

Available online at www.sciencedirect.com

ScienceDirect

Procedia - Social and Behavioral Sciences 96 (2013) 2867 – 2883

Procedia
Social and Behavioral Sciences

13th COTA International Conference of Transportation Professionals (CICTP 2013)

Economic Model for Vehicle Ownership Quota Policies and Applications in China

Shanjiang Zhu^a, Longyuan Du^b, Chenfeng Xiong^c, Lei Zhang^{d*}

^aAssistant Professor, Department of Civil, Environmental, and Infrastructure Engineering, George Mason University, Ng, Suite 1300, George Mason University, MSN-6C1, Fairfax, VA 22030, USA

^bGraduate Research Assistant, Department of Civil and Environmental Engineering, University of Maryland, 1173 Glenn L. Martin Hall, College Park, MD 20742, USA

^cGraduate Research Assistant, Department of Civil and Environmental Engineering, University of Maryland, 1173 Glenn L. Martin Hall, College Park, MD 20742, USA

^dAssistant Professor, Department of Civil and Environmental Engineering, University of Maryland, 1173 Glenn L. Martin Hall, College Park, MD 20742, USA

Abstract

Traffic congestion has caused huge economic loss and environmental pollution every year. As a demand management policy to reduce congestion, vehicle ownership quota system that directly controls the number of vehicles on the road has recently been adopted in some metropolitan areas including Beijing and Shanghai. When it comes to implementation of quota system, Beijing uses the plate lottery system, so that everyone interested in owning a vehicle can participate and there's no monetary transaction in the process. Shanghai, on the other hand, uses the plate auction system and participants bid for the limited number of vehicle plates available. This paper aims at building a theoretical model that quantitatively analyzes the benefits of such policies. This study extends the joint decision model of vehicle ownership and mileage model, and applied compensating variation method to measure the net social impact change of the different quota systems. Under this proposed framework, a numerical demonstration is conducted.

© 2013 The Authors. Published by Elsevier Ltd. Open access under [CC BY-NC-ND license](http://creativecommons.org/licenses/by-nc-nd/4.0/).

Selection and peer-review under responsibility of Chinese Overseas Transportation Association (COTA).

Keywords: Transportation economics; Vehicle ownership quota; Policy; Welfare analysis

* Corresponding author. Tel.: +1-301-405-2881

E-mail address: lei@umd.edu

1. Introduction

Over the years, along with the progress of motorization and urbanization, traffic congestion has become an ever more serious issue worldwide, especially in the metropolitan areas. According to the 2011 TTI Annual Report Schrank et al. (2011) congestion in metropolitan areas in US results in 1.94 billion gallons in wasted fuel and \$100.9 billion in total cost. In Sao Paolo, Brazil, cost of traffic congestion is nearly 20 billion US dollars in 2008. China, as one of the fast growing economies in the world, is also witnessing more severe traffic congestion. Congestion cost in Beijing, Guangzhou and Shanghai, three of the largest metropolitan cities in China, is more than 200 CNY per person per month.

Under this background, various policy tools (e.g. priority transit, high occupancy vehicle lanes, congestion pricing) have been proposed and implemented to mitigate congestion by allocating scarce road space more efficiently. Congestion pricing, as one of the means for transportation demand management, has been implemented in a number of urban areas including Singapore, London, and Stockholm. Singapore first introduced the Area License Scheme (ALS) in 1975. The scheme requires vehicles entering the central "Restricted Zone" during peak hours to purchase a license. Starting from 1998, ALS has been replaced by the new Electrical Road Pricing scheme (ERP). Using microwave technology, tolls are collected on arterials and expressways that vary by time of day. Stockholm and London have implemented similar congestion charging schemes using automatic plate number recognition technologies.

More recently, some cities, especially in developing countries, started looking at alternatives that directly control the number of vehicles on the road. In 1989, Mexico City imposed a regulation that limits each car from driving on a specific day of the week according to their license plate number ("Day without a Car") (Eskeland & Feyzioglu, 1997). Similar road space rationing policies have been implemented in Sao Paolo and Beijing during the 2008 Olympic Game (Lim, 2011).

Also, more drastic policies that restrict vehicle ownerships have been adopted by several cities. For example, Singapore implemented the Vehicle Quota System (VQS) in 1990 (Barter, 2005). The VQS sets a quota to the number of new vehicles to be registered in Singapore each year. Households need to bid for a Certificate of Entitlement (COE) through online auction system in order to own a vehicle. With this quota system and road pricing policy, fewer than 30% of Singaporeans own a vehicle. Along with the fast economic development, China is also experiencing rapid increase in motor vehicle ownership. Similar to Singapore, started from 1994, Shanghai implemented the New Plate Auction system, by auctioning the new vehicle plate license. Instead of auctioning, Beijing started implemented a plate lottery system in 2010, limiting the number of plates issued to passenger vehicles to 20,000 per month. Vehicle ownership quota as a direct travel demand management tool could be adopted by more metropolitan areas, either as a permanent policy or as a temporary policy during special events. Under this background, it is important to look into the impacts of such policies.

Both theoretical and empirical studies on road pricing have been abundant in the literature (e.g. Brownstone & Small, 2005; Verhoef, 2002; Zhang & Ge, 2004). In contrast, fewer studies have been conducted to evaluate rationing as a policy tool to address congestion problem, although its role in resource allocation has been widely discussed in other industries (e.g. Evans, 1983). A pioneering study in transportation by Daganzo (1995) shows that a Pareto optimum congestion reduction scheme can be built by combining rationing and pricing. Nakamura & Kockelman (2002) tested this theoretical framework on the San Francisco Bay Bridge corridor. The authors pointed out several limitations in current studies and recommended further research in this field. Smith & Chin (1997) concluded that the VQS in Singapore is effective in controlling traffic, but it has to be complemented by road pricing to yield higher net social impact. Wang (2010) did a qualitative investigation into different traffic demand management policies including ownership quota systems. Eskeland & Feyzioglu (1997) did an empirical study in the Mexico City "Day without a Car" policy, and found that households purchase a second car to get around this policy, making the congestion reduction effect insignificant. Zhu et al. (2012) examined the vehicle rationing and vehicle ownership policy with a theoretical model, under the assumption that users are

homogeneous. Their conclusion is that under this assumption, vehicle ownership quota will always generate a positive net social impact, while vehicle usage restriction with induced demand considered will result in a negative net social impact. Furthermore, congestion pricing will outperform both rationing policies in terms of net social impact.

This study extends the theoretical model of Zhu et al. (2012) to evaluate the welfare effects of the two quota policies and relaxed the assumption to heterogeneous user group. In order to consider the quota policy as a transportation demand management tool that mitigates traffic congestion, this study follows the indirect utility approach initiated by Dubin & McFadden (1984), and extends the model previously proposed by De Jong (1990) to consider the joint decisions of vehicle ownership and usage under the two vehicle quota systems. These demand-side models are then combined with supply-side models to capture the market equilibrium. Net social impact is then calculated for each of the quota regulations as an evaluation of the effectiveness.

2. Theoretical Framework

For a traveler who is facing various rationing policies, the decision of owning a vehicle and the decision of using a vehicle are interrelated. Households will evaluate their car buying decision based on how congested the roads are. If the operations cost gets higher, it will be less likely for a family to own a vehicle. On the other hand, after any kind of ownership quota policy is implemented, mitigation effect on traffic congestion will result in induced demand, and more households will be willing to own vehicles. Therefore, to correctly capture these behavioral dynamics in reaction to rationing policy, it is crucial to jointly model vehicle ownership and usage decisions.

The performance of the market dynamics should also depend on travel supply (the marginal cost of traveling). Effect of quota policies is dependent of the level of congestion in cities. To correctly model induced demand and capture network equilibria under different policies, we should integrate travel demand and supply models and generate market equilibrium in the analysis.

2.1. Travel demand model

This study follows the indirect utility approach initiated by Dubin & McFadden (1984) and extended by researchers including De Jong (1990); Goldberg (1998); Hensher et al. (1992); Mannering & Winston (1985); and West (2004) because of its solid foundation in consumer behavior theory (for a comprehensive review, see De Jong et al. 2004). We consider a household who seeks to maximize its utility under budget constraints. We consider two goods: vehicle usage (A) and all other goods (X). The household faces a discrete choice of owning vehicles and a continuous choice of vehicle usage conditional on the ownership choice. This joint decision gives the consumption of vehicle usage and determines vehicle ownership. The consumption of vehicle usage and all other goods yield positive marginal utility.

To derive the model, we must first specify a functional form for conditional indirect utility, which has a one-to-one correspondence with the demand function. A demand function linear in income and price has been used by Goldberg (1998); Hensher et al. (1992); Mannering & Winston (1985); West (2004). However, the demand could become negative as price increases under this specification. Also, the linear model cannot consider the option of not owning a vehicle. In contrast, De Jong (1990, 1996) adopted a double log specification, which is bounded to be positive and performs well in an early empirical study. Thus utility of not owning a vehicle can be derived. Therefore, this paper follows the second approach, while maintaining consistency in connotation with other literature whenever possible.

From De Jong (1990), we assume the demand for driving A_i units of distance (e.g. measured in annual Vehicle Mile Traveled (VMT)) by a household i with annual income Y_i is determined by:

$$\ln A_i = \alpha_i \ln(Y_i - C) - \beta_i p + e_i \quad (1)$$

Where p is the operating cost per mile for the vehicle; C represents the annualized capital cost of owning a car; e_i summarizes unobserved household socio-economic and demographic characteristics, and $e_i \sim N(0, \sigma)$. α_i and β_i are parameters.

Following Burtless & Hausman (1978), the corresponding indirect utility function is:

$$V(p, Y_i - C) = \frac{1}{\beta_i} \exp(e_i - \beta_i p) + \frac{1}{1 - \alpha_i} (Y_i - C)^{1 - \alpha_i} \quad (2)$$

We can easily verify this correspondence by applying Roy's identity to Equation 2, which gives the demand function 1. To derive the utility of owning no vehicle, De Jong (1990) observed that the optimal decision would be to not drive at all when operating cost per mile p goes to infinite. Therefore, we have:

$$\lim_{x \rightarrow \infty} V(p, Y_i) = U(0, Y_i) \quad (3)$$

Which yields:

$$U(0, Y_i) = \frac{1}{1 - \alpha_i} Y_i^{1 - \alpha_i} \quad (4)$$

Thus, the probability of owning a car is:

$$P(p) = P\left\{ \frac{1}{1 - \alpha} (Y - C)^{1 - \alpha} + \frac{1}{\beta} \exp(e - \beta p) \right\} \geq \frac{1}{1 - \alpha} Y^{1 - \alpha} \quad (5)$$

Denote:

$$\eta = \log\left[Y^{1 - \alpha} - (Y - C)^{1 - \alpha} \right] - \log(1 - \alpha) + \log \beta \quad (6)$$

Then, the probability function (5) can be simplified:

$$P(p) = 1 - \Phi\left(\frac{\eta + \beta p}{\sigma} \right) \quad (7)$$

In this study, it is assumed that households have a uniform income level Y , although this framework is capable to consider heterogeneous income level. Also we will first only consider one traveler/vehicle per household and leave the complexity of intra-household travel behavior for future research. In the following sections, the word traveler or household will be used interchangeably. Also, because of (1), average VMT for the driving population is:

$$\tilde{A}(p) = E(A | e_i \geq \eta + \beta p) = \frac{e^{\frac{\sigma^2}{2} - \beta p} \left[1 - \Phi\left(\frac{\eta + \beta p - \sigma^2}{\sigma} \right) \right]}{1 - \Phi\left(\frac{\eta + \beta p}{\sigma} \right)} \quad (8)$$

Combining (7) and (8), the aggregated demand is:

$$q(p) = H(Y - C)^{1-\alpha} e^{\frac{\sigma^2}{2} - \beta p} \left[1 - \Phi \left(\frac{\eta + \beta p - \sigma^2}{\sigma} \right) \right] \tag{9}$$

Figure 1 describes the aggregated demand curve:

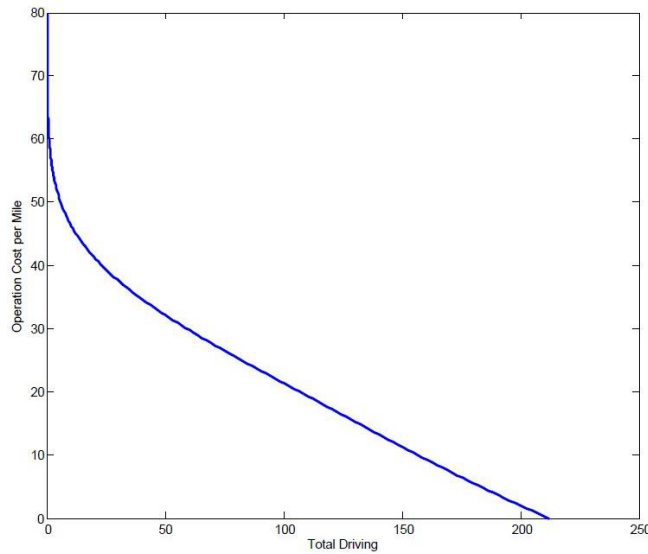


Fig. 1. Aggregate demand curve with homogenous travelers

2.2. Travel supply model

To maintain the tractability of the analysis, we consider a stylized network with only one link and one origin-destination pair. In the case of urban transportation network, when the trips are relatively evenly spread on the roads and the congestion is relatively similar among different links, such stylized network with one OD pair and a generalized capacity can be viewed as an abstract model for this urban transportation network. It should be noted that under model this framework, a real network supply model can potentially be implemented, and will be discussed briefly in Section 5.

The total demand is carried by this idealized network, with road capacity (F). The generalized travel cost p is then a function of travel demand q .

$$p = \phi t(q) \tag{10}$$

where ϕ is the value of time. To evaluate policies on the network with different levels of congestion, this study employs a generalized Bureau of Public Roads (BPR) function to describe the supply side. The total travel cost p is thus:

$$p = \phi T_0 \left(1 + \xi \left(\frac{q}{F} \right)^\phi \right) \tag{11}$$

where: T_0 captures the free flow time. ξ and φ are parameters whose values allow us to model networks with different congestibility (travel time elasticity). In the most common BPR function, $\xi = 0.15$, $\varphi = 4$. As this paper focuses on rationing policies and their impacts on market equilibrium, we assume other long term costs such as fuel price remain constant and become part of ϕT_0 .

2.3. Network equilibrium

Based on the demand and supply models introduced in the previous subsection, this subsection analyzes market equilibrium conditions. The equilibrium point (p_0, q_0) is solved by the following equation set:

$$\begin{cases} q_0 = H(Y - C)^\alpha e^{\sigma^2/2 - \beta p_0} \left[1 - \Phi\left(\frac{\eta + \beta p_0 - \sigma^2}{\sigma}\right) \right] \\ p_0 = \phi T_0 \left(1 + \xi \left(\frac{q_0}{F} \right)^\varphi \right) \end{cases} \tag{12}$$

Figure 2 illustrates the equilibrium status when demand and supply curves intersect.

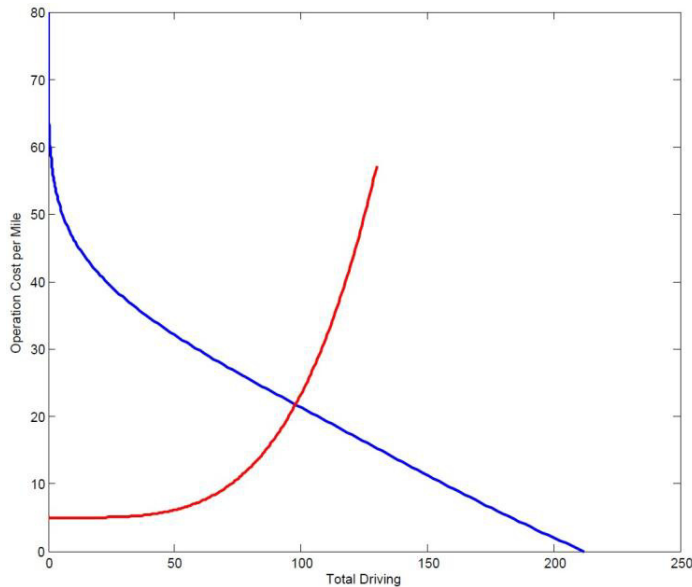


Fig. 2. Network equilibrium

3. Models for Vehicle Plate Lottery and Plate Auction Policy Analysis

In this section, two distinct ownership quota systems will be analyzed, namely the plate lottery system and plate auction system. From Equation (12), initially at equilibrium status, $HP(p_0)$ of the households are willing to own a vehicle. The quota policy will restrict the vehicle ownership, issuing $\theta HP(p_0)$ vehicle plates. Here $\theta \in (0, 1)$ will be a policy variable the authority need to decide on.

3.1. A Compensating Variation (CV) approach for welfare analysis

In previous studies, researchers have used consumer surplus (CS) to conduct social welfare analysis. Consumer surplus is an approximation for the willingness-to-pay (WTP) welfare measurement. It has a very straightforward graph implementation, which is the triangular area below demand curve and above the price line. However in this study, since Equations (2) and (4) are different in functional form and are not continuous when $q \rightarrow 0$, consumer surplus is not applicable.

Instead in this study, compensating variation (CV) from Hicks (1946) is used. It is defined as the money to be taken away from (or paid to when it is negative) the individual after an economic change, that leaves the individual with the same utility value as before. Small & Rosen (1981) extended such welfare calculation method to discrete choice models. Mannering & Winston (1987) and Winston & Mannering (1984) have applied CV in transportation research regime.

3.2. Vehicle plate lottery system

In this model, zero entry fee to the lottery is always assumed. In fact, in places where the lottery is actually implemented, the entry fee is neglect-able comparing to household income. Also, the plates obtained from the lottery are not transferable. This prevents the underground trading of the plates.

Since $1 - \theta$ portion of the households who initially own vehicles will be rationed out, the roads will get less congested. Thus, more households will be interested in the joining the lottery. A policy maker can decide the population this plate lottery is open to. In the following deduction, two different implementations will be considered. First is that this lottery will be open to the general public. This is what the Beijing authority adopts.

Case 1: Lottery open to the general public

If the authority opens the plate lottery to everyone, a new equilibrium point (p^*, q^*) will be achieved. From Equation (7), $HP(p^*)$ will enter the lottery to win the $\theta HP(p_0)$ number of vehicle plates. From Equation (8), their average VMT is $\bar{A}(p^*)$. Thus, the new aggregated demand function will be:

$$q' = \theta H(Y - C)^{1-\alpha} \left[1 - \Phi \left(\frac{\eta + \beta p_0}{\sigma} \right) \right] \cdot \frac{e^{\frac{\sigma^2}{2} - \beta p} \left[1 - \Phi \left(\frac{\eta + \beta p - \sigma^2}{\sigma} \right) \right]}{1 - \Phi \left(\frac{\eta + \beta p}{\sigma} \right)} \tag{13}$$

The new equilibrium is given by the following equation set:

$$\begin{cases} q_0^* = \theta H(Y - C)^\alpha \left[1 - \Phi\left(\frac{\eta + \beta p_0}{\sigma}\right) \right] \frac{e^{\sigma^2/2 - \beta p_0} \left[1 - \Phi\left(\frac{\eta + \beta p_0^* - \sigma^2}{\sigma}\right) \right]}{1 - \Phi\left(\frac{\eta + \beta p_0^*}{\sigma}\right)} \\ p_0^* = \phi T_0 \left(1 + \xi \left(\frac{q_0^*}{F} \right)^\rho \right) \end{cases} \tag{14}$$

Since $p_0^* < p_0$ and $P(p_0^*) > P(p_0)$, the probability of winning the lottery is:

$$\varepsilon = \frac{\theta \left[1 - \Phi\left(\frac{\eta + \beta p_0}{\sigma}\right) \right]}{(\eta + \beta p_0^*) / \sigma} < \theta \tag{15}$$

Now the task is to calculate welfare impact of the policy. For each of the households who initially do not own vehicles but enter the lottery system now and actually win it, their $e_i \in [\eta + \beta p_0^*, \eta + \beta p_0]$. Since the probability of winning the lottery is given by Equation (15), the total number of such households is:

$$\varepsilon H \left(\Phi\left(\frac{\eta + \beta p_0}{\sigma}\right) - \Phi\left(\frac{\eta + \beta p_0^*}{\sigma}\right) \right) \tag{16}$$

Their individual i 's compensating variation gain is given by the following equation:

$$\frac{1}{1 - \alpha} Y^{1 - \alpha} = \frac{1}{1 - \alpha} (Y - CV_1^i - C)^{1 - \alpha} + \frac{1}{\beta} \exp(e_i - \beta p_0^*) \tag{17}$$

Thus, the CV change from this group of households is:

$$CV_1^0 = \varepsilon H \int_{\eta + \beta p_0^*}^{\eta + \beta p_0} \left\{ Y - C - \left[Y^{1 - \alpha} - \frac{1 - \alpha}{\beta} \exp(e_i - \beta p_0^*) \right]^{\frac{1}{1 - \alpha}} \frac{1}{\sigma \sqrt{2\pi}} \right\} e^{\frac{e_i^2}{2\sigma^2}} de_i \tag{18}$$

Then consider the households who initially own vehicles, their $e_i \in [\eta + \beta p_0, +\infty)$. And the total number of such households is:

$$H \left(1 - \Phi\left(\frac{\eta + \beta p_0}{\sigma}\right) \right) \tag{19}$$

Thus, for the $1-\varepsilon$ portion of them who fail the lottery, their individual compensating variation is given by the following equation:

$$\frac{1}{1-\alpha}(Y-C)^{1-\alpha} + \frac{1}{\beta} \exp(e_i - \beta p_0) = \frac{1}{1-\alpha}(Y-CV_2^i)^{1-\alpha} \tag{20}$$

And again by aggregating individuals' CV, the total CV change from this group of households is:

$$CV_2^0 = (1-\varepsilon)H \cdot \int_{\eta+\beta p_0}^{+\infty} \left\{ Y - \left[(Y-C)^{1-\alpha} + \frac{1-\alpha}{\beta} \exp(e_i - \beta p_0) \right]^{\frac{1}{1-\alpha}} \frac{1}{\sigma\sqrt{2\pi}} \right\} e^{\frac{e_i^2}{2\sigma^2}} de_i \tag{21}$$

Then for the ε portion of them who actually win the lottery, their individual compensating variation is given by the following equation:

$$\frac{1}{1-\alpha}(Y-C)^{1-\alpha} + \frac{1}{\beta} \exp(e_i - \beta p_0) = \frac{1}{1-\alpha}(Y-C-CV_3^i)^{1-\alpha} + \frac{1}{\beta} \exp(e_i - \beta p_0^*) \tag{22}$$

By aggregating individuals' CV, the total CV change from this group of households is:

$$CV_3^0 = \varepsilon H \cdot \int_{\eta+\beta p_0}^{+\infty} \left\{ Y-C - \left[(Y-C)^{1-\alpha} + \frac{1-\alpha}{\beta} e^{e_i} (e^{-\beta p_0^*} - e^{-\beta p_0}) \right]^{\frac{1}{1-\alpha}} \frac{1}{\sigma\sqrt{2\pi}} \right\} e^{\frac{e_i^2}{2\sigma^2}} de_i \tag{23}$$

The CV change for the whole society is thus calculated by adding three portions together:

$$CV^0 = CV_1^0 + CV_2^0 + CV_3^0 \tag{24}$$

In Equation (24), the first and third terms on the right hand side are positive because of the mitigation of congestion by the implementation of the quota system. The second term is negative due to the depriving from owning a vehicle.

Case 2: Lottery open only to previous owners

Another alternative for the lottery implementation is to open the system only to those who own vehicles in the initial equilibrium. This policy can be an applicable way for the authority to control the number of vehicles in an already well developed metropolitan area. So the authority stops issuing new plates and conduct a lottery within the current driving population. Government will buy back the vehicles of households who lose the lottery. Such policy has not been implemented in any area yet. This section focuses on the theoretical point of view and serves as a comparison to the lottery system that opens to the general public.

For a given operation cost, the average VMT for household i who wins the lottery is:

$$e^{\sigma^2/2-\beta p} \left(1 - \Phi \left(\frac{\eta + \beta p_0 - \sigma^2}{\sigma} \right) \right) \tag{25}$$

Thus the new aggregated demand function is given by:

$$q' = \theta H (Y - C)^{1-\alpha} e^{\frac{\sigma^2}{2} - \beta p} \left[1 - \Phi \left(\frac{\eta + \beta p_0 - \sigma^2}{\sigma} \right) \right] \tag{26}$$

Under this policy, the equilibrium is given by the following equation set:

$$\begin{cases} q_0^* = \theta H (Y - C)^{1-\alpha} e^{\sigma^2/2 - \beta p_0^*} \left[1 - \Phi \left((\eta + \beta p_0 - \sigma^2) / \sigma \right) \right] \\ p_0^* = \phi T_0 \left(1 + \xi (q_0^* / F)^\rho \right) \end{cases} \tag{27}$$

For the household i in the θ portion who win the lottery, its compensating variation is given by:

$$\frac{1}{1-\alpha} (Y - C)^{1-\alpha} + \frac{1}{\beta} \exp(e_i - \beta p_0) = \frac{1}{1-\alpha} (Y - C - CV_\theta^i)^{1-\alpha} + \frac{1}{\beta} \exp(e_i - \beta p_0^*) \tag{28}$$

The aggregate CV change from this group of households is:

$$CV_\theta^0 = \theta H \cdot \int_{\eta + \beta p_0}^{+\infty} \left\{ Y - C - \left[(Y - C)^{1-\alpha} - \frac{1-\alpha}{\beta} e^{e_i} (e^{-\beta p_0^*} - e^{-\beta p_0}) \right]^{\frac{1}{1-\alpha}} \frac{1}{\sigma \sqrt{2\pi}} \right\} e^{\frac{e_i^2}{2\sigma^2}} de_i \tag{29}$$

For the households i in the $1 - \theta$ portion who do not win the lottery, its compensating variation is given by:

$$\frac{1}{1-\alpha} (Y - C)^{1-\alpha} + \frac{1}{\beta} \exp(e_i - \beta p_0) = \frac{1}{1-\alpha} (Y - CV_{1-\theta}^i)^{1-\alpha} \tag{30}$$

The aggregate CV change from this group of households is:

$$CV_{1-\theta}^0 = (1-\theta) H \cdot \int_{\eta + \beta p_0}^{+\infty} \left\{ Y - \left[(Y - C)^{1-\alpha} + \frac{1-\alpha}{\beta} e^{e_i - \beta p_0} \right]^{\frac{1}{1-\alpha}} \frac{1}{\sigma \sqrt{2\pi}} \right\} e^{\frac{e_i^2}{2\sigma^2}} de_i \tag{31}$$

The CV change for the whole society is thus calculated by adding these two portions together:

$$CV^0 = CV_\theta^0 + CV_{1-\theta}^0 \tag{32}$$

3.3. Vehicle plate auction system

In this section, we will consider a sealed-bid auction system that the first $\theta HP(p_0)$ best prices will win the auction, and therefore get to own a vehicle.

To quantitatively model this system, it is assumed in this work that the population size H is large, and for each individual i , he/she knows his/her own e_i as well as the distribution of e_i ($N(0, \sigma)$ in this case).

The valuation for the plate $V(e_i)$ for each individual i is:

$$V(e_i) = Y - C - \left(Y^{1-\alpha} - \frac{1-\alpha}{\beta} e^{e_i - \beta p_a^*} \right)^{\frac{1}{1-\alpha}} \tag{33}$$

where p_a is the operation cost per mile; and the * indicates equilibrium status.

When the price of the plate is $V(e_i)$, household i is indifferent between owning and not owning a vehicle.

Based on these assumptions, in the equilibrium status, the plate will go to those who has higher $V(e_i)$ which is equivalent to the higher e_i . the individuals who will get the plate has $e_i \geq e_a^*$. e_a^* is given by:

$$\frac{1 - \Phi(e_a^* / \sigma)}{1 - \Phi\left(\frac{\eta + \beta p_0}{\sigma}\right)} = \theta \tag{34}$$

e_a^* is solely determined by the initial equilibrium status before any quota policy as well as the quota portion θ .

It can be shown that under equilibrium status, households with $e_i \geq e_a^*$ will all submit $V(e_a^*)$, while those with $e_i \leq e_a^*$ will submit $V(e_a)$. As a matter of fact, since for any individual household with $e_i \geq e_a^*$, paying $V(e_a^*)$ guarantees a plate. Paying more than that amount will decrease its utility. Paying less than that amount results in failing to get the plate. Either way, this household would be worse off should it deviate with this strategy. For any individual household with $e_i \leq e_a^*$, submitting $V(e_i)$ means it will not get the plate. Paying more than $V(e_i)$ means it will be worse off than not owning a vehicle even if it wins the lottery. Paying less than $V(e_i)$, and this household still don't get the plate. This means those households won't have incentive to deviate with this strategy. Thus Nash equilibrium has been found. From empirical results, the average and lowest winning bid of the Shanghai vehicle plate auction do not differ a lot. This implies that the assumptions and

deduction of the auction model has the representation power of the real world case. Denote $V(e_a^*) = V$. The equilibrium status (p_a^*, q_a^*, V) can be calculated as:

$$\begin{cases} q_a^* = H(Y - V - C)^{1-\alpha} e^{\sigma^2/2 - \beta p_a^*} [1 - \Phi((e_a^* - \sigma^2) / \sigma)] \\ p_0^* = \phi T_0 (1 + \xi (q_a^* / F)^\varphi) \\ \frac{1}{1-\alpha} (Y - V - C)^{1-\alpha} + \frac{1}{\beta} e^{e_a^* - \beta p_a^*} = \frac{1}{1-\alpha} Y^{1-\alpha} \end{cases} \tag{35}$$

For each of the households who initially own vehicles but didn't win the auction, their $e_i \in [\eta + \beta p_0, e_a^*]$. Their individual compensating variation gain $CV_{1-\theta}^i$ is given by the following equation:

$$\frac{1}{1-\alpha} (Y - C)^{1-\alpha} + \frac{1}{\beta} \exp(e_i - \beta p_0) = \frac{1}{1-\alpha} (Y - CV_{1-\theta}^i)^{1-\alpha} \tag{36}$$

By aggregating the $CV_{1-\theta}^i$'s, the CV change from this group of households is:

$$CV_{1-\theta}^a = H \cdot \int_{\eta + \beta p_0}^{e_a^*} \left\{ Y - \left[(Y - C)^{1-\alpha} - \frac{1-\alpha}{\beta} e^{e_i - \beta p_0} \right]^{1-\alpha} \frac{1}{\sigma \sqrt{2\pi}} \right\} e^{\frac{e_i^2}{2\sigma^2}} de_i \tag{37}$$

For each of the households who wins the auction, their $e_i \geq e_a^*$. Their individual compensating variation gain CV_θ^i is given by the following equation:

$$\frac{1}{1-\alpha} (Y - C)^{1-\alpha} + \frac{1}{\beta} \exp(e_i - \beta p_0) = \frac{1}{1-\alpha} (Y - S - C - CV_\theta^i)^{1-\alpha} + \frac{1}{\beta} \exp(e_i - \beta p_0^*) \tag{38}$$

By aggregating the CV_θ^i 's, the CV change from this group of households is:

$$CV_\theta^a = H \cdot \int_{e_a^*}^{+\infty} \left\{ Y - S - C - \left[(Y - C)^{1-\alpha} - \frac{1-\alpha}{\beta} e^{e_i} (e^{-\beta p_0^*} - e^{-\beta p_0}) \right]^{1-\alpha} \frac{1}{\sigma \sqrt{2\pi}} \right\} e^{\frac{e_i^2}{2\sigma^2}} de_i \tag{39}$$

Also, the revenue generated from the auction is:

$$R = \theta S \left[1 - \Phi \left(\frac{\eta + \beta p_0}{\sigma} \right) \right] \tag{40}$$

The welfare change for the whole society is thus:

$$CV^a = CV_{\theta}^a + CV_{1-\theta}^a + R \tag{41}$$

4. Numerical Example

In this section, numerical analysis will be conducted to look into the welfare implications of the quota systems. This study uses parameters reported by De Jong (1990):

- Income elasticity of driving: $\alpha = 0.49$
- Price elasticity of driving: $\beta = 0.028$
- Average annual income $Y = 35000$
- Vehicle Price averaged in years $C = 2536$

We also set the parameters for supply-side function as: $\varphi = 4$ and $\zeta = 0.15$ in Equation (11), which is a typical BPR function. We assume the free flow operation cost γT_0 to be \$5 and capacity F to be $30H$ for convenience.

With these parameters set up, the initial equilibrium can be calculated by Equation (12). Under equilibrium status, 59% of the households will choose to own a vehicle. The operation cost per mile $p_0 = 27.5$. Previous sections has discussed welfare change calculations of the two plate lottery systems (open to general public and open only to previous vehicle owners) and the plate auction system. Since all three policies limits the number of vehicles on the road, a good comparison of the three will be the compensating variation under the same quota percentage θ .

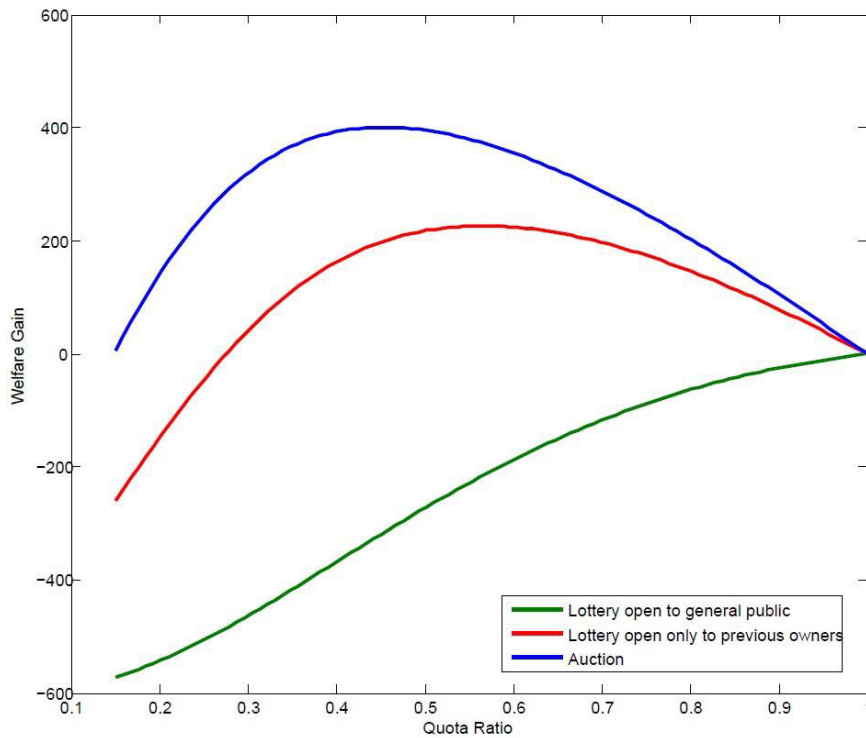


Fig. 3. Compensating variation comparison of the policies

From Figure 3, we can see that lottery that opens only to previous vehicle owners always outperform lottery that opens to the general public in terms of welfare gain. This is because more households with lower preference of driving e_i will be attracted to the lottery knowing that there will be less congestion. In fact, when the quota ratio is 0.8, which means to limit the number of vehicles to 80%, the probability of winning this lottery is only 47%. As lottery is purely random, the 53% of the households who have high preference of driving but cannot own vehicles will contribute a big welfare loss, comparing to the welfare gain generated by the 47% of the households with lower preference of driving and then win the lottery. The welfare gain from the households who don't previously own vehicles but enter and win the lottery now is $30.1H$, whereas the welfare loss from the households who previously own vehicles but don't win the lottery is as much as $321H$.

Plate auction system, comparing to both lottery systems, enjoys a higher welfare gain. The auction mechanism guarantees that households with higher preferences of driving will always get the plate. Such arrangement may result in less effective congestion mitigation, as the individuals with higher preference of driving will drive more. But since households will have to surrender part of their salary in order to get the plate, this will indirectly help reduce traffic congestion.

The mitigation effect of each of the policies can be examined in terms of the operation cost per mile at equilibrium. From Figure 4, lottery that opens to the general public is most effective in congestion mitigation, whereas auction will result in a relatively higher congestion level.

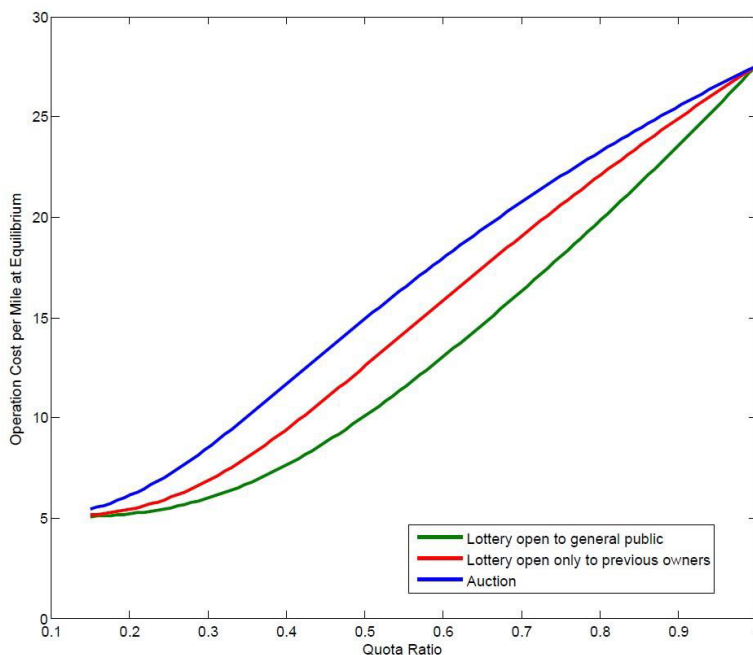


Fig. 4. Level of congestion comparison of the policies

From Figure 3, under this set of parameters, we can see that lottery system that opens to the general public will always generate negative net social impact. Next we look at another hypothetical supply model of a more congestible road with higher marginal cost. Figure 5 shows the net social impact under the set of supply curve parameters as $\varphi = 8$ and F remains $30H$. From Figure 5, we can see that lottery system that opens to general

public will be able to generate a positive welfare gain. Comparing Figure 3 and 5, we can observe that a more congestible supply model will offer a higher welfare gain.

Unlike lottery system, welfare gain of the auction system comes from the revenue collected from selling the vehicle plates. In fact, when looking all households will have a lower utility level after the auction policy is implemented. Figure 6 shows the revenue generated from auction under different quota ratios.

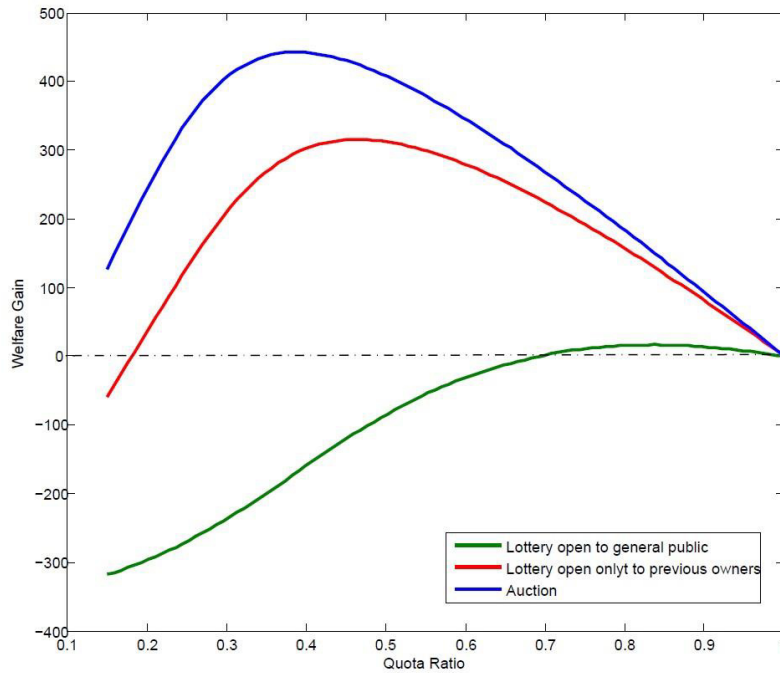


Fig. 5. Compensating variation comparison of the policies under highly congestible supply side parameters

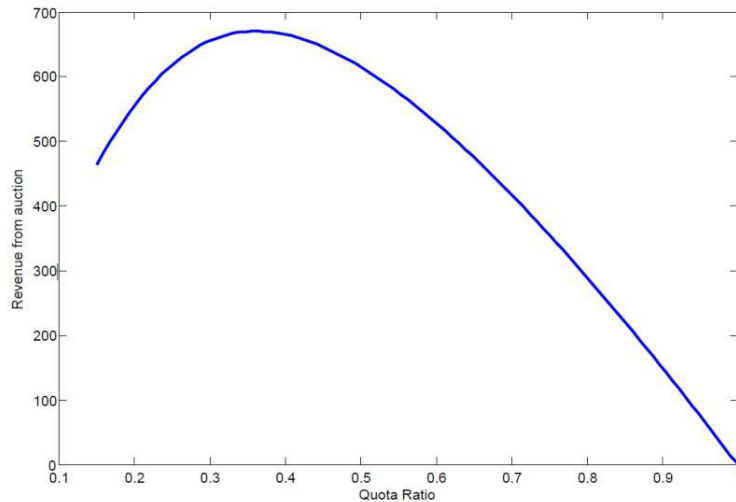


Fig. 6. Revenue generated from auction

5. Conclusion

Traffic congestion has caused huge economic loss and environmental pollution every year. As a transportation demand management policy to reduce congestion, vehicle ownership quota system, that directly controls the number of vehicles on the road, has recently been adopted in some metropolitan areas, including Beijing and Shanghai. When it comes to implementation of quota system, Beijing uses the plate lottery system, so that everyone interested in owning a vehicle can participate and there's no monetary transaction in the process. Shanghai, on the other hand, uses the plate auction system and participants bid for the limited number of vehicle plates available. This paper aims at building a theoretical model that quantitatively analyzes the benefits of such policies.

This study extends the joint decision model of vehicle ownership and mileage model, and applied compensating variation method to measure the net social impact change of the different quota systems. Under this proposed framework, a numerical example is conducted. This example shows that plate auction system will yield a higher net impact than plate lottery system. Lottery system will attract more households participating, and thus lowering the chances of winning for those with higher preference to own vehicles. This will lower the net social impact. Since the ownership quota system serves as a policy tool to fight traffic congestion, the congestion mitigation effect is also investigated. The auction mechanism allows people with higher preference for driving to own vehicles, whereas the lottery system does not distinguish between different levels preferences. Meantime, winners of the plate auction surrender part of their income in the auction, which decreases their driving. According to the numerical example, lottery system still performs better than auction system in terms of reduction in congestion. Also, under a more congestible network, all three ownership quota systems will provide a higher net social impact.

Being a theoretical model, this paper has made several assumptions to maintain tractability of the analysis. Nonetheless, under all these assumptions, empirical data has supported many of the arguments in this paper. The low variance in Shanghai vehicle plate price have confirmed that under equilibrium status, winning bidders will all pay the market clearance price. Also, this study concluded that plate lottery system will attract participants that have lower preference of owning a vehicle. From the numerical example, we see that this mechanism is harmful for the benefit of society, in terms of the aggregated net social impact. Thus, this study also proposed another alternative to conduct plate lottery, which is to limit lottery access to previous car owners only. This alternative will yield a higher net social impact comparing to the lottery that opens to everyone.

References

- Barter, P. (2005). A vehicle quota integrated with road usage pricing: A mechanism to complete the phase-out of high fixed vehicle taxes in Singapore. *Transport Policy*, 12 (6), 525 - 536.
- Brownstone, D. & Small, K. (2005). Valuing time and reliability: assessing the evidence from road pricing demonstrations. *Transportation Research Part A: Policy and Practice* 39 (4), 279 - 293.
- Burtless, G. & Hausman, J. (1978). The effect of taxation on labor supply: Evaluating the Gary negative income tax experiment. *The Journal of Political Economy*, 1103 - 1130.
- Daganzo, C. (1995). A Pareto optimum congestion reduction scheme. *Transportation Research Part B: Methodological* 29 (2), 139 - 154.
- De Jong, G. (1990). An indirect utility model of car ownership and private car use. *European Economic Review* 34 (5), 971 - 985.
- De Jong, G. (1996). Disaggregate model system of vehicle holding duration, type choice and use. *Transportation Research, Part B* 30 (4), 263 - 276.
- De Jong, G., Fox, J., Pieters, M., Daly, A., & Smit, R. (2004). A comparison of car ownership models. *Transport Reviews*, 24 (4), 397 - 408.
- Dubin, J. & McFadden, D. (1984). An econometric analysis of residential electric appliance holdings and consumption. *Econometrica: Journal of the Econometric Society*, 345 - 362.
- Eskeland, G. & Feyzioğlu, T. (1997). Rationing can backfire: the day without a car In Mexico City. *The World Bank Economic Review* 11 (3), 383.
- Evans, R. (1983). Health care technology and the inevitability of resource allocation and rationing decisions. *JAMA: the journal of the American Medical Association* 249 (16), 2208.
- Goldberg, P. (1998). The effects of the corporate average fuel efficiency standards in the US. *The Journal of Industrial Economics*, 46 (1), 1 - 33.
- Hensher, D., Smith, N., & Group, M. U. T. R. (1992). *Dimensions of automobile demand: a longitudinal study of household automobile ownership and use*. North-Holland.
- Hicks, J. (1946). *Value and capital: An inquiry into some fundamental principles of economic theory*. Oxford University Press, USA.
- Lim, L. (Accessed on April 8, 2011). License Plate Lottery Meant To Curb Beijing Traffic. National Public Radio. <http://www.npr.org/2011/01/26/133231061/license-plate-lottery-meant-to-curb-beijing-traffic>.
- Manning, F. & Winston, C. (1985). A dynamic empirical analysis of household vehicle ownership and utilization. *The RAND Journal of Economics*, 215 - 236.
- Manning, F. & Winston, C. (1987). Economic effects of voluntary export restrictions. *Blind Intersection: Policy and the Automobile Industry*, 61 - 67.
- Nakamura, K. & Kockelman, K. (2002). Congestion pricing and roadscape rationing: an application to the San Francisco Bay Bridge corridor. *Transportation Research Part A: Policy and Practice*, 36 (5), 403 - 417.
- Schrank, D., Lomax, T., & Turner, S. (2011). Ttis 2011 urban mobility report. *Texas A&M University System*.
- Small, K. & Rosen, H. (1981). Applied welfare economics with discrete choice models. *Econometrica: Journal of the Econometric Society*, 105 - 130.
- Smith, P. & Chin, A. (1997). Automobile ownership and government policy: the economics of Singapore's vehicle quota system. *Transportation Research Part A: Policy and Practice*, 31 (2), 129 - 140.
- Verhoef, E. (2002). Second-best congestion pricing in general networks. Heuristic algorithms for finding second-best optimal toll levels and toll points. *Transportation Research Part B: Methodological*, 36 (8), 707 - 729.
- Wang, R. (2010). Shaping urban transport policies in china: Will copying foreign policies work? *Transport Policy*, 17 (3), 147 - 152.
- West, S. (2004). Distributional effects of alternative vehicle pollution control policies. *Journal of Public Economics*, 88 (3-4), 735 - 757.
- Winston, C. and Manning, F. (1984). Consumer demand for automobile safety. *The American Economic Review*, 74 (2), 316 - 319.
- Zhang, H. & Ge, Y. (2004). Modeling variable demand equilibrium under second-best road pricing. *Transportation Research Part B: Methodological*, 38 (8), 733 - 749.
- Zhu, S., Du, L., & Zhang, L. (2012). An economic model for vehicle ownership quota and usage restriction policy analysis. *20th International Symposium on Transportation and Traffic Theory*, accepted.