International Workshop on Wireless Technology Innovations in Smart Grid (WTISG 2015)

The Effect of Three-Phase Voltage Imbalance at PCC on Solar Panel Output Power

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Abstract

Photovoltaic generation is one of the promising renewable sources and is deployed in more and more power systems. Like other distributed generation, solar panels are connected to distribution systems instead of transmission systems. Distribution systems are inherently unbalanced, so the power flow studies must consider three phases. The interface between solar panel and distribution system is a DC-DC converter and an inverter, which are normally designed provided that the system is balanced. This paper analyses the output power of solar panel under both balanced and unbalanced PCC voltages and found the three-phase powers are not equal and unity power factor control cannot be realized when PCC voltages are unbalanced while the controller works well when PCC voltages are balanced. The results of simulation confirm to the analysis. The work of this paper shows that simple P-Q solar panel models are not good for distribution system power flow studies, and present unity power factor controller does not work well for solar panel connected to distribution systems.

Keywords: Solar panel; inverter; distribution system power flow analysis; unbalanced voltage; output power.

This work was supported by National Natural Science Foundation of China (NSFC) under grant No. 61461055.

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1. INTRODUCTION

With increasing concerns about global warming, extreme climate, pollution from fossil fuel, and damage to environment and ecosystems, traditional generation methods are challenged. Therefore, renewables are encouraged worldwide. Different forms of incentives such as the Feed-In-Tariff (FIT) and microFIT program in Ontario (Canada)\(^1\) and similar programs in other countries are instituted by many power systems to encourage renewables. As a result, a plenty of renewables are being installed in the entire world. Photovoltaic generation, known as solar panel, is one of the most rapidly developing renewable sources after wind energy. Like other types of distributed generations, solar panels are normally connected to distribution systems instead of transmission systems\(^2\).

To determine the steady-state behavior of a distribution system, bus power balance equations should be solved. Unlike transmission systems, distribution systems are unbalanced due to the following reasons: a) the loading of a distribution system feeder is inherently unbalanced due to a large number of unequal single-phase loads, and, b) conductor spacing of three-phase line segments are asymmetrical\(^3\). Therefore distribution system power flow studies must consider the three phases. All network components have to be modeled accurately considering unbalanced three-phase system\(^4\). The simple P-Q model of solar panel, in which PQ values are balanced and are functions of solar irradiance and cell temperature\(^5-7\), were proposed in the past for power flow analysis. These models assume nominal and balanced PCC voltage. However, for a distribution system with which solar panels are connected, the effect of three-phase voltage imbalance at Point of Common Coupling (PCC) on output powers of solar panels cannot be ignored.

1.1. Interface of solar panel connected to power grid

Solar panels are always accompanied with single-phase or three-phase inverters. For the single-phase inverter, all the power generated by solar panel is injected through the inverter to one phase. However, when a solar panel is connected through a three-phase inverter, the situation is much more complicated. This paper studies on solar panels with three-phase inverters.

A common structure of solar panel three-phase interface is shown in Fig. 1, in which \(\text{Irra} \) and \(\text{Temp} \) are solar irradiance and temperature respectively. The purpose of DC-DC converter is to boost the port voltage of PV panel (normally low) to a high level for the inverter input. Normally maximum power point track (MPPT) techniques are used in the inverter or DC-DC converter to gain the maximum output power of solar panels in all kinds of situation. The MPPT technique is implemented in DC-DC converter by adjusting VPV against the current of PV panel\(^8,9\). This can be done by changing the duty cycle of switch device since \(V_C \) is maintained as a constant by inverter. In most cases, voltage source inverters (VSI) are used in solar panel implementation and pulse-width modulation (PWM) technique is adopted in inverter controlling.

![Fig. 1. A common structure of Solar Panel interface](image-url)
1.2. Controlling of the inverter of solar panel

The inverter control objectives include keeping $V_C$ constant and unity power factor control. The block diagram of the inverter control system is shown in Fig. 2, in which C is the capacitor between inverter and DC-DC converter, and $\varphi_{V_{PCC}}$ is the phase angle of PCC voltage. The proportional-integral controller is commonly used in PWM control. The variables with superscript “*” indicate the command of controller. The unity power factor control is achieved by forcing $I_{ac}$ having the same phase angle with $V_{pcc}$.

The d-q synchronous frame methods are usually used in current control loop. With this method, both of the measured three-phase PCC voltages and currents in static coordinates are transformed into d and q components in the synchronous rotating coordinates as the input of the controller. The model of inverter and its filter in the d-q frame can be written as:

$$\begin{align*}
    v_d &= L \frac{di_d}{dt} + R i_q - \omega L i_q + v_{d_{-PCC}} \\
    v_q &= L \frac{di_q}{dt} + R i_d + \omega L i_d + v_{q_{-PCC}}
\end{align*}$$

(1)

where $L$ and $R$ are the inductor and resistance of the filter between inverter and PCC. It should be noticed in (1) that $v_d$ is a function of $i_d$, as well as $i_q$ and $v_q$ is a function of $i_q$ and $i_d$, which means d axis and q axis are coupled. To achieve the independent control of $i_d$ and $i_q$, the d-q decoupling current control method shown in Fig. 3 is widely adopted. The Park’s transformation from abc to dq coordinate can be written as:

$$\begin{bmatrix}
    v_d \\
    v_q \\
    v_0
\end{bmatrix} = \frac{2}{3} \begin{bmatrix}
    \cos \omega t & \cos(\omega t - \frac{\pi}{3}) & \cos(\omega t + \frac{\pi}{3}) \\
    -\sin \omega t & -\sin(\omega t - \frac{\pi}{3}) & -\sin(\omega t + \frac{\pi}{3}) \\
    \frac{1}{2} & \frac{1}{2} & \frac{1}{2}
\end{bmatrix} \begin{bmatrix}
    v_a \\
    v_b \\
    v_c
\end{bmatrix}$$

(2)

If the voltages at PCC are balanced, they can be written as:

$$\begin{align*}
    v_a &= V \cos \omega t \\
    v_b &= V \cos(\omega t - \frac{2}{3}\pi) \\
    v_c &= V \cos(\omega t + \frac{2}{3}\pi)
\end{align*}$$

(3)

supposed that the initial phase angle of phase-a equal to 0. Putting (3) into (2) one can get:
which means the space vector of PCC voltages is coincidence with d-axis. So the q-axis current command $I_q^*$ in Fig. 3 is set to zero to obtain the current which has the same phase angle with PCC voltage. In steady state, the output q-axis current $I_q$ is zero. So the output reactive power is zero and the unity power factor control is achieved. Furthermore, since $I_d$ remains constant and $I_q$ remains zero, $I_a$, $I_b$, and $I_c$ are balanced. The output powers of three phases are also balanced considering that the voltages on PCC are balanced.

2. The three-phase Outputs of Solar Panel under Unbalanced Voltage at PCC

If the PCC voltages are unbalanced, the output reactive power cannot be kept as zero, which can be proved as following. The unbalanced three-phase voltages can be decomposed into positive, negative and zero sequence components. For the positive sequence component, the results of Park’s transformation are similar as (4) and can be written as:

$$
\begin{bmatrix}
V_{d1} \\
V_{q1}
\end{bmatrix}
= \begin{bmatrix}
V_1 \\
0
\end{bmatrix}
$$

(5)

where $V_1$ is the magnitude of the positive sequence component of PCC voltage. 0-axis component has not been taken into consider because it does not show up in the control system shown in Fig. 3. Assuming the initial phase angle of phase-α of the PCC voltage negative component is $\alpha$, the negative component can be written as:

$$
\begin{align*}
V_{a2} &= V_2 \cos(\omega t + \alpha) \\
V_{b2} &= V_2 \cos(\omega t + \alpha + \frac{\pi}{3}) \\
V_{c2} &= V_2 \cos(\omega t + \alpha - \frac{\pi}{3})
\end{align*}
$$

(6)

where $V_2$ is the magnitude of the negative sequence component of PCC voltage. Using (2) and (6) one can get:

$$
\begin{bmatrix}
V_{d2} \\
V_{q2}
\end{bmatrix}
= \begin{bmatrix}
V_2 \cos(2\omega t + \alpha) \\
- V_2 \sin(2\omega t + \alpha)
\end{bmatrix}
$$

(7)
Adding (5) and (7) together, unbalanced PCC voltage in d-q frame can be get and written as:

\[
\begin{bmatrix}
V_{d\_PCC} \\
V_{q\_PCC}
\end{bmatrix} = \begin{bmatrix}
V_1 + V_2 \cos(2\omega t + \alpha) \\
-V_2 \sin(2\omega t + \alpha)
\end{bmatrix}
\]

(8)

The voltage reference \(E_d\) and \(E_q\) which are used in the controller, on the other hand, are also measured at PCC. The control system itself, however, is a type I system which is not capable to trace a nonlinear signal. So the mean values are taken as the voltage reference in the controller, which means:

\[
\begin{bmatrix}
e_d \\
e_q
\end{bmatrix} = \begin{bmatrix}
\bar{V}_{d\_PCC} \\
\bar{V}_{q\_PCC}
\end{bmatrix} = \begin{bmatrix}
V_1 \\
0
\end{bmatrix}
\]

(9)

With Laplace transformation and the functional relationship shown in Fig. 3, two function sets can be gained:

\[
\begin{cases}
V_d^* = E_d + (K_p + \frac{K_l}{s})(I_d^* - I_d) - \omega LI_q \\
V_q^* = E_q + (K_p + \frac{K_l}{s})(I_q^* - I_q) + \omega LI_d
\end{cases}
\]

(10)

\[
\begin{cases}
(R + sL)I_d = V_d^* - V_{d\_PCC} + \omega LI_q \\
(R + sL)I_q = V_q^* - V_{q\_PCC} - \omega LI_d
\end{cases}
\]

(11)

Combining (10) with (11) we can get:

\[
\begin{cases}
I_d = \frac{E_d + (K_p + \frac{K_l}{s})I_d^* - V_{d\_PCC}}{R + sL + K_p + \frac{K_l}{s}} \\
I_q = \frac{E_q + (K_p + \frac{K_l}{s})I_q^* - V_{q\_PCC}}{R + sL + K_p + \frac{K_l}{s}}
\end{cases}
\]

(12)

Note that

\[
\begin{bmatrix}
V_{d\_PCC} \\
V_{q\_PCC}
\end{bmatrix} = \begin{bmatrix}
\frac{V_1}{s} + V_2 e^{\frac{\alpha}{s}} & \frac{s}{s^2 + 4\omega^2} \\
-V_2 e^{\frac{\alpha}{2\omega s}} & \frac{2\omega}{s^2 + 4\omega^2}
\end{bmatrix}
\]

(13)

and

\[
\begin{bmatrix}
e_d \\
e_q
\end{bmatrix} = \begin{bmatrix}
\frac{V_1}{s} \\
0
\end{bmatrix}
\]

(14)
Bring (13) and (14) into (12)

\[
\begin{align*}
I_d &= \frac{(K_p + \frac{K_i}{s})I_d^\ast - V_2 e^{\frac{\alpha}{2} \omega s}}{s^2 + 4\omega^2} \\
& \quad \frac{R + sL + \frac{K_i}{s}}{s^2 + 4\omega^2} \\
I_q &= \frac{(K_p + \frac{K_i}{s})I_q^\ast + V_2 e^{\frac{\alpha}{2} \omega s}}{s^2 + 4\omega^2} \\
& \quad \frac{R + sL + \frac{K_i}{s}}{s^2 + 4\omega^2}
\end{align*}
\]

From (15) we can find that both $I_d$ and $I_q$ have a pair of poles on the imaginary axis ($\pm j2\omega$), which makes $I_d$ and $I_q$ not constants when $t$ tends to infinite. Even when $I_q^\ast$ equals to 0, $I_q$ does not equal to 0. This means unity power factor control does not guarantee zero reactive power output. Furthermore, $I_a$, $I_b$, and $I_c$ keep changing and unbalanced since $I_d$ and $I_q$ are varying. So output active power of three-phase is unbalanced when PCC voltage is unbalanced.

3. Simulations and Analysis

The simulation model was implemented in MATLAB/Simulink environment. Unity power factor control and d-q synchronous frame method were used in the simulation model. Details of the controller are provided in Liao’s work. An example of the extent of imbalance in phase power with voltage magnitude and phase angle imbalance is shown in Table I considering a 1 MW solar panel connected to a 600 V distribution system. The solar irradiance and temperature are fixed at 1000 w/m$^2$ and 0°C during the two simulations. The solar irradiance and temperature are fixed during the two simulations.

3.1. Results

The results of simulations are shown in Table I. The data from MATLAB/Simulink simulation is transferred into per unit where power base and voltage base are set to 1/3 MVA and 600 V respectively.

<table>
<thead>
<tr>
<th>Case 1: Balanced PCC Voltage</th>
<th>Case 2: Unbalanced PCC Voltage</th>
</tr>
</thead>
<tbody>
<tr>
<td>Phase voltage at PCC</td>
<td>Phase voltage at PCC</td>
</tr>
<tr>
<td>Phase active powers</td>
<td>Phase real powers</td>
</tr>
<tr>
<td>Total active power</td>
<td>Total real power</td>
</tr>
<tr>
<td>Phase reactive powers</td>
<td>Phase reactive powers</td>
</tr>
</tbody>
</table>

Table I. Power output variations for different PCC voltages (results from MATLAB Simulink)

3.2. Analysis

From Table I we can see that the reactive powers of three phases in Case I approximate zero while the ones in Case II are deviated from zero, which means unity power factor control can gain unity power factor output when the voltages at PCC are balanced but does not work when the voltages at PCC are unbalanced.

The total real power flowing out of the panel is same in the two cases and the difference between the total powers
reaching PCC arises due to different losses. If the three-phase voltage at PCC is balanced (Case I), the per phase power outputs are equal and equal to the 1/3 of maximum power that solar panel could produces from the sunlight. However, if the PCC voltage is unbalanced, the phase power outputs are unequal. Reviewing the balanced case in Table I, the difference between three-phase powers in Case 2 is 6.7% while the phase powers are almost equal to 1/3 of solar panel power in the balanced Case 1.

4. Conclusion

MPPT techniques used in the inverter or DC-DC converter gain the maximum output power of solar panels in all kinds of situation. Barring some losses, this power shall appear at the PCC as the sum of powers flowing from three phases. If the three-phase voltage at PCC is balanced, the per phase power outputs are balanced and equal to the 1/3 of maximum power that solar panel could produces from the sunlight, and the reactive power of each phase equals to zero profiting from unity power factor control. However, if the PCC voltages are unbalanced, the output powers of three phases are not equal. Moreover, the reactive power of each phase does not equal to zero, which means that unity power factor control only works under balanced voltage. This is reasonable because the inverter control strategy is designed only for the balanced systems.

For solar panels connected with distribution systems, the PCC voltage is unbalanced in most cases, which means normally the per phase power outputs are not equal and reactive power are not zero. So using simple P-Q solar panel model can cause remarkable error in power flow studies.

The future work will focus on two directions. The first is to build a simple model of solar panel which is accurate under balanced PCC voltages as well as unbalanced PCC voltages and can be easily used in power flow analysis. The second is to develop control method which can provide balanced unity power factor output under both balanced and unbalanced PCC voltages for solar panel inverters.

References

1. http://fit.powerauthority.on.ca/