# $5^{\text {th }}$ World Conference on Educational Sciences - WCES 2013 <br> Applications of games theory in analyzing teaching process 

Graţiela Ghic ${ }^{\text {a } *}$, Carmen Judith Grigorescu ${ }^{\text {b }}$<br>${ }^{a}$ Dimitrie Cantemir Christian University, Faculty of International Business and Economics, Bucharest,Romania<br>${ }^{b}$ Dimitrie Cantemir Christian University, Faculty of Finance, Banking and Accountancy, Bucharest,Romania


#### Abstract

In the teaching activity, the problem of choosing the best solutions, methods and teaching procedures is usually difficult due to the complexity of pedagogical situations and random nature of individual particularities. This study aims at giving an answer to the question: can we plan the system of educational actions so that each action gives us maximum efficiency, reducing any unwanted results (routine, boredom ...). For this purpose we use games theory techniques. Games theory provides models of situations in which each chosen action can give us in different cases, different results with a known probability. The objective is to find the optimal mixed strategy for the professor to ensure the best possible result, giving any mixed response strategies of the students. The analysis is performed at the level of some disciplines highly mathematized: applied mathematics in economics, financial and economic analysis.


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## 1. Introduction

The mathematical models in the actual practice of the didactical process are not possible without a psychological and pedagogical basic research regarding the corresponding individual facts and phenomena. The characteristic of the pedagogical phenomena raises a lot of problems related to the quantitative research. Hence, the fact that many immaterial training factors cannot be quantitatively determined entails the necessity to use statistic correlations. The results of a teaching method application, of a means or device on different students will be always different. The exercises results depend on the productivity of the students' memory, that is, in case of doing the same exercise, the students with a good memory will acquire the knowledge better than those with a poorer memory. Further to the statistics analysis there was noticed that the knowledge acquiring, having been repeated for several times, depends $25 \%$ on the memory quality and $56 \%$ on the number of exercises. The conclusion is that the same type of exercises must be repeated more times until the students with poor memory catch up with the others. However, a hitch may occur: for a certain category of students, the unvarying repetitions decrease the interest in learning and have negative consequences over its results.

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## 2. Didactical process planning using the games theory

One of the basic conditions for a good memory is repetition. The Latin saying repetitio mater studiorum est means that repetition must not be used at random if productivity is going to be enhanced. In order to get the expected result the most favourable number of repetitions must be taken into account. While too many repetitions lead to an over-learning phenomenon, too few of them may lead to under-learning. The excessive repetitions entail saturation and protection inhibition. According to the game theory, a conflict comes up. Due to a certain factor the accumulation of repetitions improves the general learning results and due to another factor this accumulation worsens them. In this conflicting situation, a professor must find the proper strategy to ensure the best medium result of all the possible ones. The situation may be considered as a game pattern, where the parties are the professor and the students.

The conflict arises between the professor's method to store the knowledge and the students' characteristics which prevent the achievement of the expected pedagogical results through this method. Hence, the professor may use only one type of exercises to fix the specific knowledge. We call it the pure strategy no. 1. On the other hand, the professor may use only different types of exercises to fix the knowledge and this is the pure strategy no. 2. In case a professor uses each of these possible methods, a student's results depend on the category that student belongs to. For example, the correlative analysis has proven that in order to fully assimilate the primal simplex Algorithm (of medium difficulty degree) there are necessary 8 problems on average, regardless the level of the students' memory. What will happen when the professor chooses the pure strategy no. 1 and he is going to fix the knowledge using 8 exercises of the same type?

The statistical analysis made on 50 students proved that the first category of students, those inclined to memorize through uniform repetition, will acquire $100 \%$ of knowledge whereas the second category, students who cannot stand monotony, will acquire $80 \%$ of the algorithm. The following matrix of payments is obtained:

Table 1. Matrix of payments for strategy 1

|  | Students |  |  |
| :---: | :---: | :---: | :---: |
|  | Strategy 1 | Strategy 2 |  |
| Professor-strategy 1 | 100 | 80 |  |

If the professor chooses the pure strategy no.1, 8 exercise of different types, he will record losses for the first category of students. In case of frequent changes of the exercise or of its conduct, the students are confused and cannot adjust to the new situation. The following results are achieved:

Table 2. Matrix of payments for strategy 2

|  | Students |  |
| :--- | :--- | :--- |
|  |  |  |
|  | Strategy 1 | Strategy 2 |
| Professor-strategy 2 | 60 | 100 |

Gathering the results in a single matrix of payments we get the results below:
Table 3. Matrix of payments for gatering the results

|  |  |  | Students |
| :--- | :---: | :---: | :---: |
|  |  |  | Strategy 1 |
| Professor | 100 | 80 |  |
|  | strategy 1 | 60 | 100 |

The above matrix describes the conflicting situation the professor is in, due to the differences among the students' individual characteristics.

It is obvious that the results of the professor's choice are influenced by the different number of the students in the first category and the ones in the second. Hence, in the examined group there are 15 students of the first category and 5 of the second. The results of strategy no. 1 are provided by:

$$
15 \cdot \frac{100}{100}+5 \cdot \frac{80}{100}=19
$$

of 20 students with good results,
whereas the results of strategy no. $2 \quad 15 \cdot \frac{60}{100}+5 \cdot \frac{100}{100}=14$ are: of 20 students with good results.
In the above situation, strategy no. 1 is better. However, if there are 5 students of the first category and 15 of the second in the class, the following results will be achieved:

$$
5 \cdot \frac{100}{100}+15 \cdot \frac{80}{100}=17
$$

of 20 students with good results for strategy no. 1 and

$$
5 \cdot \frac{60}{100}+15 \cdot \frac{100}{100}=18
$$

of 20 students with good results for strategy no.2.
This time the strategy no. 2 is better. Therefore, in case of different proportions of the students in the first and second category there should be better applied different methods.

## 3. Case study ( Economic Mathematics and Enterprise Economic Analysis, first and third year)

In the Faculty of International Business of our University, 5 groups of students in the first year, section A, have been examined for the Economic Mathematics. The students have been divided in two categories: high-school graduates form Arts or Sciences (this fact overlapping the above described situations).
Table 4. the results of the professor's choice are influenced by the different number of the students in the first category and the ones in the second

|  | Number of high-school graduates |  | Strategy |  |
| :--- | :--- | :--- | :--- | :--- |
|  | Arts | Sciences | S1 | S2 |
| Group 1 | 15 | 5 | $19 / 20$ | $14 / 20$ |
| Group 2 | 5 | 15 | $17 / 20$ | $18 / 20$ |
| Group 3 | 10 | 10 | $18 / 20$ | $16 / 20$ |
| Group 4 | 8 | 12 | $17,6 / 20$ | $16,8 / 20$ |
| Group 5 | 12 | 8 | $18,4 / 20$ | $15,2 / 20$ |

This situation entails the following question: could it be found a combination of the methods 1 and 2 to provide the professor with fairly good results in any kind of classes, that is in case of any difference between the first and second types of students? If we formulate the problem once again the task will be: to find the professor's optimum mixed strategy to ensure the best possible result, for any mixed strategy of the students' answers.

The analysed situation has two strategies:
a.each party has 2 strategies
b.the loss suffered by one of the parties must be equivalent to the other's gain, meaning that the sum of the losses and gains is 0 (games with nul sum of $2 \times 2$ type).

According to the games theory, the following rules are good for finding the best mixed strategy:
1.the numbers in the first column of the matrix are substracted from the numbers in the second column
2. there is written the reverse ratio of the absolute values of the resulted differences.

This ratio determines the ratio regarding the application frequency for methods 1 and 2 in the case of optimum mixed strategy.

$$
\begin{aligned}
& |100-80|=20 \\
& |60-100|=40 \\
& 40: 20=4: 2
\end{aligned}
$$

This is the very ratio where we should apply strategies 1 and 2 in order to get the best result, regardless the structure of the classes from the considered point of view. Actually, there should be two types of exercise going to be repeated 4 times.

In this case, the professor's result for the first category of students is:

$$
\frac{4 \cdot 100+2 \cdot 60}{4+2}=86,6 \%
$$

For the second category of students:

$$
\frac{4 \cdot 80+2 \cdot 100}{4+2}=86,6 \%
$$

Regardless the students' pure strategy, in our case, the obtained results will certainly be of $86,6 \%$.
In group 4 , there are $40 \%$ of the first category and $60 \%$ of the second and the students' strategy is $4: 6$. This situation provides a new matrix of payments for the professor's pure strategies (we get it by multiplying the respective columns by 0,4 and 0,6$)$.

Table 5. The new matrix of payments for the professor's pure strategies

|  |  |  | Students |
| :---: | :---: | :---: | :---: |
|  |  |  | Strategy 1 |
| Professor | strategy 1 | 40 | 48 |
|  |  | strategy 2 | 24 |

Applying the professor's same mixed strategy 4:2, for the students in the first category we gave the following result:

$$
\frac{400 \cdot 40+2 \cdot 24}{4+2}=34,6 \%
$$

For the students in the second category:

$$
\frac{4 \cdot 48+2 \cdot 60}{4+2}=52 \%
$$

In the above case, we amount $86,6 \%$ good results. The S1 pure strategy provided $17,6 / 20=88 \%$ good results and the S2 pure strategy provided $16,8 / 20=84 \%$ good results.

The professor's gain of $86,6 \%$ while checking the students' pure strategies does not eliminate the possibility to get a better result in other more favorable situations. It is certain that in any combination of the students of the first and second category, be it the worst possible combination, the training results will not be lower than $86,6 \%$, in case the professor resorts to the best strategy.

At the section level there are the following results:
Table 6. The professor's gain while checking different strategies

|  | Strategy |  |  |
| :---: | :---: | :---: | :---: |
|  | S1 | S2 | S mixtă |
| Goup 1 | $88 \%$ | $84 \%$ | $86,66 \%$ |
| Group 2 | $85 \%$ | $90 \%$ | $86,66 \%$ |
| Group 3 | $90 \%$ | $80 \%$ | $86,66 \%$ |
| Group 4 | $88 \%$ | $84 \%$ | $86,66 \%$ |
| Group 5 | $92 \%$ | $76 \%$ | $86,66 \%$ |

For the third year, in the Enterprise Economic Analysis, the topics - the approach of the breakeven point (mathematically as difficult as the primal simplex algorithm) the same mixed strategy has been applied. The determined matrix of payments is:

Table 7. Matrix of payments (for the Enterprise Economic Analysis)

|  |  | Students |  |  |
| :---: | :---: | :---: | :---: | :---: |
|  |  |  | Strategy 1 |  |
| Professor | strategy 1 | 100 | 70 |  |
|  |  | strategy 2 | 40 |  |

Regarding the third year, we'd better not connect the students to the type of the high-school they graduated. We consider there have been essential changes in their ability to work with figures, their memory having been greatly developed. Therefore the professor should choose the best mixed strategy just as he cannot precisely determine the difference between the students in the first category and those in the second.

## 4. Conclusions

The didactical activity planning is highly demanding. In the teaching process, the issue of choosing the best solutions or pedagogical methods is usually difficult especially due to the many-sided pedagogical situations and to the aleatory character of their particularities. For choosing solutions risky conditions, the planning model is provided by the games theory.

In this study, simplifying the real situation to a great extent, we discovered that the best mixed strategies provide reasonable solutions for the professor so that he could efficiently plan the didactical process even before being aware of the group's knowledge level.

## References


[^0]:    * Corresponding Author: Ghic Gratiela Tel: 7656487893

    E-mail address: grati.ela@alternativeit.ro

