Dark Matter and neutrino masses from global $U(1)_{B-L}$ symmetry breaking

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ABSTRACT

We present a scenario where neutrino masses and Dark Matter are related due to a global $U(1)_{B-L}$ symmetry. Specifically we consider neutrino mass generation via the Zee–Babu two-loop mechanism, augmented by a scalar singlet whose VEV breaks the global $U(1)_{B-L}$ symmetry. In order to obtain a Dark Matter candidate we introduce two Standard Model singlet fermions. They form a Dirac particle and are stable because of a remnant Z_2 symmetry. Hence, in this model the stability of Dark Matter follows from the global $U(1)_{B-L}$ symmetry. We discuss the Dark Matter phenomenology of the model, and compare it to similar models based on gauged $U(1)_{B-L}$. We argue that in contrast to the gauged versions, the model based on the global symmetry does not suffer from severe constraints from Z' searches.

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1. Introduction

Observations of neutrino oscillations [1–4] imply that the Standard Model (SM) has to be extended in order to give mass to neutrinos. The scale for neutrino masses set by the larger of the two measured mass-squared differences is $m_\nu \sim 0.05$ eV. Another hint for physics beyond the SM comes from various cosmological and astrophysical observations, which require the existence of Dark Matter (DM), presumably a new kind of particle beyond the known ones. From WMAP CMB measurements combined with other cosmological observables one obtains for the fraction of the Dark Matter density ($\Omega_{\rm DM}$) to the total energy density of the Universe $\Omega_{\rm DM}h^2 = 0.1123 \pm 0.0035$ while for normal baryonic matter one has only about 20% thereof: $\Omega_b h^2 = 0.0226 \pm 0.0005$ [5]. Here h is the Hubble parameter in units of 100 km s^{-1} Mpc^{-1}, with $h^2 \approx 0.5$. A natural candidate for a Dark Matter particle is a weakly interacting massive particle (WIMP), since a stable particle with mass at the "weak scale" (of order 100 GeV) and a "typical" annihilation cross section acquires a relic abundance through thermal freeze-out in the early Universe with $\Omega_{\rm DM}$ in rough agreement with observations.

At first sight the two scales indicated by neutrino mass and WIMP Dark Matter are vastly different, by about 12 orders of magnitude. Nevertheless, it is tempting to consider a common origin of them. In particular, it might be possible that neutrino masses are generated by physics at the TeV scale. In such a case a connection between the neutrino mass mechanism and Dark Matter may exist, and — in the best of all worlds — both of them might

be testable at LHC. In this Letter we present a common framework linking these two mysteries and offering characteristic signatures at LHC.

There exist a number of publications which relate neutrino mass and Dark Matter; a few examples can be found in Refs. [6–25]. We provide another interesting scenario where a global $U(1)_{B-L}$ plays a central role. For the neutrino mass mechanism we depart from the Zee–Babu model [26–28], where two $SU(2)_L$ singlet scalars are introduced, one single and one double charged, and neutrino masses are generated at two-loop level. Phenomenological studies of this model have been performed, e.g., in Refs. [29–32]. We extend this model in a simple way by adding one scalar and two fermionic singlets. In the spirit of Majoron models (see e.g., [33,34]) we impose the conservation of lepton number, or actually $B - L$ (baryon minus lepton) number at the quantum level. The corresponding $U(1)$ symmetry is spontaneously broken by the non-zero vacuum expectation value (VEV) of the scalar singlet [35]. This induces the tri-linear term in the scalar potential necessary for neutrino mass generation and provides a (Dirac) mass for the Dark Matter. This leads to a common source for lepton number breaking (and hence the generation of a Majorana neutrino mass) and Dark Matter.

Any model for Dark Matter has to address the question of why the DM particle is stable on cosmological time scales. This is often achieved by introducing a Z_2 symmetry which protects the DM particle, a famous example being R-parity in Supersymmetric theories. The adhoc postulation of such a Z_2 symmetry may appear artificial, but our model provides an example where the Z_2 symmetry emerges naturally as an unbroken remnant of some larger symmetry of the theory, namely the global $U(1)_{B-L}$ symmetry. This can be considered as a specific example of a more general class of models, where the dark sector respects a global $U(1)$ (accidental or imposed), which gets spontaneously broken in such a

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way that a Z_2 symmetry remains unbroken and guarantees the stability of Dark Matter.

The plan of the Letter is as follows. We start by discussing the model in Section 2. In Section 2.1 we present the Zee–Babu model extended with the global $U(1)_{B-L}$ symmetry and in Section 2.2 we discuss the scalar sector of the model. This includes a discussion of various consequences of the massless Goldstone boson, the Majoron, from the breaking of the global $U(1)$ symmetry. We comment on implications for BBN, Higgs searches at colliders, and baryon genesis. In Section 3 we introduce the DM fermions to the model. Sections 3.1 and 3.2 deal with the relic DM abundance and direct detection prospects, respectively. In Sections 3.3 and 3.4 we briefly comment on indirect detection signals and on Majoron induced DM self-interactions. A general discussion follows in Section 4, where we also compare our model based on the global $U(1)_{B-L}$ to models using a gauged $U(1)_{B-L}$ symmetry.

2. The model

2.1. The Zee–Babu model and spontaneous breaking of $B - L$

The scalar sector of the original Zee–Babu model [26–28] contains in addition to the Standard Model Higgs doublet two complex $SU(2)_L$ singlet scalars: a singly charged scalar h^+ and a doubly charged scalar k^{++}. Then h^+ couples to left-handed lepton doublets L and k^{++} couples to right-handed leptons ℓ_R such that the contribution to the Lagrangian is

$$\mathcal{L}_{\text{lept}} = f_{ij} L_i^T \mathbf{C}^{-1} i\sigma_2 L_j h^+ + g_{ij} \ell_{Ri}^T \mathbf{C}^{-1} \ell_{Rj} k^{++} + \text{h.c.}, \qquad (1)$$

where $\mathbf{C}$ is the charge conjugation matrix, i, j label flavour indices, and the Yukawa couplings f and g are antisymmetric and symmetric, respectively. If both, h^+ and k^{++}, are assigned lepton number -2, lepton number is conserved by these interactions and therefore the theory respects a global $U(1)$ symmetry associated with $B - L$ conservation.[1] In order to break lepton number and generate a Majorana mass term for neutrinos, usually a trilinear term is introduced in the scalar potential, $\mu k^{++} h^- h^-$, which breaks $B - L$ explicitly. Here we forbid such a term by demanding that $B - L$ is an unbroken symmetry of the Lagrangian. Instead we introduce a complex scalar φ, which is singlet under the SM gauge group and has lepton number -2. Hence the VEV of φ breaks $B - L$ spontaneously [35]. The scalar potential contains a term

$$V_\mu = \lambda_\mu \varphi k^{++} h^- h^- + \text{h.c.}, \qquad (2)$$

inducing the "μ-term" once φ acquires its VEV, with

$$\mu = \lambda_\mu \frac{w}{\sqrt{2}}, \qquad \langle \varphi \rangle = \frac{w}{\sqrt{2}}. \qquad (3)$$

Light neutrino masses are generated via a two-loop diagram, which yields

$$(m_\nu)_{ab} = 16\mu f_{ac} m_c g_{cd}^* I_{cd} m_d f_{bd}, \qquad (4)$$

where m_c are charged lepton masses and I_{cd} is a two-loop integral [36]. Data from the LEP and Tevatron colliders require that the masses of charged scalars m_h and m_k are typically larger than $\mathcal{O}(100)$ GeV [37]. Hence we can neglect the masses of charged leptons compared to them. In this case, one finds

$$I_{cd} \approx I = \frac{1}{(16\pi^2)^2} \frac{1}{M^2} \frac{\pi^2}{3} \tilde{I}\left(\frac{m_k^2}{m_h^2}\right), \qquad (5)$$

where $M = \max(m_k, m_h)$ and $\tilde{I}(r)$ is a dimensionless function of order unity [31]. Using Eq. (5), the light neutrino mass matrix becomes

$$m_\nu \simeq \frac{1}{48\pi^2} \frac{\mu}{M^2} \tilde{I} f D_\ell g^\dagger D_\ell f^T, \qquad (6)$$

where the matrix $D_\ell = \text{diag}(m_e, m_\mu, m_\tau)$ contains the charged-lepton masses. Due to the antisymmetry of f, we have $\det m_\nu = 0$, and therefore, one of the neutrinos is massless as long as higher-order corrections are not considered. The neutrino phenomenology as well as other signatures of the model have been studied, e.g., in Refs. [29–32].

Light neutrino masses are suppressed by the heavy scalar masses and proportional to the scale of lepton-number violation μ, set by the $B - L$ breaking scale w, see Eq. (3). Assuming $\mu \sim M \sim \Lambda$, we see from Eq. (6) that for $m_\nu \sim 0.1$ eV the scale Λ of new physics has to be of order 1 TeV if we demand that $f \sim g \sim 0.05$. This scale Λ for generating neutrino masses is much below the scale of seesaw models due to the two loop suppression factor $1/(16\pi^2)^2 \sim 10^{-4}$. Hence, Λ is in the range of the LHC and may thus be probed soon. In particular, the double charged scalar provides a clean signature at colliders, via the decay into two like-sign leptons. Furthermore, the exchange of the charged scalars leads to enhanced signals in charged lepton-flavour violation ($\mu \to e\gamma$ or $\mu \to 3e$) with good prospects to be observed soon [30,31].

2.2. The scalar sector and the Majoron

The full scalar potential of the model is

$$\begin{aligned}
V_{\text{scalar}} &= V_\mu + \mu_1^2 \varphi^* \varphi + \mu_2^2 \phi^\dagger \phi + \mu_3^2 k^{++} k^{--} \\
&\quad + \mu_4^2 h^+ h^- + \lambda_1 (\varphi^* \varphi)^2 + \lambda_2 (\phi^\dagger \phi)^2 + \lambda_3 (k^{++} k^{--})^2 \\
&\quad + \lambda_4 (h^+ h^-)^2 + \lambda_5 (\varphi^* \varphi)(\phi^\dagger \phi) + \lambda_6 (\varphi^* \varphi)(k^{++} k^{--}) \\
&\quad + \lambda_7 (\varphi^* \varphi)(h^+ h^-) + \lambda_8 (\phi^\dagger \phi)(k^{++} k^{--}) \\
&\quad + \lambda_9 (\phi^\dagger \phi)(h^+ h^-) + \lambda_{10} (k^{++} k^{--})(h^+ h^-), \qquad (7)
\end{aligned}$$

where V_μ is given in Eq. (2), ϕ denotes the Standard Model Higgs doublet, μ_i are parameters of mass dimension one, and λ_i are dimensionless couplings. The λ_5 term in the potential induces mixing between φ and ϕ. There are two massive neutral scalars in the theory, with propagating mass states denoted by H_1 and H_2. They are related to the real part η of φ and to the real part ζ of the neutral component of ϕ by

$$\begin{pmatrix} H_1 \\ H_2 \end{pmatrix} = \begin{pmatrix} \cos\alpha & \sin\alpha \\ -\sin\alpha & \cos\alpha \end{pmatrix} \begin{pmatrix} \eta \\ \zeta \end{pmatrix}, \qquad (8)$$

where the mixing angle α is given by

$$\tan 2\alpha = \frac{\lambda_5 w v}{\lambda_1 w^2 - \lambda_2 v^2}, \qquad (9)$$

and v denotes the VEV of the Higgs doublet ϕ. The masses of H_1 and H_2 are

$$m_{H_{1,2}}^2 = \lambda_1 w^2 + \lambda_2 v^2 \pm \sqrt{(\lambda_1 w^2 - \lambda_2 v^2)^2 + \lambda_5^2 w^2 v^2}. \qquad (10)$$

The parameters $\mu_{1,2}$ in Eq. (7) can be eliminated with the help of the minimum condition for the potential. Therefore, the neutral scalar phenomenology depends only on three independent parameters (in addition to the VEVs v, w), which can be chosen to be either $(\lambda_1, \lambda_2, \lambda_5)$ or alternatively, $(m_{H_1}, m_{H_2}, \alpha)$.

Via Goldstone's theorem [38] a massless scalar has to appear in the spectrum due to the spontaneous breaking of the global $U(1)$

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[1] Note that we only write down the leptonic part of the Lagrangian. In a complete theory baryon number would have to be considered as well and sphaleron processes would break $B + L$, while $B - L$ is preserved.

symmetry. In our model the imaginary part of φ remains massless. We write

$$\varphi = \frac{1}{\sqrt{2}}(w + \eta + i\rho), \tag{11}$$

where $\eta = \sqrt{2}\,\mathrm{Re}(\varphi)$ and $\rho = \sqrt{2}\,\mathrm{Im}(\varphi)$ are real scalar fields and ρ is the Goldstone boson. Since the VEV of φ is responsible for lepton number breaking and the generation of a Majorana mass for the Dark Matter (see later) we follow the literature and call ρ a Majoron. Note that φ is a singlet under the SM gauge group and therefore there is no direct coupling of ρ to the Z boson like in triplet Majoron models [34] which are ruled out by the LEP measurement of the invisible Z decay width. The couplings of the Majoron ρ to the Higgs mass eigenstates H_1 and H_2 are obtained from the λ_1 and λ_5 terms of the potential (7):

$$\mathcal{L}_\rho = \frac{1}{2w}\left(m_{H_1}^2 \cos\alpha\, H_1 - m_{H_2}^2 \sin\alpha\, H_2\right)\rho^2. \tag{12}$$

The massless Majoron contributes to the relativistic energy density in the Universe, conventionally parameterized by the effective number of neutrino species. One thermalized scalar contributes $\Delta n_\nu = 4/7$ neutrino species, and therefore a thermal scalar degree of freedom is consistent with bounds from Big Bang Nucleosynthesis (BBN), which are in the range of $\Delta n_\nu < 1.63$ [39] or $\Delta n_\nu < 1.2$ [40] at 95% CL, depending on assumptions about the primordial abundances. Furthermore, in our model the Majoron typically decouples from the plasma at temperatures above the QCD phase transition, where the effective number of relativistic degrees of freedom is $\gtrsim 60$. Therefore, due to the entropy production at the QCD phase transition the Majoron abundance gets diluted and during BBN ρ contributes only with

$$\Delta n_\nu \lesssim \frac{4}{7}\left(\frac{10.75}{60}\right)^{4/3} \approx 0.06 \tag{13}$$

to the relativistic energy density, in good agreement with the above mentioned bounds.

The prospects for discovering the charged scalars of the Zee–Babu model at the LHC have been discussed in detail in [31]. In particular the double charged scalar has a production cross section at LHC in the range of 50 to 0.1 fb for masses between 200 GeV and 1 TeV. Its decay into two pairs of like-sign leptons provides an essentially background free signature, see also [37] for a recent analysis. Possible implications of the lepton flavour violating reactions mediated by the double charged scalar in supernova physics have been discussed in [41].

In addition to the charged scalar signatures, the presence of the Majoron will modify Higgs physics. As pointed out in [42–44] an important feature of Majoron models are invisible Higgs decays $H \to \rho\rho$, see also [45] for a recent study. In the SM, the Higgs boson dominantly decays into bottom pairs. In our model the corresponding branching ratios for the two Higgs mass eigenstates H_1 and H_2 are weighted with the Higgs mixing angle α:

$$\Gamma(H_1 \to b\bar{b}) = \frac{3\sqrt{2}G_F m_b^2 m_{H_1}}{8\pi}\left(1 - \frac{4m_b^2}{m_{H_1}^2}\right)^{\frac{3}{2}}\sin^2\alpha, \tag{14a}$$

$$\Gamma(H_2 \to b\bar{b}) = \frac{3\sqrt{2}G_F m_b^2 m_{H_2}}{8\pi}\left(1 - \frac{4m_b^2}{m_{H_2}^2}\right)^{\frac{3}{2}}\cos^2\alpha. \tag{14b}$$

The invisible decay modes into the Majoron ρ are obtained from the Lagrangian equation (12),

$$\Gamma(H_1 \to \rho\rho) = \frac{\sqrt{2}G_F}{32\pi}m_{H_1}^3\left(\frac{v}{w}\right)^2\cos^2\alpha, \tag{15a}$$

$$\Gamma(H_2 \to \rho\rho) = \frac{\sqrt{2}G_F}{32\pi}m_{H_2}^3\left(\frac{v}{w}\right)^2\sin^2\alpha, \tag{15b}$$

and therefore [44],

$$\frac{\Gamma(H_1 \to \rho\rho)}{\Gamma(H_1 \to b\bar{b})} = \frac{1}{12}\left(\frac{m_{H_1}}{m_b}\right)^2\left(\frac{v}{w}\right)^2\cot^2\alpha\left(1 - \frac{4m_b^2}{m_{H_1}^2}\right)^{-\frac{3}{2}}$$

$$\approx 250\left(\frac{m_{H_1}}{w}\right)^2\cot^2\alpha, \tag{16a}$$

$$\frac{\Gamma(H_2 \to \rho\rho)}{\Gamma(H_2 \to b\bar{b})} = \frac{1}{12}\left(\frac{m_{H_2}}{m_b}\right)^2\left(\frac{v}{w}\right)^2\tan^2\alpha\left(1 - \frac{4m_b^2}{m_{H_2}^2}\right)^{-\frac{3}{2}}$$

$$\approx 250\left(\frac{m_{H_2}}{w}\right)^2\tan^2\alpha. \tag{16b}$$

As we will see in the following, typically $m_{H_{1,2}} \sim w$, and therefore, depending on the Higgs mixing α, we expect that one or both Higgses decay dominantly into invisible Majorons. In the literature, various collider signatures have been studied for the identification of an invisible decaying Higgs boson, for example associated production of ZH [46] and $t\bar{t}H$ [47], or production in weak vector boson fusion [48].

2.3. Baryon asymmetry

In our model we do not attempt to explain the generation of the baryon number of the universe, and rely on some unspecified mechanism beyond our model. However, in models with low-scale lepton number violation often the problem occurs that any pre-existing baryon asymmetry is washed out by sphaleron processes if the lepton number violating reactions are in equilibrium before sphaleron freeze out. In our model lepton number is a global symmetry, which gets broken spontaneously by the VEV w of φ, somewhere between 200 GeV and 2 TeV, depending on parameter values, see Fig. 1. Hence, at temperatures above w lepton number is conserved and a pre-existing asymmetry is not affected. Only at temperatures $T \lesssim w$ lepton number violating processes occur via reactions involving the term $\lambda_\mu w k^{++}h^-h^-$. If sphaleron processes are still in equilibrium at that time, a pre-existing baryon asymmetry will be washed out. Sphaleron processes freeze out at the electro-weak phase transition at temperatures $T \sim v$. Since generically $w \sim v$ both phase transitions (the $B - L$ as well as the electro-weak ones) happen at similar temperatures. It remains a quantitative question of how much of the baryon asymmetry is erased by the detailed interplay of both phase transitions, which is beyond the scope of our Letter. Note that there is some parameter space where w is slightly less than v (compare Fig. 1), and in this case $B - L$ would be broken only after sphaleron freeze out.

3. Dark Matter

Having a global $B - L$ symmetry which is motivated by neutrino masses, it is tempting to ask if it could play also a role in stabilizing Dark Matter. Therefore we introduce fermions N_i, which are singlets under the SM gauge group, but charged under $U(1)_{B-L}$ in such a way that the Yukawa interaction with the SM Higgs doublet ϕ, $\bar{L}_j\tilde{\phi}N_i$ ($\tilde{\phi} \equiv i\sigma_2\phi^*$), is forbidden. Hence, our N_i cannot have lepton number $+1$ and are therefore not "right-handed neutrinos" in the conventional sense. Still we want the mass term for N_i to be generated by spontaneous lepton number breaking from the term

$$\mathcal{L}_N = \frac{1}{2}h_{ij}\varphi N_i^T \mathbf{C}^{-1}N_j + \text{h.c.} \tag{17}$$

This can be achieved by introducing two N fields, N_1 and N_2, and assigning them lepton numbers q_1 and q_2, respectively, such that $q_1 + q_2 = 2$ with $q_1 \neq q_2 \neq 1$; for example $q_1 = 1/2$ and $q_2 = 3/2$. Then no Yukawa term with the lepton doublets is allowed and Eq. (17) leads to a mass matrix

$$M_N = \begin{pmatrix} 0 & m_\chi \\ m_\chi & 0 \end{pmatrix} \quad \text{with } m_\chi = \lambda_\chi \frac{w}{\sqrt{2}}, \tag{18}$$

and $\lambda_\chi \equiv h_{12}$. Hence, N_1 and N_2 form a Dirac particle, with a pair of degenerate mass eigenstates with mass m_χ and opposite CP parity:

$$\chi_1 = \frac{1}{\sqrt{2}}(N_1 + N_2), \qquad \chi_2 = \frac{i}{\sqrt{2}}(N_1 - N_2). \tag{19}$$

They are stable because of an accidental Z_2 symmetry which emerges as an unbroken remnant of the global $U(1)_{B-L}$.[2] The interaction equation (17) becomes diagonal in the χ_i fields:

$$\mathcal{L}_N = \frac{1}{2} \lambda_\chi \varphi (\chi_1^T \mathbf{C}^{-1} \chi_1 + \chi_2^T \mathbf{C}^{-1} \chi_2) + \text{h.c.} \tag{20}$$

Note that in general global symmetries and non-gauge discrete symmetries (i.e., discrete symmetries which are not remnants of broken gauge symmetries) are expected to be violated by quantum gravity effects, e.g. [49], see [50] for a recent discussion. Planck scale suppressed higher dimensional operators are therefore expected in the low energy effective theory which would provide a mass to the Majoron [51] and might also induce DM decay in our model. If there is a dimension 6 operator suppressed by the Planck mass m_{pl} inducing DM decay, the corresponding lifetime exceeds the age of the Universe. However, a Dark Matter decay operator of dimension 5 would be problematic and should therefore be strongly suppressed or absent, since an estimate for the lifetime yields $\tau \sim m_{pl}^2 / m_\chi^3 \approx 10^7$ s for $m_\chi \sim 100$ GeV and there would be no DM left today. In the following we assume that Planck scale suppressed operators induce DM decays only with lifetimes larger than the age of the Universe. We comment on the effects of a finite mass for the Majoron in Section 4

The only particle to which χ_i can couple is the scalar φ, with the coupling λ_χ which is related to the DM mass via the VEV w, see Eq. (18). Furthermore, the coupling of φ to the SM is provided via the Higgs portal proportional to λ_5 (or the mixing angle α). Therefore, the two parameters λ_χ and α will play an important role for DM phenomenology, as we are going to discuss in the following.

3.1. Relic Dark Matter abundance

The relic DM density $\Omega_{DM} h^2$ is determined by the thermal freeze-out of χ_i in the early Universe. It is roughly inversely proportional to the thermal average of the total annihilation cross section times the relative velocity: $\Omega_{DM} h^2 \simeq 3 \times 10^{-27}$ cm^3 s$^{-1}/\langle \sigma v_{rel} \rangle$. Note that the two DM particles χ_1 and χ_2 have identical couplings to φ. Hence the annihilation cross sections are the same and they will contribute in equal parts to the total DM density. Two DM particles χ_i can annihilate into a quark–antiquark pair, into SM gauge bosons, into the massive scalars $H_{1,2}$, into the Zee–Babu scalars k^{++}, h^+, as well as in the Majoron ρ. As

examples we show the cross sections for annihilation via s-channel $H_{1,2}$ exchange into $b\bar{b}$, $W^+ W^-$ and $k^{++} k^{--}$, as well as the t- and u-channel annihilation into $\rho\rho$:

$$\sigma_{b\bar{b}} v_{rel} \approx \lambda_\chi^2 \sin^2 2\alpha \frac{y_b^2 v_{rel}^2}{1024\pi m_\chi} \cdot \frac{(m_{H_1}^2 - m_{H_2}^2)^2}{(s - m_{H_1}^2)^2 (s - m_{H_2}^2)^2}$$
$$\times (s - 4m_b^2)^{3/2}, \tag{21a}$$

$$\sigma_{WW} v_{rel} \approx \lambda_\chi^2 \sin^2 2\alpha \frac{g^4 v^2 v_{rel}^2}{2048\pi m_\chi} \cdot \frac{(m_{H_1}^2 - m_{H_2}^2)^2}{(s - m_{H_1}^2)^2 (s - m_{H_2}^2)^2}$$
$$\times \sqrt{(s - 4m_W^2)} \left[1 + \frac{1}{2} \left(\frac{s}{2m_W^2} - 1 \right)^2 \right], \tag{21b}$$

$$\sigma_{kk} v_{rel} = \lambda_6^2 \frac{m_\chi v_{rel}^2}{32\pi} \frac{[(s - m_{H_2}^2) \cos\alpha - (s - m_{H_1}^2) \sin\alpha]^2}{(s - m_{H_1}^2)^2 (s - m_{H_2}^2)^2}$$
$$\times \sqrt{s - 4m_{k^{++}}^2}, \tag{21c}$$

$$\sigma_{\rho\rho} v_{rel} = \lambda_\chi^4 \frac{v_{rel}^2}{1536\pi m_\chi^2}. \tag{21d}$$

We expanded $s = 4m_\chi^2/(1 - v_{rel}^2/4)$ in $v_{rel} \ll 1$ to show explicitly the velocity suppression of the annihilation, however we keep s in the kinematical terms as well as in the denominators showing the resonant behavior of the s-channel cross section as a function of the DM mass m_χ. For simplicity we neglect here also the width of the resonances, which are however included in the numerical calculations presented below. In Eq. (21a), y_b and m_b are the b-quark Yukawa coupling and mass, respectively. The annihilation cross section of DM into a ZZ pair can be obtained from Eq. (21b) by replacing $g \to g' = g/(\sqrt{2} \cos\theta_W)$ and $m_W \to m_Z$. The annihilation cross section into $h^+ h^-$ can be obtained from Eq. (21c) by replacing $\lambda_6 \to \lambda_7$ and $m_{k^{++}} \to m_{h^+}$. Note that the full annihilation cross section $\chi\chi \to \rho\rho$ receives also contributions from an s-channel diagram, whereas Eq. (21d) shows only the t- and u-channel contribution for simplicity. This is the only case without the suppression by the Higgs masses $m_{H_{1,2}}$.

All annihilation cross sections are proportional to the coupling λ_χ, however, in Eq. (21c) λ_χ appears together with the VEV w and has been absorbed into the DM mass via Eq. (18). Annihilations into SM particles are controlled by the Higgs portal, and therefore proportional to the mixing parameter $\sin^2 2\alpha$. In contrast, annihilations into the Majoron and the Zee–Babu scalars survive even in the case of no mixing ($\sin 2\alpha = 0$, or $\lambda_5 = 0$). Hence, these annihilation channels still provide a mechanism to generate the correct relic abundance at the electro-weak scale for $\sin 2\alpha = 0$, despite of χ_i's being completely decoupled from the SM.

We use the micrOMEGAs software package [52] to calculate the relic density $\Omega_{DM} = \Omega_{\chi_1} + \Omega_{\chi_2}$. For a given set of parameters, the left column of plots in Fig. 1 shows the relative contribution of the various annihilation channels to the relic abundance. The middle and the right panels of the figure show the coupling λ_χ and the singlet VEV w which are needed to obtain the correct relic abundance. Note that for a given DM mass m_χ, λ_χ and w are simply related by Eq. (18). We have chosen representative values for the two scalar masses $m_{H_1} = 300$ GeV and $m_{H_2} = 120$ GeV, but we have verified that our conclusions do not depend on this specific choice and hold within the full range of "reasonable" values for the Higgs masses. The couplings $\lambda_\mu, \lambda_{3,4,6,7,8,9,10}$ have been set to one. Again we have checked that random values in the range from 0.1 to 1 give qualitative similar results.

The upper two rows of plots in Fig. 1 correspond to a relatively large Higgs mixing angle of $\sin\alpha = 0.7$ and there is a large

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[2] Note that there are more unbroken accidental symmetries in the Lagrangian. For example there is a Z_3 symmetry $N_1 \to \omega N_1, N_2 \to \omega^2 N_2$ with $\omega^3 = 1$. Another example is an additional $U(1)$ symmetry with opposite charges for N_1 and N_2 but all other fields uncharged. Those accidental symmetries emerge due to the $B - L$ charge assignments of $N_{1,2}$ and they ensure that no Majorana mass term is generated for them after $B - L$ breaking.
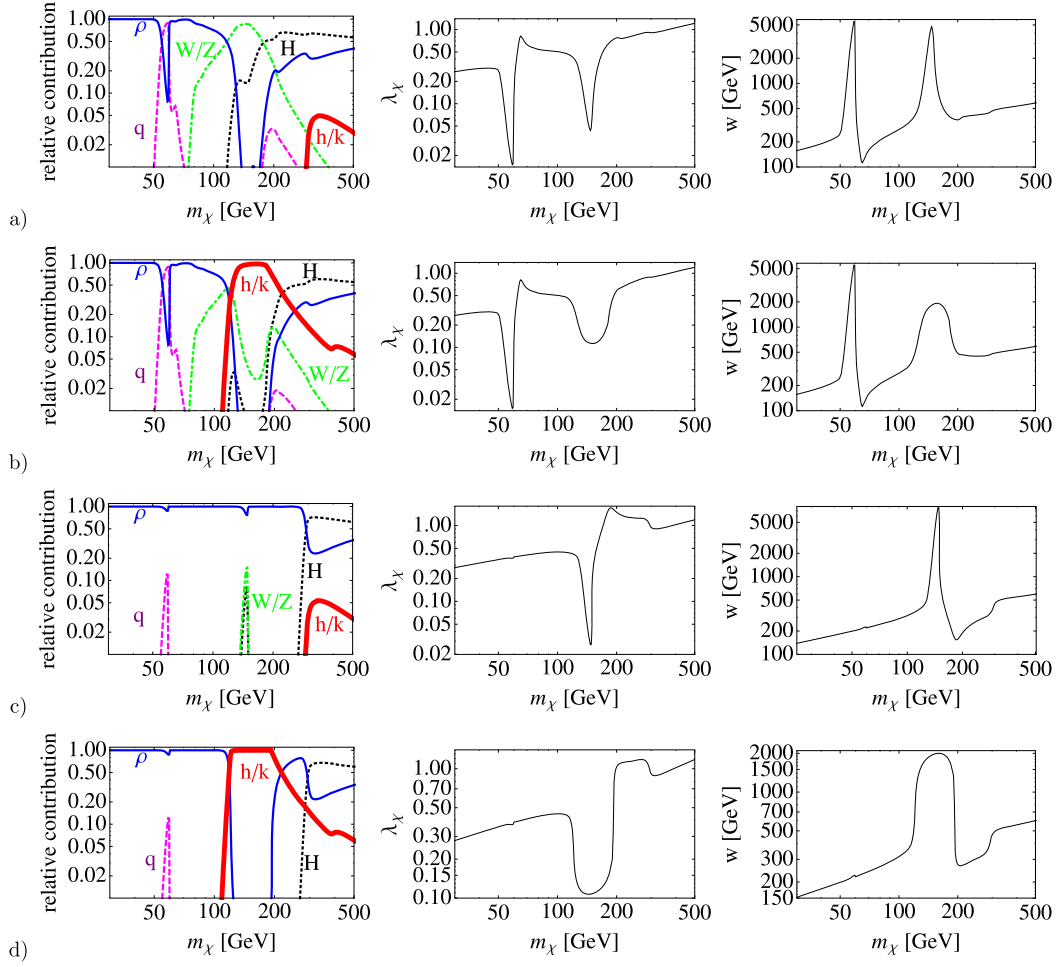
Fig. 1. Left: Relative contribution of annihilation channels to the relic DM abundance. Shown are annihilations into H_1 and H_2 ("H"), quarks ("q"), W and Z bosons ("W/Z"), Majorons ("ρ") and Zee–Babu scalars ("h/k"). Middle and right columns show the DM scalar coupling λ_χ and singlet VEV, respectively, which lead to the correct relic DM density. We assume $m_{H_1} = 300$ GeV, $m_{H_2} = 120$ GeV, and all Zee–Babu scalar couplings are set to unity. Rows a) and b) are for a Higgs mixing angle $\sin\alpha = 0.7$, rows c) and d) for $\sin\alpha = 0.01$. Rows a) and c) are for Zee–Babu scalar masses $m_{h^+} = 300$ GeV, $m_{k^{++}} = 800$ GeV, rows b) and d) for $m_{h^+} = 120$ GeV, $m_{k^{++}} = 400$ GeV.

coupling of DM to the SM. We find that depending on the DM mass various annihilation channels are important and the two resonances at $m_\chi \simeq m_{H_{1,2}}/2$ are clearly visible. In the lower two rows we use the same parameters but a small Higgs mixing $\sin\alpha = 0.01$. In this case the mass eigenstate H_1 practically coincides with the singlet φ and therefore only the resonance corresponding to H_1 exchange occurs. Furthermore, the coupling to SM particles is suppressed and the relic density is provided only by annihilations into the massless Majoron or, if kinematically accessible, into H_1 and the Zee–Babu scalars $k^{\pm\pm}$, $h^\pm$.

In Fig. 1 we compare also two assumptions on the masses of the Zee–Babu scalars. For masses larger than the Higgs masses annihilations into $k^{\pm\pm}$ and $h^\pm$ are sub-dominant, but if one of the charged scalars is lighter than one of the neutral scalars they can dominate DM annihilations, as visible in rows b) and d). Note also the modified shape of the curves close to the resonance if the dominant annihilation channel is into charged scalars. While the shape of the resonance of the cross section itself remains the same, the different shape for λ_χ which gives the correct relic abundance follows from a different dependence of the annihilation cross section on λ_χ and m_χ visible from Eq. (21c), which shows an m_χ proportionality, compared to the typical λ_χ^2/m_χ proportionality of other channels.

The middle and right columns of plots in Fig. 1 show that away from the resonances the correct relic abundance is obtained for

couplings in the perturbative range, $0.5 \lesssim \lambda_\chi \lesssim 1$, and VEVs between 200 and 500 GeV. Close to the resonances the enhancement of the annihilation cross section has to be compensated by a large VEV (which corresponds to small λ_χ) in order to maintain the correct relic abundance. Note that if the symmetry of the model was gauged, there would be a Z' gauge boson with a mass set by w. Since Z' searches require the mass of such a new gauge boson to be above few TeV, e.g. [53–56], gauged versions of this model would be confined to the resonance regions, see Ref. [17] for an example of such a model. It is an advantage of the global symmetry considered here, that the breaking scale can be lower, and therefore this model does not suffer from the need of tuning the DM mass close to half of the mass of one of the Higgs mass states.

3.2. DM direct detection

DM scattering on nuclei relevant for direct detection is mediated via t-channel exchange of the Higgs mass eigenstates H_1 and H_2. Hence, scattering is spin-independent. The elastic scattering cross section σ_p of χ off a proton p is obtained as

$$\sigma_p = \frac{\lambda_\chi^2 \sin^2 2\alpha}{4\pi} m_{\text{red}}^2 \left(\frac{1}{m_{H_1}^2} - \frac{1}{m_{H_2}^2} \right)^2 g_{Hp}^2, \tag{22}$$
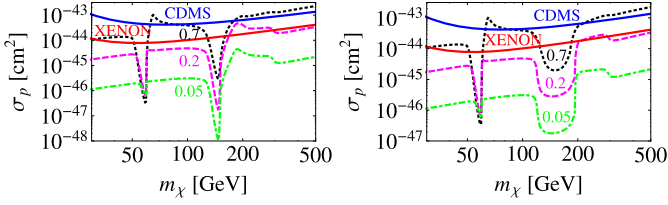
Fig. 2. Elastic scattering cross section σ_p of χ off a proton p for $m_{H_1} = 300$ GeV and $m_{H_2} = 120$ GeV and mixing angles $\sin\alpha = 0.7, 0.2, 0.05$, according to the label. The singlet VEV w has been chosen in order to obtain the correct relic DM abundance. The left (right) panel corresponds to masses of the Zee–Babu scalars of $k^{++} = 800$ (400) GeV and $h^+ = 300$ (120) GeV. Also shown are the CDMS [57] and XENON100 [58] exclusion limits.

where $m_{red} = m_p m_\chi / (m_p + m_\chi)$ is the reduced mass of the DM–proton system and

$$g_{Hp} = \frac{m_p}{v}\left[\sum_{q=u,d,s} f_q^{(p)} + \frac{2}{27}\left(1 - \sum_{q=u,d,s} f_q^{(p)}\right)\right], \qquad (23)$$

see e.g., [52] where also values for $f_q^{(p)}$ are given. We observe from Eq. (22) that the cross section is proportional to the Higgs mixing, since for $\alpha = 0$ the DM particle is decoupled from the SM and therefore the scattering cross section will vanish. Fig. 2 shows the DM cross section on a proton obtained with microMEGAs [52], where for given $m_{H_{1,2}}, \alpha$ and m_χ the coupling λ_χ (or equivalently the VEV w) has been chosen such that the correct relic abundance is obtained. Therefore the resonances in the annihilation cross section appear also in the scattering cross section. Although the Zee–Babu scalars k^{++} and h^+ do not contribute directly to the scattering cross section, they affect the annihilation cross section relevant for the relic abundance, and therefore also DM–nucleus scattering depends indirectly on their masses, compare left and right panel of Fig. 2. While part of the parameter space for large values of $\sin\alpha$ is already excluded by present bounds [57–59], the cross section can always be suppressed by making α small. Since generically mixing should be sizable, one may expect observable signals in direct detection searches from this model, although they are not guaranteed.

3.3. DM indirect detection

Let us comment briefly on possible signatures of our model for indirect searches for DM. The dominating branching fractions of the various DM annihilation channels are illustrated in Fig. 1. Generically our DM particles annihilate into a combination of gauge bosons and Higgs, leading to a rather typical mix of final annihilation products, with generic features common to many DM candidates produced thermally. In cases where DM annihilations into the Majoron dominate, visible annihilation products can be highly suppressed, which will lead to very small signals for indirect detection.

If the charged scalars are not too heavy there might be a region in the parameter space where DM annihilates dominantly into $k^{\pm\pm}$ and/or $h^\pm$, which further will decay into charged leptons and neutrinos. In this case one might think that our model could explain anomalies in the cosmic electron/positron fluxes [60,61], while respecting tight constraints for anti-protons. Note however, that a DM explanation of those excesses requires annihilation cross sections several orders of magnitude larger than the one needed for the thermal relic density. In our model no mechanism exists to enhance the cross section. Sommerfeld enhancement, for example, would require typically light (GeV-like) mediator particles, which are not present in our model. Therefore, attempting to explain those anomalies within our framework would require an exten-

sion of the model. However, those anomalies may very well be of astrophysical origin.

3.4. DM self-interactions

As has been noted already in [33] the exchange of the massless Majoron can lead to a long-range force. Hence, it is important to check whether this leads to conflicts with bounds on DM self-interactions, see e.g. [62–64]. Due to the pseudo-scalar nature of the Majoron, the interaction is spin-dependent and the induced potential $V(r)$ falls off like $1/r^3$, in analogy to the one-pion exchange potential in nuclear physics [33,65]:

$$V(r) = \frac{3\lambda_\chi^2}{16\pi\, w^2 r^3}\big[3(\hat{r}\vec{\sigma_1})(\hat{r}\vec{\sigma_2}) - \vec{\sigma_1}\vec{\sigma_2}\big], \qquad (24)$$

where $\vec{\sigma_1}, \vec{\sigma_2}$ denote the spins of the two interacting particles. Using analogous estimations as in [62], we have checked that a potential of this type leads to less than one hard-scattering process during the age of the Universe for a typical DM particle in the halo of a Milky Way-like galaxy. Therefore, the Majoron induced self-interactions are in agreement with the corresponding bounds.

4. Discussion and conclusions

We have discussed in this Letter a common framework for neutrino mass and Dark Matter where both are related to the breaking of a global $U(1)$ symmetry associated with the conservation of $B - L$. From the perspective of neutrino masses, we studied a modification of the Zee–Babu model, where neutrinos obtain their masses at two-loop level after $U(1)_{B-L}$ breaking. The single and double charged scalars of this model provide interesting signatures at collider experiments as well as in searches for charged lepton-flavour violation [29–31]. It is then tempting to use the same global $U(1)_{B-L}$ symmetry in order to stabilize Dark Matter. To provide a Dark Matter candidate we introduced two Majorana fermions with non-standard $B - L$ quantum numbers, such that a Yukawa term with the SM Higgs doublet is forbidden. They form a Dirac particle and are stable because of an accidental Z_2 symmetry. Thanks to the non-standard quantum numbers mentioned above in this framework the Z_2 symmetry emerges as an unbroken remnant of $U(1)_{B-L}$ and has not to be imposed by hand.

Due to the spontaneous breaking of the global $U(1)$ symmetry we obtain a massless Goldstone boson, the Majoron. Quantum gravity effects are expected to break global symmetries and hence Planck suppressed operators may provide a mass to the Majoron. As discussed in [51] there are certain constraints on operators from the requirement that the Majoron does not over-close the Universe. On the other hand, if Majoron masses are induced in the keV range, the Majoron could even provide a warm component to the Dark Matter [66,67] with interesting signatures due to its decay [68,69]. Here we have neglected this contribution to Dark Matter, and furthermore, we have assumed that possible Planck scale suppressed operators leading to DM decay maintain a lifetime of the Dark Matter particle large compared to the age of the Universe.

There exists some literature on models for Dark Matter in the framework of gauged $U(1)_{B-L}$, see e.g. [17,18,70,23,71] and it is interesting to compare with the scenario with a global $U(1)_{B-L}$ discussed here. In the gauged version the requirement of anomaly cancellation provides a natural motivation to introduce right-handed neutrinos while we had to postulate new fermions N_i. In general the right-handed neutrinos will have the usual Yukawa interactions with the SM Higgs doublet, and hence, an additional Z_2 symmetry has to be postulated if one (or more) of

them should be the Dark Matter, while the protective symmetry arises more naturally in our case. Furthermore, a Z' gauge boson appears in the gauged version, which leads to additional constraints due to Z' searches at LEP, Tevatron, and LHC, e.g. [53–56]. For instance, from LEP-II Ref. [53] finds $w = M_{Z'}/(2g_{B-L}) \gtrsim 3$ TeV. Comparing this limit with the value of w needed to obtain the correct Dark Matter relic abundance in Fig. 1, we find that the collider bound can be satisfied only close to the s-channel resonances, in agreement with [17]. We have checked that also additional contributions to the annihilation cross section due to Z' exchange as well as $Z' - B$ kinetic mixing cannot change this result. Since the Dark Matter mass and the scalar masses are unrelated in these models the requirement for the resonance, $m_H \approx 2m_\chi$, appears as an unnatural coincidence. A recent example for a model with neutrino masses at one loop and a gauged $U(1)$ broken to a remnant Z_2 has been given in [24].

Before concluding let us stress that the model discussed here may serve as a specific example for a more general class of models, where Dark Matter is charged under some global $U(1)$ symmetry and coupled to the Standard Model via Higgs-mixing of a "dark scalar" whose VEV breaks the symmetry. Many aspects of the phenomenology discussed here will apply to such models. Furthermore, for models based on $B - L$, the neutrino mass mechanism is not specific to the Zee–Babu model and one may think of various other possibilities to generate neutrino masses due to the $U(1)_{B-L}$ breaking linked to Dark Matter in the way described here.

Acknowledgements

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