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# Nonequivalent Seiberg–Witten maps for noncommutative massive $U(N)$ gauge theory

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**Abstract**

Massive vector fields can be described in a gauge invariant way with the introduction of compensating fields. In the unitary gauge one recovers the original formulation. Although this gauging mechanism can be extended to noncommutative spaces in a straightforward way, nontrivial aspects show up when we consider the Seiberg–Witten map. As we show here, only a particular class of its solutions leads to an action that admits the unitary gauge fixing.


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## 1. Introduction

The idea that space–time may be noncommutative at very small length scales is not new [1]. Originally this has been thought just as a mechanism for providing space with a natural cut off that would control ultraviolet divergences. However, the interest on this topic increased a lot in the last years motivated mainly by important results coming from string theory that indicate a possible noncommutative structure for space–time (see [2,3] for a review and a wide list of important references). The presence of an antisymmetric tensor background along the D-brane [4] world volumes (space–time region where the string endpoints are located) is an important source for noncommutativity in string theory [5,6].

In noncommutative space–time of dimension $D$ the coordinates $x^\mu$ are replaced by Hermitian generators $\hat{x}^\mu$ of a noncommutative $C^*$-algebra over space–time functions satisfying

$$[\hat{x}^\mu, \hat{x}^\nu] = i\theta^{\mu\nu}, \tag{1.1}$$

where $\theta^{\mu\nu}$ is usually taken as a constant antisymmetric matrix of dimension $D$.

In order to define noncommutative quantum field theories one can rather than working with noncommuting functions of the operators $\hat{x}^\mu$, replace the ordinary

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products everywhere by the Moyal star product

$$\phi_1(x) \star \phi_2(x) = \exp\left(\frac{i}{2}\theta^{\mu\nu}\partial_\mu^x \partial_\nu^y\right)\phi_1(x)\phi_2(y)\Big|_{x=y} \tag{1.2}$$

and then consider usual functions of $x^\mu$. Since the space–time integral of the Moyal product of two fields is equal to the usual product (when boundary terms do not contribute), the noncommutativity does not affect the free part of the action but the vertices. This implies many interesting features of noncommutative quantum field theories as discussed in [2,3].

Gauge theories can be extended to noncommutative spaces by considering actions that are invariant under gauge transformations defined in terms of the Moyal structure. However, the form of these gauge transformations imply that the algebra of the generators must be closed not only under commutation but also under anticommutation. So $U(N)$ is usually chosen as the symmetry group for noncommutative extensions of Yang–Mills theories in place of $SU(N)$, although other symmetry structures can also be considered [7–9].

Once one has a noncommutative gauge theory, in the sense that the field polinomia in the action and their gauge structure are constructed by using Moyal products, it is possible to generate a map from this noncommutative theory to an ordinary one, as shown by Seiberg and Witten [2]. Interesting aspects of the general form of this map can be found in [10]. The mapped Lagrangian is usually written as a nonlocal infinite series of ordinary fields and their space–time derivatives but the noncommutative Noether identities are however kept by the Seiberg–Witten map.

It is sometimes useful to transform global symmetries in gauge symmetries by the introduction of pure gauge “compensating fields” [11]. This procedure can be used, for example, as a tool for calculating anomalous divergencies associated with global currents [12]. Another use of compensating fields is to allow a gauge invariant formulation for a massive vector field. In this Letter we will investigate the extension to noncommutative spaces of this kind of gauging process. We will see that it is possible to define a noncommutative version of a gauged vector field with mass and also that a Seiberg–Witten map can be constructed. When we introduce a gauge invariance that was not originally present it is in general possible to return to the original theory by a particular gauge fixing of this new symmetry. This condition, expected to hold also at noncommutative level, will represent a criterion for choosing the appropriate Seiberg–Witten map among the general solutions.

This Letter is organized as follows: in Section 2 we discuss the noncommutative massive vector field theory. In Section 3 we present the general structure of the Seiberg–Witten map, that means: we derive the general set of equations it has to satisfy. Different solutions for the map are then presented in Section 4. We reserve Section 5 for some concluding remarks.

## 2. Gauging the noncommutative $U(N)$ Proca field

The action for the ordinary $U(N)$ Proca (massive vector) field is given by

$$S[a] = \mathrm{tr}\int d^4x\left(-\frac{1}{2}f_{\mu\nu}f^{\mu\nu} + m^2 a_\mu a^\mu\right), \tag{2.1}$$

where the curvature tensor is defined by

$$f_{\mu\nu} = \partial_\mu a_\nu - \partial_\nu a_\mu - i[a_\mu, a_\nu] \tag{2.2}$$

and the vector field $a_\mu$ take values in the $U(N)$ algebra, with generators $T^A$, assumed to be normalized as

$$\mathrm{tr}\big(T^A T^B\big) = \frac{1}{2}\delta^{AB} \tag{2.3}$$

and satisfying the (anti)commutation relations

$$\begin{aligned} \big[T^A, T^B\big] &= i f^{ABC} T^C, \\ \big\{T^A, T^B\big\} &= d^{ABC} T^C. \end{aligned} \tag{2.4}$$

We take $f^{ABC}$ and $d^{ABC}$ as completely antisymmetric and completely symmetric, respectively.

The theory described by (2.1) is not gauge invariant because of the presence of the mass term. As it is well known, it is possible to gauge the above theory with the introduction of compensating fields. In the Lagrangian formalism, this can be directly done with the introduction of scalar fields $g$ which transform as $U(N)$ group elements. The procedure is very simple and consists in replacing the field $a_\mu$ by a kind of invariant collective field $\tilde{a}_\mu = \tilde{a}_\mu(a, g)$ defined as [11,12]

$$\begin{aligned} \tilde{a}_\mu &= g^{-1} a_\mu g + i g^{-1}\partial_\mu g \\ &= g^{-1}(a_\mu - b_\mu) g, \end{aligned} \tag{2.5}$$

where

$$b_\mu = -i\partial_\mu g g^{-1} \tag{2.6}$$

is a "pure gauge" compensating vector field since its curvature, constructed as in (2.2), vanishes identically. As $a_\mu$, $b_\mu$ also takes values in the $U(N)$ algebra.

If we write $\tilde{a}_\mu$ instead of $a_\mu$ in action (2.1), we get directly

$$S[a,g] = \mathrm{tr}\int d^4x\left(-\frac{1}{2}f_{\mu\nu}f^{\mu\nu} + m^2(a_\mu - b_\mu)\big(a^\mu - b^\mu\big)\right) \tag{2.7}$$

which is now invariant under the gauge transformations

$$\bar{\delta}a_\mu = \partial_\mu\alpha - i[a_\mu,\alpha] \equiv D_\mu\alpha,$$
$$\bar{\delta}g = i\alpha g \tag{2.8}$$

as can be verified. We are denoting the gauge variation by $\bar{\delta}$ since we will reserve the symbol $\delta$ for the gauge variation of the noncommutative case, which will be shortly introduced. For completeness, we note that the above definitions imply that

$$\bar{\delta}b_\mu = \partial_\mu\alpha - i[b_\mu,\alpha] \equiv \bar{D}_\mu\alpha. \tag{2.9}$$

The gauge algebra of all of these fields closes as

$$[\bar{\delta}_1,\bar{\delta}_2]y = \bar{\delta}_3 y, \tag{2.10}$$

$y$ representing $a_\mu$, $g$ or $b_\mu$. The parameter composition rule then is given by

$$\alpha_3 = i[\alpha_2,\alpha_1]. \tag{2.11}$$

As expected, the original theory is recovered in the unitary gauge $g = 1$. There is no obstruction to implement this model also at the quantum level, even if there are arbitrary couplings with fermions [11], since candidates to anomalies are compensated by appropriate Wess–Zumino terms constructed with the fields $a_\mu$ and $g$.

The gauge invariant action given by (2.7) can be extended to a noncommutative space. Let us represent the corresponding noncommutative fields by capital letters and introduce Moyal products whenever usual ordinary products appear in the original ordinary theory. We get the noncommutative version for the action (2.7)

$$S = \mathrm{tr}\int d^4x\left(-\frac{1}{2}F_{\mu\nu}F^{\mu\nu} + m^2(A_\mu - B_\mu)\big(A^\mu - B^\mu\big)\right), \tag{2.12}$$

where now the curvature is given by

$$F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu - i[A_\mu \overset{\star}{,} A_\nu] \tag{2.13}$$

and the infinitesimal gauge transformations (2.8) are replaced by

$$\delta A_\mu = D_\mu\epsilon = \partial_\mu\epsilon - i[A_\mu \overset{\star}{,} \epsilon],$$
$$\delta F_{\mu\nu} = -i[F_{\mu\nu} \overset{\star}{,} \epsilon],$$
$$\delta G = i\epsilon \star G. \tag{2.14}$$

Note that we are using the same symbol to denote ordinary and noncommutative covariant derivatives but we believe that there will be no misunderstanding. The compensating field $B_\mu$ is now

$$B_\mu \equiv -i\partial_\mu G \star G^{-1} \tag{2.15}$$

and transforms accordingly

$$\delta B_\mu = \bar{D}_\mu\epsilon = \partial_\mu\epsilon - i[B_\mu \overset{\star}{,} \epsilon]. \tag{2.16}$$

Its noncommutative curvature, defined in analogy with (2.13), vanishes identically as in the ordinary case. As expect, the noncommutative gauge transformations listed above also close in an algebra

$$[\delta_1,\delta_2]Y = \delta_3 Y, \tag{2.17}$$

$Y$ representing $A_\mu$, $G$ or $B_\mu$. The composition rule for the parameters now is given by

$$\epsilon_3 = i[\epsilon_2 \overset{\star}{,} \epsilon_1] \tag{2.18}$$

and belongs to the algebra due to (2.4). In the above expressions $G$ is an element of the noncommutative $U(N)$ group. This means that the composition rule is also to be operated with the Moyal product. For instance the inverse to $G$ is defined by $G^{-1}\star G = 1$ which implies different features when compared with the usual (commutative) $U(N)$ group. If one writes down explicitly expressions like (2.15), (2.16) or (2.18), it is easy to see that they will involve both the structure functions $f^{ABC}$ and $d^{ABC}$ present in Eq. (2.4). With these remarks in mind, we see that there is also no problem for implementing the unitary gauge $G=1$. This can be seen by using directly the finite form of the gauge transformations (2.14):

$$A'_\mu = U^{-1}\star A_\mu \star U + iU^{-1}\star \partial_\mu U,$$
$$G' = iU^{-1}\star G. \tag{2.19}$$

This guarantees that the physical content of the Proca model is not affected by the introduction of the compensating fields. We observe that the Hamiltonian treatment of these points has been done for the simpler noncommutative $U(1)$ case [13], along the BFFT procedure [14].

## 3. General structure of the Seiberg–Witten map

Let us consider now the Seiberg–Witten map linking the massive noncommutative $U(N)$ gauge theory described in the previous section and a corresponding higher derivative theory defined in terms of usual commutative products and ordinary fields. Following the same notation employed in the last section, the noncommutative variables will be represented by capital letters, here generically denoted by $Y$. The corresponding ordinary ones, represented by small letters, will be generically denoted by $y$. We assume that the gauge transformations $\delta Y$ of the noncommutative variables listed in the last section can be obtained through the underlying gauge structure of the corresponding ordinary theory. The construction of the Seiberg–Witten map starts by imposing for all fields that

$$\delta Y = \bar\delta Y[y]. \tag{3.1}$$

The explicit form of this map comes solving the above equations when one assumes that the noncommutative parameters $\epsilon$ are functions of the commutative parameters $\alpha$ and ordinary fields $y$. Although we are taking the same form of the gauge transformations displayed in (2.8) and (2.14), the form of the mapped action will be different from (2.7) if the map is nontrivial. Now, the transformations above also close in an algebra:

$$[\bar\delta_1,\bar\delta_2]A_\mu[y]$$
$$= D_\mu\big(\bar\delta_1\epsilon_2[y]-\bar\delta_2\epsilon_1[y]+i\big[\epsilon_2[y]\stackrel{\star}{,}\epsilon_1[y]\big]\big)$$
$$= D_\mu\epsilon_3[y],$$

$$[\bar\delta_1,\bar\delta_2]G[y]$$
$$= i\big(\bar\delta_1\epsilon_2[y]-\bar\delta_2\epsilon_1[y]+i\big[\epsilon_2[y]\stackrel{\star}{,}\epsilon_1[y]\big]\big)G[y]$$
$$= i\epsilon_3[y]G[y], \tag{3.2}$$

where the indices 1, 2 and 3 represent the dependence of $\epsilon$ in $\alpha_1$, $\alpha_2$ and $\alpha_3$. For instance, $\epsilon_3[y]\equiv\epsilon[\alpha_3,y]$. From the equations above we find the composition rule for the noncommutative parameter $\epsilon[y]$ given by

$$\epsilon_3[y] = \bar\delta_1\epsilon_2[y]-\bar\delta_2\epsilon_1[y]+i\big[\epsilon_2[y]\stackrel{\star}{,}\epsilon_1[y]\big] \tag{3.3}$$

in place of (2.18). Eq. (3.2) is not new in the literature [2,8] but will be crucial for the results that we will derive.

Now let us obtain the general equations that must be satisfied by the Seiberg–Witten map. Assuming, as usual, that the gauge transformation parameter can be expanded to first order in $\theta^{\mu\nu}$ as $\epsilon[y]=\alpha+\epsilon^{(1)}[y]$, we get from (3.3) that

$$\bar\delta_1\epsilon_2^{(1)}-\bar\delta_2\epsilon_1^{(1)}-i\big[\alpha_1,\epsilon_2^{(1)}\big]+i\big[\alpha_2,\epsilon_1^{(1)}\big]-\epsilon_3^{(1)}$$
$$=-\frac{1}{2}\theta^{\mu\nu}\{\partial_\mu\alpha_1,\partial_\nu\alpha_2\}. \tag{3.4}$$

This relation will be important in finding the Seiberg–Witten map for the gauge parameter. We will see in the next section that it allows more than one solution for $\epsilon^{(1)}$. Assuming as well that to first order in $\theta$ the field is expanded as $A_\mu=a_\mu+A_\mu^{(1)}$, the field strength $F_{\mu\nu}=f_{\mu\nu}+F_{\mu\nu}^{(1)}$ and that $G=g+G^{(1)}$, it is not difficult to deduce from (2.8), (2.14) and (3.1) that

$$\bar\delta A_\mu^{(1)}+i\big[A_\mu^{(1)},\alpha\big]$$
$$=\partial_\mu\epsilon^{(1)}+i\big[\epsilon^{(1)},a_\mu\big]-\frac{1}{2}\theta^{\alpha\beta}\{\partial_\alpha\alpha,\partial_\beta a_\mu\} \tag{3.5}$$

and as a consequence, the field strength transformation satisfy

$$\bar\delta F_{\mu\nu}^{(1)}+i\big[F_{\mu\nu}^{(1)},\alpha\big]$$
$$=i\big[\epsilon^{(1)},f_{\mu\nu}\big]-\frac{1}{2}\theta^{\alpha\beta}\{\partial_\alpha\alpha,\partial_\beta f_{\mu\nu}\}. \tag{3.6}$$

Also from the same equations we get

$$\bar\delta G^{(1)}-i\alpha G^{(1)}=-\frac{1}{2}\theta^{\mu\nu}\partial_\mu\alpha\partial_\nu g+i\epsilon^{(1)}g \tag{3.7}$$

for the compensating field $G$. The corresponding vector field, writing in first order in $\theta$ that $B_\mu=$

$b_\mu + B_\mu^{(1)}$, satisfies

$$\bar\delta B_\mu^{(1)} + i\big[B_\mu^{(1)}\alpha\big] = \partial_\mu \epsilon^{(1)} + i\big[\epsilon^{(1)}, b_\mu\big] - \frac{1}{2}\theta^{\alpha\beta}\{\partial_\alpha\alpha, \partial_\beta b_\mu\}. \quad (3.8)$$

Instead of solving the above equation, we observe that the map for $G$ induces directly a map for $B_\mu$. From (2.15) one can show that

$$B_\mu^{(1)} = -i\partial_\mu g\big(G^{-1}\big)^{(1)} - i\partial_\mu G^{(1)}\big(g^{-1}\big) + \frac{1}{2}\theta^{\alpha\beta}\partial_\alpha\partial_\mu g\,\partial_\beta\big(g^{-1}\big) \quad (3.9)$$

solves (3.8). Now, by using the equations for the gauge transformations defined above, it is not difficult to verify that action (2.12) written as

$$S = \mathrm{tr}\int d^4x\,\Big(-\frac{1}{2}f_{\mu\nu}f^{\mu\nu} + m^2(a_\mu - b_\mu)\big(a^\mu - b^\mu\big) - f^{\mu\nu}F_{\mu\nu}^{(1)} + m^2\big\{a^\mu - b^\mu, A_\mu^{(1)} - B_\mu^{(1)}\big\}\Big) \quad (3.10)$$

up to $O(\theta^2)$, is indeed gauge invariant. This result is of course independent of the particular maps one obtains from (3.5), (3.7) or (3.8).

## 4. Different solutions of Seiberg–Witten map

Let us now look for the solutions of the Seiberg–Witten map. The general solution of (3.4) when the compensating field sector is not present is [10]

$$\epsilon^{(1)} = \frac{1}{4}\theta^{\mu\nu}\{\partial_\mu\alpha, a_\nu\} + \lambda_1\theta^{\mu\nu}[\partial_\mu\alpha, a_\nu], \quad (4.1)$$

where $\lambda_1$ is an arbitrary constant. The first term corresponds to the particular solution of Eq. (3.4) and the second term is the solution of the homogeneous part of the same equation. It is possible from (3.5) and (4.1) to find an explicit form for the map of the connection as [10]

$$A_\mu[a] = a_\mu - \frac{1}{4}\theta^{\alpha\beta}\{a_\alpha, \partial_\beta a_\mu + f_{\beta\mu}\} + \sigma\theta^{\alpha\beta}D_\mu f_{\alpha\beta} + \frac{\lambda_1}{2}\theta^{\alpha\beta}D_\mu[a_\alpha, a_\beta] + O\big(\theta^2\big), \quad (4.2)$$

where $\sigma$ is also an arbitrary constant associated with the homogeneous solution of (3.5) when one uses (4.1). We observe that if we consider only the particular solution ($\lambda_1 = 0$) for the gauge parameter, Eqs. (3.7) and (4.1) give us

$$G[a, g] = g - \frac{1}{2}\theta^{\alpha\beta}a_\alpha\Big(\partial_\beta g - \frac{i}{2}a_\beta g\Big) + \gamma\theta^{\alpha\beta}f_{\alpha\beta}g + O\big(\theta^2\big), \quad (4.3)$$

where $\gamma$ is arbitrary. At this point we note that it if we choose the ordinary unitary gauge $g = 1$ the corresponding noncommutative mapped group element keeps a dependence on $a_\mu$ and cannot be suppressed from the theory as can be seen from the above expression. However by considering the complete solution (4.1) and taking $\lambda_1 = -1/4$ it is possible to eliminate one of the problematic terms in (4.3) to obtain

$$G[a, g] = g - \frac{1}{2}\theta^{\alpha\beta}a_\alpha\partial_\beta g + \gamma\theta^{\alpha\beta}f_{\alpha\beta}g + O\big(\theta^2\big). \quad (4.4)$$

If we now choose $\gamma = 0$, $G$ goes to $g$ in the unitary gauge. Also, from (3.9),

$$B_\mu^{(1)} = \frac{1}{2}\theta^{\alpha\beta}\big((\bar D_\mu b_\alpha)b_\beta - \bar D_\mu(a_\alpha b_\beta)\big) \quad (4.5)$$

when one uses (4.4) with $\gamma = 0$. Observe, however, that the expression for $A_\mu^{(1)}$ coming from (4.2), with $\lambda_1 = -1/4$ does not vanish for any $\sigma$. We will show in what follows that when we consider the $g$ sector, it is possible to construct a Seiberg–Witten map that can be completely suppressed in the unitary gauge. We are considering a theory involving the pure gauge field $b_\mu$ besides the usual gauge field $a_\mu$. So, the space of solutions for $\epsilon^{(1)}, G^{(1)}, A_\mu^{(1)}, B_\mu^{(1)}$ representing the noncommutative field extensions is actually greater than the one studied in detail in [10]. One can check that now instead of (4.1) we get

$$\epsilon^{(1)} = \frac{1}{4}(1-\rho)\theta^{\mu\nu}\{\partial_\mu\alpha, a_\nu\} + \lambda_1\theta^{\mu\nu}[\partial_\mu\alpha, a_\nu] + \frac{1}{4}\rho\theta^{\mu\nu}\{\partial_\mu\alpha, b_\nu\} + \lambda_2\theta^{\mu\nu}[\partial_\mu\alpha, b_\nu] \quad (4.6)$$

when one also considers the compensating field sector. Observe that the first and third terms play a complementary role as a particular solution of Eq. (3.4). The other terms represent homogeneous solutions. In Eq. (4.6) $\rho$, $\lambda_1$ and $\lambda_2$ are arbitrary.

From (3.7) and (4.6) we get now

$$G[a,g] = g - \frac{1}{2}(1-\rho)\theta^{\alpha\beta}a_\alpha\left(\partial_\beta g - \frac{i}{2}a_\beta g\right) + i\lambda_1\theta^{\alpha\beta}a_\alpha a_\beta g + \gamma\theta^{\alpha\beta}f_{\alpha\beta}g + i\left(\lambda_2 - \frac{\rho}{4}\right)\theta^{\alpha\beta}b_\alpha b_\beta g + O(\theta^2). \tag{4.7}$$

Since $b_\alpha$ vanishes identically when $g$ goes to 1, it is possible to implement an unitary gauge for $G(g)$ if we choose $\lambda_1 = \frac{1}{4}(\rho - 1)$ and $\gamma = 0$, leaving $\lambda_2$ free. This choice, however, does not make $A^{(1)} \to 0$ when $g \to 1$, as can be observed from (3.5) and (4.6). Additionally imposing that $\rho = 1$ and $\lambda = 0$, we verify that $A^{(1)} \to 0$ when $g \to 1$. In this last case

$$B_\mu^{(1)} = \frac{1}{4}\theta^{\alpha\beta}\{\bar{D}_\mu b_\alpha, b_\beta\} = \frac{1}{4}\theta^{\alpha\beta}\{\partial_\alpha b_\mu, b_\beta\} \tag{4.8}$$

and

$$A_\mu^{(1)} = \frac{1}{4}\theta^{\alpha\beta}\{b_\alpha, D_\mu b_\beta - 2\partial_\beta a_\mu\} \tag{4.9}$$

and indeed both expressions vanish in the unitary gauge. This is in accordance with the fact that the original Proca model is not a gauge theory.

Now that the structure of this map has been found, it is only algebraic work the construction of the corresponding mapped action. From (2.13)

$$F_{\mu\nu} = f_{\mu\nu} + D_\mu A_\nu^{(1)} - D_\nu A_\mu^{(1)} + \frac{1}{2}\theta^{\alpha\beta}\{\partial_\alpha a_\mu, \partial_\beta a_\nu\} \equiv f_{\mu\nu} + F_{\mu\nu}^{(1)} \tag{4.10}$$

up to $O(\theta^2)$, and discarding terms that come from the homogeneous part of (3.6) [10] that do not vanish if $g = 1$. Now, the mapped action can be written as in (3.10) with $B_\mu^{(1)}$, $A_\mu^{(1)}$ and $F_{\mu\nu}^{(1)}$ given by (4.8)–(4.10).

This action is invariant under the transformations (2.8) and (2.9) since condition (3.1) defining the Seiberg–Witten map is satisfied by construction. This guarantees that the Noether identities are kept by the map. Also, the unitary gauge: $g = 1$, $b_\mu = 0$ can be implemented in a consistent way recovering the noncommutative Proca model action given by (2.12), with $B_\mu = 0$ and $A_\mu = a_\mu$, in $O(\theta^2)$.

## 5. Conclusion

We discussed here how to build up a noncommutative extension for a gauged massive vector $U(N)$ field theory. The ordinary (commutative) theory can be gauge fixed to the so-called unitary gauge where the standard massive vector field theory is recovered. Although the same mechanism can be easily extended to the noncommutative theory, nontrivial aspects appear when one considers the Seiberg–Witten map of that theory. Taking into account the compensating field sector as well as the terms that come from the homogeneous equations that define the Seiberg–Witten map, we have found several nonequivalent solutions. One of them consistently admits the implementation of the unitary gauge fixing for all the fields.

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