Robust $\mathcal{H}_\infty$ control of Magnetic Levitation system based on parallel distributed compensator

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Abstract This paper concerns with designing of a robust $\mathcal{H}_\infty$ controller for Magnetic Levitation system exposed to external disturbances. First, the Maglev is modeled in a discrete form of Takagi–Sugeno fuzzy (TSF) system. Then the nonlinear fuzzy controller is synthesized based on such a technique called Parallel Distributed Compensation (PDC) schemes and sufficient conditions are developed to control and guarantee the robust stabilization of a complex nonlinear system. So the proposed algorithm is used to enhance the performance and to ensure the stability of the system. Furthermore, the design conditions and criteria for quadratic stability of the TSF system are formulated as a Linear Matrix Inequality (LMI) to robustly stabilize the position of the iron ball in a Maglev system in the existence of disturbances. The proposed technique will guarantee $\mathcal{H}_\infty$ performance to be less than one and attenuate the effect of exogenous disturbances. Moreover, the comparison results for the control of Maglev will show the capability of the proposed approach.
analysis and design closed-loop controlled systems. Based on TSF model, a simple feedback control is synthesized by a technique called parallel distributed compensation (PDC) to control and stabilize complex nonlinear systems. The design concept of PDC is to develop a control system by using a feedback controller for each rule in TSF model [2–4]. Hence, the designed fuzzy controller reflects the structure of the related TSF model.

The aim of this paper is to propose a robust controller to highly nonlinear Magnetic Levitation system using TSF model control in the discrete form. By using the PDC concept, a $H_\infty$ controller will be applied to Maglev exposed to external disturbances. Thus, the proposed controller is TSF control system that the stability analysis is achieved and then LMI conditions are presented to calculate the gain of the controller. The novelties here is that a proposed controller will robustly stabilize the Maglev system for both current-controlled and voltage-controlled schemes exposed to external disturbances using straightforward algorithms. Finally, simulation results are given to prove that the proposed technique ensures the stability conditions and guarantees robustness against external disturbance for a complex nonlinear system. The paper is organized as follows. First, the related work is presented in Section 2. Then some preliminaries and problem formulation are given in Section 3. Section 4 shows the fuzzy controller design. In Section 5, the simulations are given to emphasize the efficiency of the proposed method and compare it with other PDC schemes for voltage and current controlled Maglev system. Finally, the paper is concluded in Section 6.

2. Related work

Recently, PDC scheme has been intensively used in solving many academic and industrial problems because the concept of PDC is simple and easy to develop. Such as, the TSF model of an inverted pendulum is presented and the PDC scheme is developed to stabilize it in [3]. In [5], switched PDC is synthesized for R/C Hovercraft. Baranyi et al. [8] propose a PDC controller for stabilization of a real 3-DOF RC helicopter to get a good speed response. A practical approach of a Takagi–Sugeno fuzzy (TSF) modeling and control for F16 aircraft is presented in [9]. The design and implementation of PDC controller for temperature control are introduced in [10]. In [11], the fuzzy controller is presented utilizing the PDC technique and it is synthesized in the experimental tank level system. PDC-fuzzy based controller is proposed for chaotic systems in [12].

The literature reveals that there have been significant problems in controllability, stability analysis and robustness of nonlinear systems. Therefore, a lot of efforts have been made in developing stabilization conditions and designing PDC controllers for guaranteeing not only the stability, but also the robustness performances of closed-loop nonlinear systems. Thus a significant research works have been exploited in the field of $H_\infty$ robust control system for fuzzy system via TSF model [3,6,7]. The use of Lyapunov function in terms of LMI technique is popular in designing stability conditions for the TSF model with parallel distributed compensation (PDC) [4,13]. By utilization of a convex program, the performance conditions and stability criteria can be checked and then the parameter of a fuzzy model control system can be obtained by solving the LMIs. Although LMI-based approaches have been successful, the obtained conditions lead to conservatism and the local stability problem. Thus, considerable works have been investigated to develop effective methods so as to reduce the conservatism that comes from the usage of Lyapunov function, for example, piecewise Lyapunov function [14–16], Fuzzy Lyapunov functions [17–20], switched fuzzy system [5–7,21] and nonquadratic membership-dependent Lyapunov function [22].

Nowadays Magnetic Levitation (Maglev) has various usages in many fields such as Maglev trains, wind turbines,Launching Rockets, Electrodynamics’ Suspension and the centrifuge of nuclear reactor [23]. These systems are highly nonlinear and unstable open loop systems because of magnetic force. Therefore, designing a robust controller to stabilize the Magnetic Levitation is a challenging task. Thus, great efforts have been applied for constructing the high-performance feedback controllers to stabilize and control the Maglev systems. In the last decades, various studies try to manipulate the Maglev systems such as [24] a multilevel fuzzy logic control strategy is proposed, and in [25] switched controller is applied; adaptive fuzzy neural Network has been utilized by [26]; real-time PID control with PSO gain selections is implemented in [27]; optimal fuzzy control design using neural fuzzy inference networks is proposed in [28]. Nevertheless, a lot of the existing techniques that manage the nonlinearity of the Maglev system usually need complicated algorithms. Therefore, TSF model control has been applied to deal with the nonlinearity of Maglev as it is simpler and more effective technique [13]. In [13], a fuzzy model control is employed using the non-PDC to regulate the Maglev system for voltage-controller scheme. But, this controller has some shortcomings. First the controlled model needs more time to converge to the equilibrium point. And there are a lot of LMI variables which take much time to calculate and evaluate the gain. In this paper, we will develop a robust $H_\infty$ fuzzy controller based on PDC to control and stabilize discrete time Maglev system. The proposed controller is a state feedback controller and LMI conditions are presented to calculate the gain of the controller. The resulting algorithms are much simpler with fewer variables involved which result in more efficient computationally. The LMI conditions are included by slack matrix variables to minimize the conservatism arising from using the Lyapunov function and to improve the performance as compared to corollary [6].

Therefore, the goal of the proposed PDC is to robustly control the position of the iron ball in Maglev in the presence of disturbances for both current-controlled and voltage-controlled schemes. The convex optimization algorithms were solved using a MATLAB toolbox YALMIP [29] with the solver SeDuMi [30]. MATLAB environment has some successful applications to simulate systems and validate the algorithms [5–9,26–32]. The main advantage of using YALMIP is much faster solver than LMI toolbox and it is so simple to use.

Notation: The superscript ‘‘T’’ represents the transpose of a matrix. The notation ‘‘*’’ in symmetric matrix is used as a transposed element in the symmetric position.

3. Preliminaries and problem formulation

The TSF model is represented by fuzzy IF-THEN rules and described a nonlinear system by linear time-invariant subsystems connected by nonlinear membership functions [1]. In this
paper, we studied discrete form of TSF system with disturbances [2]. So the l-th rule of the discrete fuzzy system models (DFS) is given as follows:

$$\text{IF } \delta_l(n) \text{ is } M_l^1 \text{ and } \ldots \text{ and } \delta_p(n) \text{ is } M_l^p \text{ then }$$

$$\begin{align*}
x(n+1) &= Ax(n) + B_u w(n) + B_d u(n) \\
y(n) &= C x(n) + D_u w(n) + D_d u(n)
\end{align*}$$

(1)

where \( l = 1, 2, \ldots, q \) where \( q \) is the number of inference rules; \( M_l^j \) are the fuzzy sets \( (j = 1, 2, \ldots, p) \) where \( p \) is the number of fuzzy sets in each rule; \( x(n) \in R^l \) is the state vector; \( u(n) \in R^m \) is the control signal; \( y(n) \in R^p \) is the output vector; \( w(n) \in R^t \) is the energy-bounded disturbance; \( A_l, B_{ul} , B_{dl}, C_l, D_{ul} , D_{dl} \) is the l-th local model of TSF (1) with appropriate dimensions and \( \delta_l(n), \ldots, \delta_p(n) \) are known premise variables.

The global model of a discrete-time TSF system is inferred by using product inference engine and a weighted average method defuzzifier, as follows:

$$x(n+1) = \sum_{l=1}^{q} \eta_l(n) (A_l x(n) + B_{ul} w(n) + B_{dl} u(n))$$

(2)

$$y(n) = \sum_{l=1}^{q} \eta_l(n) (C_l x(n) + D_{ul} w(n) + D_{dl} u(n))$$

(3)

where \( \delta_l(n) = [\delta_1(n), \delta_2(n), \ldots, \delta_p(n)]^T \) and

$$\eta_l(n) = \frac{\prod_{j=1}^{p} M_l^j(\delta_l(n))}{\sum_{l=1}^{q} \prod_{j=1}^{p} M_l^j(\delta_l(n))}$$

(4)

For all \( n \). The term \( M_l^j(\delta_l(n)) \) is the grade of membership of fuzzy sets \( (\delta_l(n)) \) at \( M_l^j \). For brevity expression, we will denote \( \eta_l(n) = \eta_l(\delta_l(n)) \). We have:

$$\begin{align*}
\sum_{l=1}^{q} \eta_l(n) &= 1 \\
\eta_l(n) &\geq 0 \quad \forall l
\end{align*}$$

(5)

For the above TSF model, Fuzzy controller can be constructed using the Parallel Distributed Compensation (PDC) concept. The PDC control technique is synthesized by designing a linear compensator to control each of fuzzy rules. So the fuzzy controller uses the same fuzzy sets of TSF model.

Control rule 1: IF \( \delta_l(n) \text{ is } M_l^1 \text{ and } \ldots \text{ and } \delta_p(n) \text{ is } M_l^p \text{ then } u(n) = -F_l x(n), \quad l = 1, 2, \ldots, q \)

where \( F_l (l = 1, 2, \ldots, q) \) is the gain of feedback controller. Hence, the PDC controller can be obtained by

$$u(n) = -\sum_{l=1}^{q} \eta_l(n) F_l x(n),$$

(6)

By substituting (6) in (3) and (4), we have

$$\begin{align*}
x(n+1) &= [A(q(n)) - B_u(q(n)) F_l(x(n))] x(n) + B_d(q(n)) w(n) \\
y(n) &= [C(q(n)) - D_u(q(n)) F_l(x(n))] x(n) + D_d(q(n)) w(n)
\end{align*}$$

(7)

where

\[
\begin{pmatrix}
A(q(n)) - \sum_{i=1}^{q} \eta_i(n) A_i & B_u(q(n)) - \sum_{i=1}^{q} \eta_i(n) B_{ul} & B_d(q(n)) - \sum_{i=1}^{q} \eta_i(n) B_{dl} \\
C(q(n)) - \sum_{i=1}^{q} \eta_i(n) C_i & D_u(q(n)) - \sum_{i=1}^{q} \eta_i(n) D_{ul} & D_d(q(n)) - \sum_{i=1}^{q} \eta_i(n) D_{dl}
\end{pmatrix}
\]


4. PDC controller design

In this part, state feedback controller is presented using PDC technique. The proposed controllers will be used to control and stabilize unstable system. The proposed PDC will exploit an LMI-based approach to calculate the controller’s gain to solve the problem of output feedback controller for discrete-time TSF systems.

Lemma [6]. Stability of closed loop system (7) is fulfilled with \( \mathcal{H}_\infty \) performance level \( \gamma > 0 \) if for \( 1 \leq l, j \leq q \), if \( \Pi_{ll} < 0 \) and \( Q = Q^T > 0 \), following two LMI’s conditions hold:

$$\Pi_{ll} < 0, \quad 1 \leq l \leq q,$$

(8)

$$\frac{1}{q - 1} \Pi_{ll} + \frac{1}{2} (\Pi_{lj} + \Pi_{jl}) < 0, \quad 1 \leq l \neq j \leq q$$

(9)

where

$$\Pi_{lj} = \begin{bmatrix} Q - E - E^T & * & * \\ * & -\gamma I & * \\ * & * & -\gamma I \end{bmatrix}, \quad 1 \leq l \neq j \leq q,$$

Remark 1. The main problem in this PDC algorithm is the conservatism. So to reduce it, the slack matrix is introduced to the LMI’s conditions. The proposed algorithm will improve the performance and give better result as it will be justified in the simulation results. So based on the lemma, LMI’s conditions for designing \( \mathcal{H}_\infty \) controller are given in the following theorem:

Theorem. For a given \( \mathcal{H}_\infty \) performance level \( \gamma > 0 \), if there exists a symmetric matrix \( Q = Q^T > 0 \), \( \Psi_{yj} = \Psi_{jy}^T, E > 0 \), \( Y_j \), \( 1 \leq l, j \leq q \), satisfying the following LMI’s:

$$\Psi_{yj} \geq 0,$$

(10)

$$\epsilon \Psi_{yj} \geq 0, \quad 1 \leq l, \ j \leq q, \quad \epsilon \in \{- 1, 1\}$$

(11)

$$\Gamma_{lj} < 0, \quad 1 \leq l \leq q,$$

(12)

$$\frac{1}{q - 1} \Gamma_{lj} + \frac{1}{2} (\Gamma_{lj} + \Gamma_{jl}) < 0, \quad 1 \leq l \neq j \leq q$$

(13)

where

$$\Gamma_{lj} = \begin{bmatrix} Q - E - E^T & * & * \\ * & -\gamma I & * \\ * & * & -\gamma I \end{bmatrix} + \Psi_{yj}, \quad 1 \leq l \neq j \leq q,$$

(14)

Then the controller (6) with

$$K_i = Y_i Q^{-1}$$

(15)

Provides the TSF system is asymptotically stable with guaranteed \( \mathcal{H}_\infty \) performance level \( \gamma \).
Proof. Since $\Pi_B$ should be negative definite from Lemma [6] and condition (9) should also be negative definite to guarantee the stability of the system, so with add positive slack matrices $\Psi_{\beta}, \Psi_{\kappa}$ to both conditions without change in the negative definiteness, therefore the theorem will be ensured the condition of stability will be remained with less conservative than Lemma [6]. Notice that if we choose $\Psi_{\beta} = \Psi_{\kappa} = 0$, $1 \leq l, j \leq q$ in the theorem, then the LMIs in theorem will be the same PDC scheme as in corollary in [6].

Remark 2. The theorem is presented in LMI terms is straightforward to obtain the gains of state feedback controller. So it can be simply solved using MATLAB toolbox YALMIP [29] with the efficient solver SeDuMi [30]. And the time needed to compute the solution is a little different in the existence of the slack Matrix.

5. Simulation results

In this section, the proposed scheme is employed in a Maglev system to ensure that the system is quadratically stable and more robust. The discrete-time TSF system of voltage-controlled and current-controlled Magnetic Levitation is presented. Next, fuzzy controller is introduced and the corresponding simulations are presented. Fig. 1 shows Magnetic Levitation (Maglev) system model. The proposed technique is used to control the position of iron ball for both current-controlled and voltage-controlled Maglev systems exposed to exogenous disturbances.

Where $m$ is the mass of the iron ball; $h$ is the gap between the ball and the magnet; $g$ is the acceleration gravity; $f$ is a force of electromagnetism; $i$ is current of the electromagnet; $v$ is the electromagnetic voltage on the coil.

5.1. Voltage-controlled Maglev system

After applying the Kirchhoff’s voltage law and the Newton’s second law, the nonlinear equations of motion of Maglev is given as [25]

$$\begin{bmatrix}
m \dddot{x}_2 + f &= Mg + f_d \\
\ddot{v} + \frac{2}{\alpha} \dddot{v} &= -iR + \frac{2}{\alpha} \frac{\dddot{x}_2}{\alpha} \\
f &= \beta \left( \frac{v}{t} \right)^2
\end{bmatrix}$$

(16)

where $\beta$ is electromagnetic constant ($\beta = 0.25 \mu_0 N^2 A$); $\mu_0$ is the permeability of air; $N$ is the number of coil’s turns; $A$ is cross-sectional area of magnetic; $f_d$ is the exogenous disturbance force.

In this scheme the voltage on the coil is considered to be the control input. The three state variables are assumed to be the position of ball $h$, the speed of the ball $\dot{h}$ and the current $i$ and the control variable is the voltage $v$. Since the gap is required to be kept fixed at $h_0 = 4$ mm, so the state variables will be

$$x_1(t) = h - 0.004, \quad x_2(t) = \dot{h}, \quad x_3(t) = i - 0.004\beta, \quad \dot{x}_1 = x_2, \quad \dot{x}_2 = -\frac{\beta}{M} \left( \frac{x_1}{x_1} \right)^2 + g + \frac{1}{M} f_d, \quad \dot{x}_3 = \frac{x_2 x_3}{x_1} - \frac{x_1 x_2}{2\beta} R + \frac{N_1}{2\beta} u$$

(17)

The discrete-time TSF system model of Magnetic Levitation with sampling time $T = 0.5$ ms:

$$\begin{align*}
x(n + 1) &= \sum_{i=1}^{4} \eta_i(n)[A_i x(n) + B_i w(n) + C_i u(n)] \\
y(n) &= \sum_{i=1}^{4} \eta_i(n) C_i x(n)
\end{align*}$$

The parameters of local system are

$$\begin{bmatrix}
A_1 &= \begin{bmatrix} 1 & 0.0005 & 0 \\
-0.3267 & 1 & 0.9996 \\
-0.3267 & 0.0005 & 0 \end{bmatrix}, \\
A_2 &= \begin{bmatrix} 0.0005 & 0 \\
-0.3267 & 0.9996 \\
-0.3267 & 0.0005 \end{bmatrix}, \\
A_3 &= \begin{bmatrix} 0.0005 & 0 \\
-0.3267 & 0.9996 \\
-0.3267 & 0.0005 \end{bmatrix}, \\
A_4 &= \begin{bmatrix} 0.0005 & 0 \\
-0.3267 & 0.9996 \\
-0.3267 & 0.0005 \end{bmatrix}, \\
B_{u1} &= B_{u2} = B_{u3} = B_{u4} = \begin{bmatrix} 0 \\
0.000000333 \\
0 \end{bmatrix}, \\
B_{u5} &= B_{u6} = \begin{bmatrix} 0 \\
0.0003886 \\
0.0002331 \end{bmatrix}, \\
B_{u7} &= B_{u8} = \begin{bmatrix} 0 \\
0.0003886 \\
0.0002331 \end{bmatrix}, \\
B_{u9} &= B_{u10} = \begin{bmatrix} 0 \\
0.0003886 \\
0.0002331 \end{bmatrix}, \\
B_{u11} &= B_{u12} = \begin{bmatrix} 0 \\
0.0003886 \\
0.0002331 \end{bmatrix}, \\
B_{u13} &= B_{u14} = \begin{bmatrix} 0 \\
0.0003886 \\
0.0002331 \end{bmatrix}, \\
C_1 &= C_2 = C_3 = C_4 = [1 \ 0 \ 0]
\end{bmatrix}$$

And where $w(n)$ is the discrete form of $f_d(t)$. The premise variables are $\delta_1 = x_1(n)$ and $\delta_2 = x_3(n)$. And so the membership functions are described as follows:

$$M_1^1(\delta_1(n)) = M_2^2(\delta_1(n)) = \frac{\delta_1(n) + 0.001}{0.002},$$

$$M_3^1(\delta_1(n)) = M_4^2(\delta_1(n)) = \frac{0.001 - \delta_1(n)}{0.002},$$

$$M_1^1(\delta_2(n)) = M_2^2(\delta_2(n)) = \frac{\delta_2(n) + 1}{2},$$

$$M_3^1(\delta_2(n)) = M_4^2(\delta_2(n)) = \frac{1 - \delta_2(n)}{2}$$

The grade of membership functions is

$$\eta_i(n) = \frac{\prod_{j=1}^{4} M_j^j(\delta_i(n))}{\sum_{i=1}^{4} \prod_{j=1}^{4} M_j^j(\delta_i(n))}$$

Consequently, by using a modeling language YALMIP with SeDuMi solver, we solve the LMI of theorem and the controller gain matrices for PDC (6) are obtained as follows:

$$\begin{align*}
F_1 &= 10^3 \times [-6.4070 - 0.0289 0.0060], \\
F_2 &= 10^3 \times [-1.4964 - 0.0733 - 0.0050], \\
F_3 &= 10^3 \times [-4.9099 - 0.2377 - 0.0217], \\
F_4 &= 10^3 \times [-3.3630 - 0.1871 - 0.0136]
\end{align*}$$

The state with zero initial condition and the exogenous disturbance $w(n) = 10^{-3} (0.2 + 5n) \sin(10n)$ are chosen. So the simulation results of the state response of the closed loop for
each of the proposed theorem and [13] are depicted in Fig. 2. The results of using the proposed theorem and non-PDC [13] with initial value of gap $x_1(0) = 0.0005$ m and $x_1(0) = 0.001$ m exposed to external disturbance are given in Figs. 3 and 4 respectively. The proposed algorithm gives less maximum overshoot than that in [13]. The simulation results address that the proposed theorem minimizes the effect of external disturbances and has outperformance over non-PDC. In [13], a fuzzy model control by non-PDC method to regulate the Maglev system for voltage-controller scheme and this controller has shortcomings. A considerable number of LMI variables need much time to solve the problem and evaluate the gain which results in a weak solution. And the response just takes a longer time to converge to its equilibrium. While the proposed algorithms are much simpler with much fewer variables involved so more efficient computationally. Also, it is obvious from Fig. 2 for an energy bounded disturbance, the proposed algorithm provides better performance in terms of less overshoot that’s because introducing the slack matrices (10) in the new algorithm gives less conservative results and produces higher controller’s signal (high voltage) without losing the stability behavior which makes the system converges to its equilibrium faster and leads to better performance. Even if the initial value is changed, the system still has a stable behavior with good performance and there is no overshoot in the gap and the variation of the gap is smoother as shown in Figs. 3 and 4, that is because the new algorithms are more relaxed conditions. Thereby it is concluded that the proposed PDC system is an effective and a proper technique. Moreover, when the controller’s gain for the proposed algorithm is higher, the voltage will be a little bit more which make the current in the case of proposed algorithm not exceed 1A. Therefore, the proposed algorithm is more practical and more suitable than that of [13] and can be synthesized to control the Maglev exposed to external disturbances.

5.2. Current-controlled Maglev system

In this case the current in the coil will be the control input. The Maglev’s dynamic equations [25] are presented as

$$m \dot{h} + f = M g + f_d$$

$$f = \beta \left( \frac{i}{h} \right)^2$$  \hspace{1cm} (18)

Define the state variable as follows:

$$x_1 = h \quad x_2 = \dot{h}$$

Then the state equation

![Figure 2 State response for zero initial condition.](image-url)
Consider a following discrete-time Magnetic Levitation TSF system model, which is based on Tustin’s method of numerical example in [28] with sampling time $T = 0.2$ s.

$$\begin{align*}
x_1(n+1) &= x_2(n) + \frac{1}{T} \left( \frac{x_1(n)}{x_1(n)} \right)^2 - u(n), \\
x_2(n+1) &= g - \frac{k}{M} x_2(n), \\
u(n) &= i(n)
\end{align*}$$

(19)

$$\begin{align*}
x(n+1) &= \sum_{i=1}^{2} \delta_i(n) [A_i x(n) + B_i u(n) + B_{wi} w(n)] \\
y(n) &= \sum_{i=1}^{2} \delta_i(n) [C_i x(n) + D_i u(n)]
\end{align*}$$

Where the grades of membership functions are as follows:

$$M_i(x(n)) = \exp \left[ - \frac{(x(n) - m_i)^2}{\sigma_i^2} \right], \quad i = 1, 2$$

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**Figure 3** State response for initial gap = 0.5 mm.

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**Figure 4** State response for initial gap = 1 mm.
Applying the theorem of the proposed PDC technique, we get the following:

$$F_1 = \begin{bmatrix} -103.921 \\ -1.8200 \end{bmatrix}, \quad F_2 = \begin{bmatrix} -323.0482 \\ -5.5537 \end{bmatrix} \quad \gamma = 0.2117$$

The state trajectories of the closed loop controlled system are depicted in Figs. 5 and 6 with initial value $x_0 = [10^{-3} 0]^T$. Fig. 7 shows the controller input $u(n)$ (the current of electromagnetic) and it indicates that the control signal is higher in the case of proposed algorithms that result in rapid changing of the position of the iron ball as referred in Fig. 6. The proposed controller stabilizes and compensates the overall system.
Based on the comparison between the proposed method and corollary in [6], it should be noted that the main problem of the algorithm in corollary [6] is the conservatism. To reduce it, the slack matrix is introduced to the LMIs conditions. Thus, this matrix will relax the conservatism and improve the performance without increasing the complexity of the LMI. Then, the efficiency of the controller will be increased and the robustness will be ensured using the proposed PDC. So the iron ball will converge to its equilibrium state quickly. Accordingly, it is noted that the response obtained from proposed PDC is better than PDC and the obtained $H_\infty$ performance equals 0.2117, while the value using PDC equals 0.5019 which indicates that the proposed algorithm gives less conservatism than that of lemma. As in Fig. 8, the controlled system response of the proposed algorithm decays quickly and approaches the origin if change in the initial position to 10 mm and exposure to disturbance equal to $w = e^{-n/2} \sin(10n)$ that's because of the high controller input signal and the stabilization condition in the theorem.

So based on the lemma, LMI conditions for designing $H_\infty$ controller are given in the following theorem: the proposed robust $H_\infty$ state feedback control method gives less overshoot and less settling time for the states of the controlled nonlinear system as compared to the method in [6] because slack matrix in proposed algorithm reduces the conservativeness of stability condition and then the performance of the system gets better. Furthermore, the Maglev is exposed to changing in amplitude of exogenous disturbance $w = \rho \sin(10n) \times \exp(-n/2)$ and ini-

![Figure 7](image1.png) State response of control input $u$.

![Figure 8](image2.png) The controlled position with initial value = 10 mm.
tial position of iron ball equals to 1 mm to notice the effect of disturbances on the controlled system and compare the results with other PDC scheme. Where $q$ is the gain of the disturbance. So the performance of the proposed will be addressed by changing the amplitude of the disturbance. With increasing the amplitude of the disturbance, the proposed PDC controller is compared with PDC [6]. Therefore, Figs. 9 and 10 show that the proposed PDC is more tolerable to disturbance than other PDC scheme. Note that despite high disturbance amplitude, the system is quadratically stable.

6. Conclusions

This paper synthesizes robust $H_\infty$ controller for a Maglev system using the TSF model control technique for both current-controlled and voltage-controlled schemes exposed to external disturbances. The TSF controller is applied based on the PDC scheme in a discrete form and the sufficient conditions and criteria by adding matrix variable have been derived in set of Linear Matrix Inequality (LMI) to ensure the robustness of the nonlinear system. The results show that the proposed
approach stabilizes the complex nonlinear system and guarantees $\delta_{\infty}$ performance of Maglev system for both schemes. The comparison results for the control of Magnetic Levitation system prove that the proposed PDC has a superior performance in all respects and outperforms the other techniques.

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References


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