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INFORMATION AND DECISION IN OPTIMAL INVENTORY PROCESSES

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Abstract—According to developments in management information systems, more investigation is required to adapt the fundamental features that American management information systems have to the Japanese technical climate. One important problem is to decide the kind and the accuracy of management information systems. If complete information is desired regarding a system in each stage of control, some time and cost will be entailed. Otherwise, if with incomplete information a decision is quickly made, we must put up with using a probability that controls a non-optimum system. We do not have the complete accuracy for both the information and decision. This is analogous to Heisenberg's uncertainty principle. In this paper, we discuss the relation between the information and decision in optimal inventory processes from this viewpoint.

1. INTRODUCTION

According to management information system development, more investigation is required before adapting the fundamental features of the American management information system to the Japanese technical climate. One important problem is to decide the types and accuracy of such a management information system. If complete information is desired regarding a system in each stage of control, some time and cost will be entailed. Otherwise, if incomplete information is used to make a decision quickly, we must put up with using a probability that will control a non-optimum system. We do not have complete accuracy for both the information that is available and decisions that are made. This is analogous to Heisenberg's uncertainty principle. This paper discusses the relation between information and decision in an optimal inventory process from this viewpoint.

Additionally, we introduce the general principle of balance. We thus possess two weapons, namely the principle of optimality in dynamic programming and the principle of balance in a management information system. In the third section, this principle of a balance will be applied to the development of the relation between information and decision in optimal inventory processes. Then, problems regarding quantity approximation, time approximation, demand approximation, the criterion approximation and system structure approximation are summarized. The fourth section discusses the stability of the optimal inventory equation and presents a design for an optimal inventory system.

Finally, we point out that one source of imprecision stems from both randomness and fuzziness, and conclude with a discussion of some areas for further research.

2. PRINCIPLE OF BALANCE

The stochastic properties of quantum mechanics are based on the uncertainty principle. A balance relation is pointed out wherein it is theoretically impossible to measure with the same accuracy at the same time a pair of quantities, called a conjugate quantity.

The phenomenon that is analogous to this principle in physics exists in many fields. Let us generally call this the principle of balance and discuss the relationship between this principle and several phenomena.

For example, the approximating linear prediction theory due to Wiener leads to the problem of minimizing the quadratic form

$$E = \sum_{k=0}^{N} \left(a_k - \sum_{l=0}^{M} A_l a_{k-l} \right)^2,$$

over the real quantities A_l, where the quantities a_k are given real numbers. E is the prediction error. The prediction error decreases and the structure complex increases when M is increased. It is an important practical question to decide how large to make M to balance the prediction error and the structure complex.

2.1. *Principle of optimality in dynamic programming*

The principle of optimality in dynamic programming indicates that the optimal policy should harmonize the balance between costs involved in deciding present and future values on a new state reduced by its decision, because dynamic programming involves multi-stage decision processes. Information for the future is necessary in order to make a decision in the present. The principle of optimality is an exact mathematical expression for this idea.

Let us assume R multi-stage decision processes. We shall be concerned with criteria possessing a structure which permits us to focus our attention solely upon the past and present history of the process in a search for values of policies. Then, to construct the optimal policy of the Rth stage, whatever the initial state and initial decision are, the remaining $(R-1)$th decisions must constitute an optimal policy with regard to the state resulting from the decision on the first stage. We must determine the first decision in order to determine the balance between gain in the first stage and gains in remaining $(R-1)$ stages.

2.2. *Principle of balance in information and decision*

If complete knowledge of the system is deemed necessary at any stage, then an appreciable time is usually required to accumulate this data. During this time, the system is uncontrolled. That is to say that time is one of the most valuable resources we have; it is unique in the fact that it cannot be reversed or replaced. It takes time to make decisions and then to implement those decisions. If, however, we make a decision quickly, using incomplete information about the system, there is a non-negligible probability that a non-optimal action will be taken. We cannot have complete accuracy in both information about the system, there is a non-negligible probability that a non-optimal action will be taken. We cannot have complete accuracy in both information and control. This is the uncertainty principle in a management information system.

3. INFORMATION AND DECISION IN OPTIMAL INVENTORY PROCESSES

This section discusses some applications of the principle of balance in regard of information and decision in multi-stage stochastic inventory control processes. Multi-stage stochastic inventory control processes will be introduced in Section 4. At first, if we observe the exact inventory quantities, then we have the right decision and the optimum expected cost, but we must accordingly allow for the cost of more observation. This sort of approximation relates to the quantity aspects. Secondly, instead of keeping records and placing orders at each period, it may be better to observe and order at intervals of a few periods, even when this delay necessitates paying a penality charge for getting items quickly. This type of approximation relates to the component of time. Also, there are some approximation problems in regard to determining demand information, optimum criterion and inventory system structure, etc.

3.1. *Approximation of observation [1, 2]*

A major problem in modern management is that of keeping records. However, sometimes, at a certain point, the cost of keeping records is greater than the gain that is obtained by using these records. These factors provide the motivation for a study of the approximation of observation of inventory quantity. It is necessary to decide on the degree of observation approximation that harmonizes with the observation cost and the gain obtained by using approximation information. We have obtained the following results, using both analytic and computational studies [1, 2].

(1) optimal choices between degree of observation M and degree of policy N depend on the unit costs for this inventory process;

(2) inventory processes are as sensitive to M as to N

and

(3) inventory processes are as sensitive to the backlogging problem as to the lost sales problem, etc.

We can determine the degree of approximation that balances the cost of observation and the total expected cost, if the approximate observation quantity is used.

3.2. Approximation in time [3, 4]

In introducing the basic optimal inventory equation, explicit use was made of the assumption that observations and orders are made at each period. However, this assumption may be questionable. Instead of keeping records in every period, it may be better to count the number of items when the supply is low, and even to pay a penalty charge for getting items quickly when the supply is very low. The problem that we want to study is that of determining the time to examine the number of items remaining in stock.

The results of analytic and computational studies are given in Ref. [4]. As we might have expected, the shortage and the total expected cost increase with increasing variability of the time interval in decision. Thus, we can determine the time of observation and control that balances the cost of observation and the expected cost, which are obtained by using an approximate time.

3.3. Approximation of demand information

The first step away from completely deterministic demands and a step of considerable import, is the classical theory of probability with its introduction of random variables. We want to indicate the existence of high levels of uncertainty. We can consider the following three cases.

3.3.1. Stochastic problem. The case when the stochastic feature is known.

3.3.2. Adaptive problems. The case where the demand distribution contains unknown stochastic parameters.

3.3.3. Game theoretic problem. The case when the stochastic feature is unknown.

In Ref. [5], we have compared the solutions for cases when the probability density functions of demand are assumed to be exactly known, adaptively known and game theoretically known. The total expected cost increases as the completeness of information decreases.

3.4. Approximation in criterion [6]

The problem of establishing the inventory system effectiveness criterion is a very fundamental one.

Let us discuss the following problems of a multi-stage nature, namely average cost per period and probability criterion.

3.4.1. Multi-stage problem. We often say that we are planning for the next year, and that we wish to minimize the multi-stage expected costs or maximize the multi-stage expected profits. It is clear that managers need not plan for next year only and that, in fact, they must consider many years in advance. In this case, the optimal policy in one period does not always mean the optimal policy is multi-stage periods. However, under some assumption, the former coincides with the latter.

3.4.2. Average cost per period. In the stationary approach, we select a particular (s, S) policy, calculate the long-run costs based on this policy, and then select the policy variables so as to minimize long-run cost. Let the minimum cost be denoted by k. In the dynamic programming approach, the technique depends on the minimum cost function $C_n(x)$. If the interest rate is zero, then, as period n becomes infinite, $C_n(x)$ will tend toward infinity. It seems plausible that there will be some connection between

$$\lim_{n\to\infty} [C_n(x)]/n \quad \text{and} \quad k.$$

3.4.3. Probability criterion [6]. Let us discuss the criterion which minimizes the probability that the inventory over all stages exceeds a fixed level. The profits in the probability criterion are as follow. At first, this is simple, because we do not require an estimation of the cost functions. Secondly, we have the same policy characterized by the principle of constant stock level as the criterion of cost functions.

3.5. System structure approximation [6]

Consider an inventory system that has many benefits. Under individual inventory control, each location puts in their orders separately and is concerned only with its own welfare. Under its centralized inventory control procedure, by contrast, quantity orders are made simultaneously for all locations in the network. There are immediate advantages and disadvantages to controlling such an inventory system centrally.

Since information about the entire supply network is recorded at a central location, decisions can be made effectively and expediently in emergencies, but the resulting decisions are more complex. An important question is the determination of how many benefits that are optimal in order to achieve centralized control.

4. INVENTORY SYSTEM DESIGN

4.1. Introduction

Most authors who have written on the subject of inventory control have made the assumptions either that we have obtained or that we shall have information used to make the necessary decision. In this section, to the contrary, let us determine the kind and accuracy of the information, on the assumption that we know how to decide when to have some information.

There are two types of costs with regard to information. One is the observation cost, which is entailed in obtaining information. The other is the error cost, owing to the approximation of information. Out aim is to minimize the sum of these costs. A model of our inventory control process is the multi-stage stochastic inventory problem. Let $L(y)$ be given by:

$$L(y) = \int_0^y h(y-\xi)\phi(\xi)\,\mathrm{d}\xi + \int_y^\infty p(\xi - y)\phi(\xi)\,\mathrm{d}\xi, \tag{1}$$

where $L(y)$ represents the sum of the expected inventory cost

$$\int_0^y h(y-\xi)\phi(\xi)\,\mathrm{d}\xi,$$

and the expected penalty cost

$$\int_0^\infty p(\xi - y)\phi(\xi)\,\mathrm{d}\xi$$

in each period, and $\phi(\xi)$ is the demand probability density in each period, given that, at the beginning of a period, the sum of the initial inventory on hand and the stock to be received in a period is y. We define the functional $f_n(x)$ as the total expected discounted cost over n periods, where x is the inventory on hand at the beginning of the first period. We have observed that $f_n(x)$ can be written for all x in the following functional equation.

$$f_n(x) = \min_{y \geqslant x}\left[c(y-x) + L(y) + \alpha\int_0^\infty f_{n-1}(x-\xi)\phi(\xi)\,\mathrm{d}\xi\right]. \tag{2}$$

In equation (2) $c(y)$ is the ordering cost and α denotes the discount factor ($0 < \alpha < 1$).

At first, we shall review the stability of the inventory process which is fundamental to design issues. In the following, we introduce the inventory processes design and the inventory policies control problems.

4.2. Optimal inventory equation stability [6]

The numerical error arises in the following ways: the quantities $c(y-z)$, $L(y)$, $y-z$ can introduce round-off errors in regard to their observation and optimal policy while the quantity

$$\int_0^\infty f_{n-1}(y-\xi)\phi(\xi)\,\mathrm{d}\xi$$

may require an interpolation in its calculation.

The search for the minimum over y may introduce an error, since we allow only a finite set of allowable y-values. It follows that the computational determination of the sequence $\{f_n(x)\}$ generates a new multistage process $\{\psi_n(x)\}$ governed by the relation,

$$\psi_n(x) = \min_y \left[c(y-x) + L(y) + \alpha \int^\infty \psi_{n-1}(y-\xi)\phi(\xi)\,\mathrm{d}\lambda \right] + u_n(x), \tag{3}$$

where $u_n(x)$ represents the accumulation of the different errors mentioned above.

We hope that, by taking some x and z, we can keep $|u_n(x)|$ sufficiently small, and that in turn, implies that $|f_n(x) - \psi_n(x)|$ is small. If so, our process in stable; if not, any results obtained numerically must be regarded with a jaundiced eye. Let us now demonstrate quite simply that we do have stability in the calculation of $\{f_n(x)\}$. Let y be a minimizing value in the original functional equation,

$$f_n(x) = \min_y \left[c(y-x) + L(y) + \alpha \int_0^\infty f_{n-1}(y-\xi)\phi(\xi)\,\mathrm{d}\xi \right] \tag{4}$$

and let w be a minimizing value in equation (3). Then

$$\begin{aligned} f_n(x) &= c(y-x) + L(y) + \int_0^\infty f_{n-1}(y-\xi)\phi(\xi)\,\mathrm{d}\xi \\ &\leqslant c(w-x) + L(w) + \int_0^\infty f_{n-1}(w-\xi)\phi(\xi)\,\mathrm{d}\xi \\ \psi_n(x) &= c(w-x) + L(w) + \int_0^\infty \psi_{n-1}(w-\xi)\phi(\xi)\,\mathrm{d}\xi + u_n(x) \\ &\leqslant c(y-x) + L(y) + \int_0^\infty \psi_{n-1}(y-\xi)\phi(\xi)\,\mathrm{d}\xi + u_n(x). \end{aligned} \tag{5}$$

Hence,

$$\begin{aligned} f_n(x) - \phi_n(x) &\leqslant \int_0^\infty [f_{n-1}(w-\xi) - \psi_{n-1}(w-\xi)]\phi(\xi)\,\mathrm{d}\xi - u_n(x) \\ &\geqslant \int_0^\infty [f_{n-1}(y-\xi) - \psi_{n-1}(y-\xi)]\phi(\xi)\,\mathrm{d}\xi - u_n(x) \end{aligned} \tag{6}$$

and thus,

$$|f_n(x) - \psi_n(x)| \leqslant \max\left[\int_0^\infty |f_{n-1}(w-\xi) - \psi_{n-1}(w-\xi)|\phi(\xi)\,\mathrm{d}\xi, \int_0^\infty |f_{n-1}(y-\xi) - \psi_{n-1}(y-\xi)|\phi(\xi)\,\mathrm{d}\xi\right] + |u_n(x)|. \tag{7}$$

Let us now assume that we have taken care that $|u_n(x)| < \epsilon$ for all $x > 0$. Then,

$$\max_x |f_n(x) - \psi_n(x)| \leqslant \max_x |f_{n-1}(x) - \psi_{n-1}(x)| + \epsilon \tag{8}$$

and thus,

$$\max_x |f_n(x) - \psi_n(x)| n\epsilon. \tag{9}$$

4.3. *Error cost, using approximation policies*

Let us consider the lost sales problem. We have the optimum expected cost per period.

$$K(\bar{x}) = -cx + (c-p)\bar{x} + (p+h)\int_0^{\bar{x}} \Phi(\xi)\,\mathrm{d}\xi + pm, \tag{10}$$

where the optimal level $\bar{x}$ may be received as,

$$\bar{x}\,|\,\Phi(x) = \frac{p-c}{p+h-\alpha c} \tag{11}$$

and $\Phi(x)$ is probability distribution of demand,

$$\int_0^x \phi(\xi)\,\mathrm{d}\xi = \Phi(x),$$

$$p(x) = px,$$

$$c(x) = cx,$$

$$h(x) = hx$$

and

$$E(\xi) = m.$$

Now, if we use p, c and h when true values are p', c' and h', then we have the expected cost per period

$$\bar{K}(\bar{x}) = -c'x + (c'-p')\bar{x} + (p'+h')\int_0^{\bar{x}} \Phi(\xi)\,\mathrm{d}\xi + p'm. \tag{12}$$

However, in this case, the true optimal expected cost is

$$\bar{\bar{K}}(\bar{\bar{x}}) = -c'x + (x'-p')\bar{\bar{x}} + (p'+h')\int_0^{\bar{x}} \Phi(\xi)\,\mathrm{d}\xi + p'm, \tag{13}$$

where

$$\bar{\bar{x}}\,|\,\Phi(x) = \frac{p'-c'}{p'+h'-\alpha c}. \tag{14}$$

So, we have the error expected cost $[\bar{K}(\bar{x}) - \bar{\bar{K}}(\bar{\bar{x}})]$.

4.4. *Parameter value control [7]*

Let us assume that W is the cost of changing inventory policies. When the values of p, h and c are varied, the region without changing optimal policy is as follows:

$$\bar{K}(\bar{x}) - \bar{\bar{K}}(\bar{\bar{x}})w. \tag{15}$$

5. NUMERICAL EXAMPLE

We illustrate the foregoing by assuming various values of p, h, c, α and m. For example, when $p = 6.2$, $h = 1.0$, $c = 0.2$, $\alpha = 0.9$ and $m = 4.0$, the values of $[\bar{K}(\bar{x}) - \bar{\bar{K}}(\bar{\bar{x}})]$ for p' and h' are as presented in Table 1.

As the system and its environment change, we have to control the solution in order to conserve at high level the system operation. We have to find these changes and adjust the solution. This table express the method that find the time of the solution adjustment.

6. DISCUSSION

We have shown that the environment for approximation of observations and policies affects our inventory processes.

Table 1. Values of $\bar{K}(\bar{x}) - \bar{\bar{K}}(\bar{x})$ for p' and h', when $p = 6.2$, $h = 1.0$, $c = 0.02$, $\alpha = 0.9$ and $m = 4.0$

	h'															
p'	0.50	0.60	0.70	0.80	0.90	1.00	1.10	1.20	1.30	1.40	1.50	1.60	1.70	1.80	1.90	2.00
3.0	0.00	0.00	0.01	0.02	0.03	0.04	0.06	0.07	0.09	0.10	0.12	0.14	0.15	0.18	0.20	0.22
3.2	0.00	0.00	0.01	0.02	0.02	0.03	0.05	0.06	0.07	0.10	0.11	0.13	0.14	0.16	0.19	0.21
3.4	−0.00	0.00	0.01	0.01	0.02	0.03	0.04	0.06	0.07	0.08	0.10	0.12	0.13	0.15	0.17	0.19
3.6	−0.00	0.00	0.00	0.01	0.02	0.03	0.03	0.05	0.06	0.08	0.09	0.11	0.13	0.14	0.16	0.17
3.8	−0.00	0.00	0.00	0.01	0.01	0.02	0.03	0.04	0.06	0.07	0.08	0.09	0.12	0.13	0.15	0.17
4.0	0.00	−0.00	0.00	0.01	0.01	0.02	0.03	0.04	0.05	0.06	0.08	0.09	0.11	0.13	0.14	0.16
4.2	−0.00	−0.00	0.00	0.00	0.01	0.01	0.02	0.03	0.04	0.06	0.07	0.08	0.10	0.12	0.13	0.14
4.4	0.00	−0.00	0.00	0.00	0.01	0.01	0.02	0.03	0.04	0.05	0.07	0.08	0.09	0.10	0.13	0.14
4.6	0.00	0.00	0.00	0.00	0.01	0.01	0.01	0.03	0.04	0.04	0.06	0.07	0.08	0.10	0.11	0.13
4.8	0.01	0.00	−0.00	0.00	0.00	0.01	0.01	0.02	0.03	0.04	0.05	0.07	0.08	0.09	0.10	0.12
5.0	0.01	0.00	−0.00	0.00	0.00	0.01	0.01	0.02	0.03	0.04	0.05	0.05	0.07	0.09	0.10	0.11
5.2	0.01	0.00	−0.00	0.00	0.00	0.00	0.01	0.01	0.03	0.03	0.04	0.05	0.07	0.08	0.09	0.10
5.4	0.01	0.00	0.00	−0.00	0.00	0.00	0.01	0.01	0.02	0.03	0.04	0.05	0.06	0.07	0.09	0.10
5.6	0.01	0.01	0.00	−0.00	0.00	0.00	0.01	0.01	0.02	0.03	0.04	0.04	0.05	0.07	0.08	0.09
5.8	0.02	0.01	0.00	−0.00	0.00	0.00	0.00	0.01	0.01	0.02	0.03	0.04	0.05	0.06	0.08	0.09
6.0	0.02	0.01	0.00	−0.00	−0.00	0.00	0.00	0.01	0.01	0.02	0.03	0.04	0.05	0.05	0.06	0.08

Much of the decision making in the real world takes place in an environment in which the goals, the constraints and the consequences of possible actions are not known precisely. To deal quantitatively with imprecision, the traditional approach is to employ the concepts and techniques of probability theory and, more particularly, the tools provided by decision theory, control theory and information theory. This is now questionable especially in view of the developments in the field of fuzzy sets theory [8].

There is a differentiation between randomness and fuzziness, with the latter being a major source of imprecision in many decision processes. By fuzziness, we mean a type of impression which is associated with fuzzy sets, that is, classes in which there is no sharp transition from membership to non-membership. There are many facets of the theory of decision making in a fuzzy environment which require more thorough investigation. This is also the case with inventory systems.

7. AREAS FOR FURTHER RESEARCH

Although considerable progress has been made in improving our understanding of inventory models, a variety of important problems remain unsolved [9]. Some of these problems include the following.

7.1. Approximately optimal policies

Consider a situation where one is determining good ordering policies for a fixed life commodity when there is a positive lead time for placing an order and when inventory levels are reviewed continuously as in perishable inventory theory. The problem is extremely difficult, because the state variable would have to be a possibly infinite vector of on hand and on order stocks, since the age of any particular batch of items in stock could take on any one of a continuum of values [4, 10].

7.2. Large scale inventory system

The study of large scale inventory systems has a major role to play in management science. We meet the challenge of incommensurability, and vector-valued criteria. We can expect the new theory of "*fuzzy systems*" to play a vital role in this area [11]. For example, delays is an important feature in inventory systems. Optimal delay can be defined as the fuzzy set of alternatives resulting from the confluences of the goals and constraints.

7.3. Probability criterion

Analyze the functional equations corresponding to the inventory process under the assumption that we desire to minimize the probability that the cost exceeds a given quantity α [12, 13].

7.4. Optimal expansion of capacity

Critical to planning facility investment for a growing area is the determination of when capacity should be added and how much. Applications to inventory theory are important [14].

7.5. *Multi-objective version [15]*

The problem with which we have dealt is the multi-objective extension of Bellman's standard problem, although a similar analysis is possible for similar problems.

7.6. *Water management*

Certain water management problems can be viewed as inventory problems under conditions of uncertain supply and controlled demand [16].

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