustained actions.

At a typical in-service moment $M_s$, the instantaneous curvature due to tension stiffening $\delta \kappa_{0,t}$ is DE in Figure 5b and the time-dependent tension stiffening curvature after the period of sustained loadings $\delta \kappa_{c}(t)$ is D'E'. Tension stiffening reduces under sustained loading, primarily due to time-dependent cracking, shrinkage-induced degradation of bond at the concrete-reinforcement interface and tensile creep between the cracks in the tensile concrete. It is generally believed that tension stiffening reduces rapidly after first loading and reduces to about half its instantaneous value with time (Bischoff, 2001; Scott and Beeby, 2005; Gilbert & Wu, 2009).

4.3 Design Predictions of Average Curvature and Deflection

Clearly, for a cracked member, deformation will be underestimated if the analysis assumes every cross-section is uncracked. On the other hand, deformation will be overestimated, sometimes grossly overestimated, if every cross-section is assumed to be fully-cracked. An excellent method for determining deflection is to calculate the cracked and uncracked curvatures at frequent cross-sections along the member and then to calculate the average curvature at each section using Equation 4 (taken from Eurocode 2, 2004):

$$\kappa_{\text{ave}} = \zeta \kappa_{cr} + (1 - \zeta) \kappa_{uncr}$$  \hspace{1cm} (4)

where $\zeta$ is a distribution coefficient given by:

$$\zeta = 1 - \left( \frac{M_{cr,t}}{M_s^*} \right)^2$$  \hspace{1cm} (5)

and where $M_{cr,t}$ is the cracking moment at the time under consideration and $M_s^*$ is the maximum in-service moment that has been imposed on the member at or before the time instant at which deflection is being determined. For long-term deflection calculations, $M_{cr,t}$ may be taken as 70% of the short-term value. With the curvature diagram thus determined, the deflection can be obtained by numerical integration. If the curvature is determined at the mid-span ($\kappa_{0t}$) and at the left and right ends ($\kappa_{L}$ and $\kappa_{R}$) of a span of length $L$, the mid-span deflection ($v_{M}$) at the time under consideration may be conveniently obtained from Equation 6:
\[ v_M = \frac{L^2}{96} \left( \kappa_L + 10 \kappa_M + \kappa_R \right) \]  

(6)

5. COMPARISONS OF CALCULATED AND MEASURED DEFLECTION

5.1 Experimental program

The final long-term deflections calculated using the procedure outlined in the previous sections are here compared with the measured final deflections of twelve prismatic, one-way, singly reinforced concrete specimens (6 beams and 6 slabs) that were tested by Gilbert and Nejadi (2004) under constant sustained service loads for periods in excess of 400 days. The specimens were simply-supported over a span of 3.5 m with cross-sections shown in Figure 7. All specimens were cast from the same batch of concrete and moist cured prior to first loading at age 14 days. Details of each test specimen are given in Table 1.

Table 1: Details of the test specimens (Gilbert and Nejadi, 2004).

<table>
<thead>
<tr>
<th>Beam</th>
<th>No. of bars</th>
<th>(d_b) mm</th>
<th>(A_{st}) mm(^2)</th>
<th>(c_b) mm</th>
<th>(c_r) mm</th>
<th>(s_b) mm</th>
<th>(M_{cr}) kNm</th>
<th>(M_{sus}) kNm</th>
<th>(\sigma_{st1}) MPa</th>
<th>(M_u) kNm</th>
<th>(M_{sus}/M_u) (%)</th>
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<tbody>
<tr>
<td>B1-a</td>
<td>2</td>
<td>16</td>
<td>400</td>
<td>40</td>
<td>40</td>
<td>154</td>
<td>14.0</td>
<td>24.9</td>
<td>227</td>
<td>56.2</td>
<td>44.3</td>
</tr>
<tr>
<td>B1-b</td>
<td>2</td>
<td>16</td>
<td>400</td>
<td>40</td>
<td>40</td>
<td>154</td>
<td>14.0</td>
<td>17.0</td>
<td>155</td>
<td>56.2</td>
<td>30.2</td>
</tr>
<tr>
<td>B2-a</td>
<td>2</td>
<td>16</td>
<td>400</td>
<td>25</td>
<td>25</td>
<td>184</td>
<td>13.1</td>
<td>24.8</td>
<td>226</td>
<td>56.2</td>
<td>44.1</td>
</tr>
<tr>
<td>B2-b</td>
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<td>16</td>
<td>400</td>
<td>25</td>
<td>25</td>
<td>184</td>
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<td>16.8</td>
<td>153</td>
<td>56.2</td>
<td>29.8</td>
</tr>
<tr>
<td>B3-a</td>
<td>3</td>
<td>16</td>
<td>600</td>
<td>25</td>
<td>25</td>
<td>92</td>
<td>13.7</td>
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<td>81.5</td>
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</tr>
<tr>
<td>B3-b</td>
<td>3</td>
<td>16</td>
<td>600</td>
<td>25</td>
<td>25</td>
<td>92</td>
<td>13.7</td>
<td>20.8</td>
<td>129</td>
<td>81.5</td>
<td>25.5</td>
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<table>
<thead>
<tr>
<th>Slab</th>
<th>No. of bars</th>
<th>(d_b) mm</th>
<th>(A_{st}) mm(^2)</th>
<th>(c_b) mm</th>
<th>(c_r) mm</th>
<th>(s_b) mm</th>
<th>(M_{cr}) kNm</th>
<th>(M_{sus}) kNm</th>
<th>(\sigma_{st1}) MPa</th>
<th>(M_u) kNm</th>
<th>(M_{sus}/M_u) (%)</th>
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<tr>
<td>S1-a</td>
<td>2</td>
<td>12</td>
<td>226</td>
<td>25</td>
<td>40</td>
<td>308</td>
<td>4.65</td>
<td>6.81</td>
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<td>13.9</td>
<td>49.0</td>
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<tr>
<td>S1-b</td>
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<td>226</td>
<td>25</td>
<td>40</td>
<td>308</td>
<td>4.65</td>
<td>5.28</td>
<td>195</td>
<td>13.9</td>
<td>38.0</td>
</tr>
<tr>
<td>S2-a</td>
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<td>25</td>
<td>40</td>
<td>154</td>
<td>4.75</td>
<td>9.87</td>
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<td>12</td>
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<td>25</td>
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<td>154</td>
<td>4.75</td>
<td>6.81</td>
<td>171</td>
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</tr>
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<td>S3-a</td>
<td>4</td>
<td>12</td>
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<td>25</td>
<td>40</td>
<td>103</td>
<td>4.86</td>
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<tr>
<td>S3-b</td>
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<td>4.86</td>
<td>8.34</td>
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<td>26.4</td>
<td>31.6</td>
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In addition to the time-varying deflection that was measured throughout each test using LVDTs, the time-dependent development of cracking, including the crack spacing and crack width was measured in each specimen throughout the test. The measured elastic modulus and compressive strength of the concrete at the age of first loading were \(E_c = 22820\) MPa and \(f_c = 18.3\) MPa, whilst the creep coefficient and shrinkage strain associated with the 400 day period of sustained loading were \(\varphi(t, r) = 1.71\) and \(\varepsilon_{sh} = -825\mu e\). The loads on all specimens were sufficient to cause primary cracks to develop in the region of maximum moment at first loading. In Table 2, the sustained in-service moment at mid-span, \(M_{sus}\), is given, together with the stress in the tensile steel at mid-span, \(\sigma_{st1}\), due to \(M_{sus}\) (calculated on the basis of a fully cracked section); the calculated ultimate flexural strength, \(M_u\) (assuming a characteristic yield stress of the
reinforcing steel of 500 MPa); the ratio $M_{sus}/M_u$; and the cracking moment, $M_{cr}$, (calculated assuming a tensile strength of concrete of $0.6 \sqrt{f_c(t)}$, where $f_c(t)$ is the measured compressive strength at the time of loading in MPa).

![Cross-sections of test specimens.](image)

Figure 7: Cross-sections of test specimens.

### 5.2 Sample deflection calculations – Beam B2-a

Typical calculations for the maximum final deflection at mid-span are provided here for Beam B2-a. The sustained moment at mid-span is $M_s = 24.8$ kNm.

**Instantaneous Deflection:**

The second moments of area of the uncracked transformed section and the fully-cracked transformed section are $I_{uncr} = 823 \times 10^6$ mm$^4$/m and $I_{cr} = 212 \times 10^6$ mm$^4$/m, respectively. The cracking moment is $M_{cr} = Z f_{ct} f = 5.09 \times 10^6 \times 2.57 \times 10^{-6} = 13.1$ kNm/m. For the calculation of immediate deflection due to loads applied at any time after shrinkage induced cracking has occurred, $\beta = 0.5$ and from Equation 1, $I_{ef} = 237 \times 10^6$ mm$^4$/m. The instantaneous curvature at mid-span is $(\kappa_0)_M = M_s/E I_{ef} = 4.59 \times 10^6$ mm$^4$/m and the instantaneous deflection at mid-span due to the full service load is obtained from Equation 6:

$$\left(\nu_{H}\right)_{\text{max}} = \frac{3500^2}{96} (0 + 10 \times 4.59 \times 10^{-6} + 0) = 5.86 \text{ mm}.$$

**Time-Dependent Deflection:**

For long-term calculations, $M_{cr,t} = 0.7M_{cr} = 9.2$ kNm and, from Equation 5, the distribution coefficient is $\zeta = 1 - (9.2/24.8)^2 = 0.864$

**Due to Creep:**

In this laboratory test, the entire service load is sustained and therefore $M_{sus} = 24.8$ kNm/m. From Equation 3, the creep modification factors $\alpha$ for the cracked cross-section is $\alpha = 6.34$ and for the uncracked cross-section is $\alpha = 1.21$. The final creep-induced curvatures at mid-span for a cracked and an uncracked section are $(\kappa_{cr})_{cr} = (\kappa_0)_{M} \varphi (t, r)/6.34 = 1.24 \times 10^{-6}$ mm$^{-1}$ and $(\kappa_{cr})_{uncr} = (\kappa_0)_{M} \varphi (t, r)/1.21 = 6.27 \times 10^{-6}$ mm$^{-1}$. From Equation 4, the creep induced curvature at mid-span is $(\kappa_{cr})_M = 0.864 \times 1.24 \times 10^{-6} + (1 - 0.864) \times 6.27 \times 10^{-6} = 1.95 \times 10^{-6}$ mm$^{-1}$ and the final creep-induced deflection is obtained from Equation 6:

$$\left(\nu_{H}\right)_{cr} = \frac{3500^2}{96} (0 + 10 \times 1.95 \times 10^{-6} + 0) = 2.49 \text{ mm}.$$
Due to Shrinkage:

Equation 2 gives the shrinkage-induced curvature for a cracked and an uncracked section, \((\kappa_{sh})_{cr} = 3.07 \times 10^{-6} \text{ mm}^{-1}\) and \((\kappa_{sh})_{uncr} = 0.92 \times 10^{-6} \text{ mm}^{-1}\), and the shrinkage-induced curvature at mid-span is given by Equation 4: \((\kappa_{sh})_M = 0.864 \times 3.07 \times 10^{-6} + (1 - 0.864) \times 0.92 \times 10^{-6} = 2.785 \times 10^{-6} \text{ mm}^{-1}\). For the uncracked, section at each support, \((\kappa_{sh})_L = (\kappa_{sh})_L = 0.92 \times 10^{-6} \text{ mm}^{-1}\). The shrinkage induced deflection may be approximated using Equation 6:

\[
(v_M)_{sh} = \frac{350^2}{96} \left( (0.92 \times 10 \times 2.79 + 0.92) \times 10^{-6} \right) = 3.78 \text{ mm}.
\]

The Final Long-term Deflection:

The final long-term deflection at mid-span \((v_C)_{max}\) is therefore:

\[
(v_M)_{max} = (v_M)_{l,max} + (v_M)_{cr} + (v_M)_{sh} = 5.86 + 2.49 + 4.78 = 12.1 \text{ mm}
\]

This compares well with the measured final deflection of B2-a of 12.4 mm.

The calculated final deflection of each of the test specimens is compared to the measured value in Table 2. In general, the agreement between the measured and the calculated deflection is excellent.

<table>
<thead>
<tr>
<th>Specimen</th>
<th>Measured Final long-term deflection (mm)</th>
<th>Calculated</th>
<th>Measured / Calculated</th>
</tr>
</thead>
<tbody>
<tr>
<td>B1-a</td>
<td>12.1</td>
<td>12.0</td>
<td>1.01</td>
</tr>
<tr>
<td>B1-b</td>
<td>7.4</td>
<td>8.3</td>
<td>0.89</td>
</tr>
<tr>
<td>B2-a</td>
<td>12.4</td>
<td>12.1</td>
<td>1.02</td>
</tr>
<tr>
<td>B2-b</td>
<td>7.9</td>
<td>8.6</td>
<td>0.92</td>
</tr>
<tr>
<td>B3-a</td>
<td>13.3</td>
<td>13.0</td>
<td>1.02</td>
</tr>
<tr>
<td>B3-b</td>
<td>7.9</td>
<td>8.9</td>
<td>0.88</td>
</tr>
<tr>
<td>S1-a</td>
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<td>27.2</td>
<td>0.92</td>
</tr>
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<td>20.2</td>
<td>0.99</td>
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<td>S2-b</td>
<td>21.9</td>
<td>22.2</td>
<td>0.98</td>
</tr>
<tr>
<td>S3-a</td>
<td>32.5</td>
<td>30.2</td>
<td>1.08</td>
</tr>
<tr>
<td>S3-b</td>
<td>22.9</td>
<td>23.6</td>
<td>0.97</td>
</tr>
</tbody>
</table>

Mean 0.97
Coefficient of Variation 6.0%

6. CONCLUDING REMARKS

The in-service behaviour of reinforced concrete flexural members under sustained service loads has been discussed and procedures for calculating in-service deflection, both short-term and long-term, have been outlined. The approaches effectively and efficiently include the dominating effects of cracking, tension stiffening, creep and shrinkage and they are ideally suited for design. The methods have been illustrated by example and shown to be both mathematically tractable and reliable.

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References