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## A finite difference model of a PV-PCM system

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### Abstract

The performances of a photovoltaic panel (PV) are defined according to the “peak power” that identifies the maximum electric power supplied by the panel when it receives an insolation of 1 kW/m<sup>2</sup> and the temperature of the cell is maintained at 25°C. These conditions are only nominal since the solar radiation has a variable intensity and also the panel is subjected to thermal excursions; due to these reasons the real power efficiency of the panel is considerably lower than that obtainable in the nominal conditions.

The study focused on assessing a method to reduce the peak temperatures of PV systems using Phase Change Materials (PCM). To this aim it was created a numerical model capable to describe the behaviour of a PV-PCM systems. The results were validated and compared with real data obtained from a test facility.

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*Keywords:* Phase change material; FDM; heat storage, PV panel

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### 1. Introduction

Among Renewable Energy Sources, wind and solar energy are among the most important and available natural resources [1]. During the last decade there was a widespread use of photovoltaic (PV) systems not only for decentralized production in advanced countries but also in developing countries, where the most likely alternative, once saturated hydroelectric resources, is the use of poor fuels (coal and lignite, peat, etc. which are very polluting) and biomass. Furthermore, the use of PV technology is also very attractive when applied to buildings that try to be energy-independent [2].

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A key element of a wider dissemination of PV systems is represented by high power conversion efficiency. Concerning this point, the temperature of the PV cell is one of the main important parameters that affects the energy performance of the PV panel.

### Nomenclature

A	area [m <sup>2</sup> ]
c <sub>p</sub>	specific heat [J/kgK]
f	liquid fraction
h <sub>c</sub>	convective heat transfer coefficient [W/m <sup>2</sup> K]
h <sub>e</sub>	external convective heat transfer coefficient [W/m <sup>2</sup> K]
I <sub>T</sub>	solar irradiance [W/m <sup>2</sup> ]
L	latent fusion heat [kJ/kg]
n	node
p	step
q	heat flux [W/m <sup>2</sup> ]
T	Temperature [°C]
t	time [s]
α	thermal diffusivity [m <sup>2</sup> /s]
λ	thermal conductivity [W/mK]
ρ	density of control volume [kg/m <sup>3</sup> ]
Δx	thickness of domain [m]

The performance of a PV panel in fact are defined by manufactures according to the "peak power", which identifies the maximum electric power supplied by the PV panel when it receives a insolation of 1 kW/m<sup>2</sup> and the cell temperature is maintained at 25 °C. In real conditions the performance of a PV panel is different from that declared under the nominal conditions and the conversion efficiency decreases when the temperature of the cell decreases.

Phase Change Materials (PCMs) are employed to reduce operative temperature of the PV system for increasing the conversion efficiency of the PV panel. Indeed the idea to couple the PCMs with the photovoltaic technology arises from features of these materials to absorb large amounts of heat (keeping almost constant the temperature) when the heat is not required. Indeed overheating causes a drop in efficiency of the photovoltaic cells. The absorbed heat should be then released to the surrounding air during the night when the panel does not produce electrical power. The application of PCM in PV systems has been experimentally and numerically studied by several authors [3-7].

## 2. The heat storage and transfer problem in a PCM layer

The PCMs are materials able to store large amounts of heat because:

- have a high latent heat of liquefaction;
- are solid at ambient temperature.

Thanks to these features PCM absorbs thermal energy by undergoing a phase transition and will release the absorbed energy later, when the peak temperature will be over.

There are different substances, with high latent heat, which allow the storage of big amounts of heat during a solid-liquid phase transition and the consequent release of the same heat amount during the inverse process. For pure substances this phase change takes place at constant temperature and for certain materials the process of melting and solidifying can be repeated over a very high number of cycles with no change in their physical or chemical properties. Different kinds of paraffin have been tested in our experimental application.

The specific heat of a PCM varies as a function of its temperature, reaching its maximum value during the phase transition. In general the changing phase is not completely isothermal, although the temperature remains within a limited range of variation.

Analytical method may be used, in particular cases, to obtain exact mathematical solution for steady conduction problems. These solutions have been generated for an assortment of simple geometries and boundary conditions and they are well documented in the literature [8],[9],[10]. However, more often than not, thermal problems involve geometries and/or boundary conditions that preclude such solutions. The authors decided to use the finite differences method to solve the problem in a not stationary mode and mono-dimensional geometry. This method represents one of the easiest ways to search for a numerical solution of the heat transfer problem, because it is possible to reduce a partial differential equations system into an algebraic equations system. The method can be easily coded into software and the thermal analysis can be performed easily and quickly.

## 3. The finite difference method

The finite difference method is based on the approximation of spatial and temporal derivative of heat diffusion equation with relations between space and time finite differences.

In contrast to an analytical solution a numerical solution enables determination of the temperature at only discrete point. The original domain can be subdivided in a finite number of parts, assigning to each a reference point at its center (node).

Each  $n$  node represents a certain region, and its temperature is a measure of the average temperature of the region. The value of this derivative at the  $n$  nodal point may be approximated as:

$$\left. \frac{\partial^2 T}{\partial x^2} \right|_n \approx \frac{\left. \frac{\partial T}{\partial x} \right|_{n+1/2} - \left. \frac{\partial T}{\partial x} \right|_{n-1/2}}{\Delta x} \quad (1)$$

The temperature gradients may in turn be expressed as a function of the nodal temperatures:

$$\left. \frac{\partial T}{\partial x} \right|_{n+1/2} \approx \frac{T_{n+1} - T_n}{\Delta x} \quad (2)$$

$$\left. \frac{\partial T}{\partial x} \right|_{n-1/2} \approx \frac{T_n - T_{n-1}}{\Delta x} \quad (3)$$

substituting Equations (5) and (6) into (4):

$$\left. \frac{\partial^2 T}{\partial x^2} \right|_n \approx \frac{T_{n+1} + T_{n-1} - 2T_n}{(\Delta x)^2} \quad (4)$$

Concerning the time discretization, dividing the observation time  $t$  of the phenomenon in a finite number of time steps  $p$  with amplitude  $\Delta t$ :  $t = p \cdot \Delta t$ ; therefore, the time derivative present in the second member of heat diffusion equation can be replaced with the finite difference approximation by using the “central difference” operator:

$$\left. \frac{\partial T}{\partial t} \right|_n \approx \frac{T_n^{p+1} - T_n^p}{\Delta t} \quad (5)$$

where  $p + 1$  is the current time step and  $p$  is the previous time step. Finally, in on dimensional system the heat diffusion equation at the finite difference method becomes:

$$\frac{\Delta^2 T}{\Delta x^2} = \frac{1}{\alpha} \frac{\Delta T}{\Delta t} \quad (6)$$

where  $\alpha$  is the thermal diffusivity; and replacing Eqs. (4) and (5) into (6), is obtained:

$$\frac{T_{n+1} + T_{n-1} - 2T_n}{(\Delta x)^2} = \frac{1}{\alpha} \frac{T_n^{p+1} - T_n^p}{\Delta t} \quad (7)$$

In other words, explicit finite difference method can calculate the state of a system at a later time from the state of the system at the current time.

#### 4. PV-PCM model

Considering a PV panel coupled with PCM system, the energy balance must take into account the presence of the phase-change material. Schematically, the energy exchanges in a PV-PCM system can be exemplified by the Fig. 1. Thanks to the presence of a simple geometry it was possible to choose a one-dimensional approach, considering only a heat flow orthogonal to the PV plane.

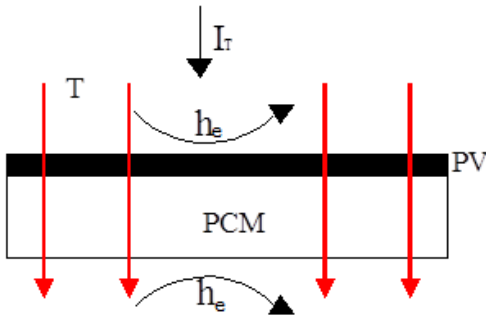


Fig. 1. PV-PCM schema of the PV-PCM system

where  $T$  is the external temperature,  $I_T$  is the solar irradiance, PV is the photovoltaic panel, PCM is the phase change layer and  $h_e$  is the external convective coefficient.

In detail, our test system is composed by a:

- tempered glass sheet with a thickness of 3.2 mm (Glass layer);
- 1 mm of PET plastic panel on which are “printed” the silicon cells; the silicon cells are considered having negligible thickness (PET layer);
- an optional layer of air interposed between the panel and the heat storage system representing a possible imperfect contact (Air layer); a plastic layer that takes into account the bag that contains the PCM (bag layer);
- the PCM layer.

The PV-PCM system is presented as a multi-layer plate invested by solar radiation and exchanging heat with the external environment by convection and radiation. Depending on the properties of the PCM and on the amount of energy captured from the panel, the layer of PCM can partially or totally melt during the maximum insolation, and returns that amount of energy, possibly solidifying again, during the night. The first hypothesis is that the phase change is isothermal. In case of non-isothermal transition, having the value of  $c_p$  with temperature, the problem is reduced to a heat conduction case. The occurrence of a phase change that blocks the temperature at a given value is a more interesting case and implies the determination of the PCM liquid fraction.

This hypothesis is not very far from reality because many PCMs are characterized by isothermal phase change, while some paraffin and eutectic mixtures have a very short range of temperature during transition

## 5. One-dimensional thermal analysis of an isothermal phase change by using explicit finite difference approach

If we suppose that the phase change is isothermal, it is possible to define the liquid fraction as:

$$f_i = \begin{cases} 1 & \text{if } T > T_m \\ f_i(t) & \text{if } T = T_m \\ 0 & \text{if } T < T_m \end{cases} \quad (8)$$

where  $T_m$  is the temperature of the phase changing.

### 5.1. Balance equation at superficial nodes

In case of border nodes, it is possible to write the energy balance of the domain that pertains these nodes. Assuming that on the external surface there is a generic heat flux  $q$  and a convective process, the discretized explicit form of the thermal balance will be:

$$\rho c_p A \frac{\Delta x}{2} (T_0^{p+1} - T_0^p) = h_c A (T_\infty^p - T_0^p) + \frac{\lambda A}{\Delta x} (T_1^p - T_0^p) + q^p A \quad (9)$$

where  $\rho$  is the density of the control volume,  $c_p$  is the specific heat at constant pressure,  $A$  is the area,  $\Delta x$  is the thickness of the domain,  $\lambda$  is the thermal conductivity,  $T_0^{p+1}$  is the temperature of the superficial node at the present time,  $T_0^p$  is the temperature of the superficial node at the past time,  $h_c$  is the convective heat transfer coefficient,  $T_\infty^p$  is the temperature of the air at the past time,  $T_1^p$  is the temperature of the first internal node following the superficial one at the past time and  $q^p$  is the external heat flux at the past time.

When there is not a phase change, the time variation of the liquid fraction is null and the equation useful to calculate the superficial temperature is:

$$T_0^{p+1} = T_0^p + 2 \frac{h_c \Delta t}{\rho c_p \Delta x} (T_\infty^p - T_0^p) + 2 \frac{\lambda \Delta t}{\rho c_p \Delta x^2} (T_1^p - T_0^p) + 2 \frac{q^p \Delta t}{\rho c_p \Delta x} \quad (10)$$

In case the phase change has just started, it is possible to make some observations. First of all, if the phase change is isothermal, during this process the temperature is locked to the value  $T_m$  as long as the liquid fraction is 1. Furthermore, an additional term has to be evaluated to consider the sensible heat that the control volume has adsorbed to overcome the previous temperature and reach the phase change temperature in one time step

When the phase changing is starting the superficial temperature is  $T_m$  and the liquid fraction is:

$$T_0 = T_m$$

$$f_0^{p+1} = f_0^p + 2 \frac{h_c \Delta t}{\rho L \Delta x} (T_\infty^p - T_0^p) + 2 \frac{\lambda \Delta t}{\rho L \Delta x^2} (T_1^p - T_0^p) + 2 \frac{q^p \Delta t}{\rho L \Delta x} - \frac{c_p}{L} (T_m - T_0^p) \quad (11)$$

with  $L$  as the latent fusion heat.

If the phase changing is occurring the superficial temperature is  $T_m$  and the liquid fraction is

$$T_0 = T_m$$

$$f_0^{p+1} = f_0^p + 2 \frac{h_c \Delta t}{\rho L \Delta x} (T_\infty^p - T_0^p) + 2 \frac{\lambda \Delta t}{\rho L \Delta x^2} (T_1^p - T_0^p) + 2 \frac{q^p \Delta t}{\rho L \Delta x} \quad (12)$$

when the phase changing is just ending the superficial temperature is again free to float but we must compute an additional term that computes the necessary latent heat to end the phase changing. The time variation of liquid fraction became null and the superficial temperature can be calculated with:

$$T_0^{p+1} = T_0^p + 2 \frac{h_c \Delta t}{\rho c_p \Delta x} (T_\infty^p - T_0^p) + 2 \frac{\lambda \Delta t}{\rho c_p \Delta x^2} (T_1^p - T_0^p) + 2 \frac{q^p \Delta t}{\rho c_p \Delta x} - \frac{L(1-f_0^p)}{c_p} \quad (13)$$

when the substance has already ended the transition and is fully liquid the superficial temperature is:

$$T_0^{p+1} = T_0^p + 2 \frac{h_c \Delta t}{\rho c_p \Delta x} (T_\infty^p - T_0^p) + 2 \frac{\lambda \Delta t}{\rho c_p \Delta x^2} (T_1^p - T_0^p) + 2 \frac{q^p \Delta t}{\rho c_p \Delta x} \quad (14)$$

## 5.2. Internal nodes

Even in case of internal node it is possible to write the energy balance of the associated control volume. In the hypothesis of only conductive heat flux coming from the previous and next nodes, the thermal balance in explicit form will be:

$$\rho c_p A \Delta x (T_i^{p+1} - T_i^p) = \frac{\lambda A}{\Delta x} (T_{i-1}^p - T_i^p) + \frac{\lambda A}{\Delta x} (T_{i+1}^p - T_i^p) \quad (15)$$

When there is not phase change, the time variation of the liquid fraction is null and the temperature of the internal node is:

$$T_i^{p+1} = T_i^p + \frac{\lambda \Delta t}{\rho c_p \Delta x^2} (T_{i-1}^p - 2T_i^p + T_{i+1}^p) \quad (16)$$

with similar considerations it is possible to state that, when the phase change is just started the temperature of the internal node will be  $T_m$  and the liquid fraction:

$$T_i = T_m$$

$$f_i^{p+1} = f_i^p + \frac{\lambda \Delta t}{\rho L \Delta x^2} (T_{i-1}^p - 2T_i^p + T_{i+1}^p) - \frac{c_p}{L} (T_m - T_i^p) \quad (17)$$

when the phase change is occurring the temperature of the internal node is  $T_m$  and the liquid fraction:

$$T_i = T_m$$

$$f_i^{p+1} = f_i^p + 2 \frac{\lambda \Delta t}{\rho L \Delta x^2} (T_{i-1}^p - 2T_i^p + T_{i+1}^p) \quad (18)$$

when the phase change is just ended the time variation of the liquid fraction is null and the temperature of the internal node is:

$$T_i^{p+1} = T_i^p + \frac{\lambda \Delta t}{\rho c_p \Delta x^2} (T_{i-1}^p - 2T_i^p + T_{i+1}^p) - \frac{L(1-f_i^p)}{c_p} \quad (19)$$

when the phase change is already ended and the substance is fully liquid the temperature of the internal node will be:

$$T_i^{p+1} = T_i^p + \frac{\lambda \Delta t}{\rho c_p \Delta x^2} (T_{i-1}^p - 2T_i^p + T_{i+1}^p) \quad (20)$$

This set of equations permits to solve the problem of the one-dimensional thermal analysis of an isothermal phase change by using explicit finite difference approach.

## 6. Experimental system

To perform the measurements, an experimental system was made up and situated on the top of the Energy Department of University of Palermo, (38°07' N, 13°22' E). It consists of a silicon PV panel, a precision resistance set and a Delta Ohm pyrometer mod. LP PYRA 02 AV, which is a first class according to ISO 9060, linked to an Advantech ADAM 6024 module. A Davis Vantage PRO2 Plus Weather station was used to collect the measurements of air temperature and relative humidity, wind speed and direction, horizontal global solar irradiance and atmospheric pressure.

Two identical PV panels by Kyocera manufacturers' have been chosen (KC175GHT-2), one of which is coupled to PCM. To secure a proper PV-PCM system configuration it was decided to install PCM in the bottom part of PV panel using a perforated metal mesh bolted into the frame of the panel. The silicon temperature was measured using thermocouples (type T, copper-constantan) installed into little holes made in the PET (Polyethylene Terephthalate) rear film of the panel, in order to improve the thermal contact with the cell silicon back face.

To study the influence of PCM, as previously described, were monitored two identical PV panels, one of which was applied to the PCM. The field measures were launched in in the summer months as these are characterized by higher irradiance values.

The electrical load also has been changed frequently in order to observe the system response to load variations. Therefore, numerical simulations were performed on different days and the results were compared with measured data. In detail, the comparison was made between temperature and electrical power, measured and calculated.

In the following the thermo-physically characteristics of the PCMs used in this work; the graphs below show some of the results that were obtained comparing the trends of the measured and simulated temperatures.

Table 1. Thermo-physically characteristics of the PCM

Characteristic	PCM	
Transition phase	26-28	°C
Solid density	0.87	kg/l
Liquid density	0.75	kg/l
Heat Capacity	179	kJ/kg
Specific enthalpy of phase change	1.8-2.4	kJ/kgK
Conductibility	0.2	W/mK
Volumetric change	10	%

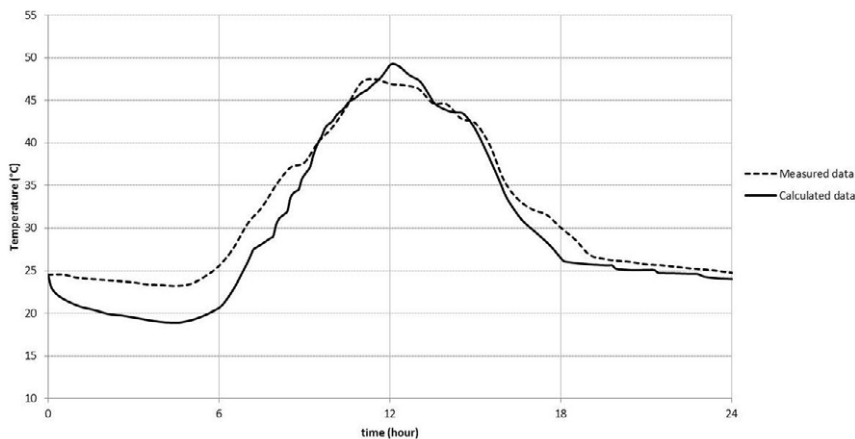


Fig.2. Temperature trend of the measured and simulated PV-PCM system, in Palermo, 2010 June 19

The figure 2 represents the temperature trends of measured and calculated data of PV-PCM system in a summer day in Palermo. It is possible to see that there is a good correspondence of the two curves during the day and a larger distance during the night probably due to an incorrect evaluation of the radiative exchange. Anyway for the all-day MAE (mean absolute error) is 1.67 °C; so if we consider only the sunny period of the corrected MAE is 0.61°C.



## 7. Comments and conclusions

Looking at the graphs represented before, it is possible to see as the calculated temperature trends is in good agreement with measured temperatures, validating the reliability of the calculation model.

The calculated temperatures at night are significantly lower than the comparable measured: this is an indication of a probably incorrect estimation of radiative heat exchange during the night, which were estimated under the assumption of the sky always clear.

The results show that the proposed model is valid and can be usefully used to determine the thermal behavior of a multilayer wall in which there is a phase change material. The good agreement between experimental measurements and numerical predictions have shown that the algorithm, although simplified and in one-dimensional geometry, can be used to determine the trend in temperature of a multilayer wall accompanied by a PCM.

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