Z′ mediated flavor changing neutral currents in B meson decays

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Abstract

We study the effects of an extra U(1)′ gauge boson with flavor changing couplings with fermion mass eigenstates on certain B meson decays that are sensitive to such new physics contributions. In particular, we examine to what extent the current data on Bd → φK and Bd → η′K decays may be explained in such models, concentrating on the example in which the flavor changing couplings are left-chiral. We find that within reasonable ranges of parameters, the Z′ contribution can readily account for the anomaly in SφKS but is not sufficient to explain large branching ratio of Bd → η′K with the same parameter value. SφKS and Sη′KS are seen to be the dominant observables that constrain the extra weak phase in the model.

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1. Introduction

CP-violation has been a puzzling phenomenon in the studies of elementary particle physics since the first observation of its effects in hadronic kaon decays almost four decades ago [1]. In the standard model (SM), CP-violation is due entirely to the Cabibbo–Kobayashi–Maskawa (CKM) mechanism [2,3], describing the mismatch between the unitary transformations relating the up and down type quark mass eigenstates to the corresponding weak eigenstates. The CKM matrix involves a single weak phase along with three mixing angles. The validity of the CKM picture is further strengthened by the fact that recent sin2β measurements from time-dependent CP asymmetries of decay modes involving the b → c ¯s subprocess [4] agree well with the range of the weak phase β from many other constraints [5]. However, it is still unknown whether there are any other sources that may give rise to CP-violating effects. Good places to search for deviations from the SM predictions are decay processes that are expected to be rare in the SM, which may reveal new physics through interference effects. In particular, discrepancies among the time-dependent CP asymmetries of different B decay modes may show evidence for new physics [6–11].

Recently, an anomaly was reported in the time-dependent CP asymmetry measurement of the Bd → φKS decay mode. Within the framework of the SM, this process should also provide us with information on the weak phase β, up to about 5% theoretical uncertainty [6,12]. However, the averaged value of SφKS reported by the BaBar and Belle groups is [4]

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This result is only about 1.3σ away from the corresponding quantity measured by the $B \to J/\psi K_S$ mode. $S_{J/\psi K_S} = 0.736 \pm 0.049$ [4]. However, the scale factor $S = 2.11$ suggests a discrepancy between the two experimental groups.\(^1\) Before this discrepancy is settled, the difference between $S_{\phi K_S}$ and $S_{J/\psi K_S}$ suggests the possibility of new physics contributions. From the theoretical point of view, the $B \to \phi K_S$ decay is a loop-induced process involving $b \to s \bar{s}s$ penguin operators in the SM. Therefore, it is susceptible to new physics contributions even if they are suppressed by a large mass parameter which characterizes the new physics scale.

In addition to model-independent approaches [6,12,14,15], many studies have been made to explain the anomaly in supersymmetric and related models [16,17]. Such an effect can also be explained using models in which the bottom quark is mixed with heavy mirror fermions with masses of the order of the weak scale [18]. It is the purpose of this work to show that a new physics effect of similar size can be obtained from some models with an extra $Z'$ boson.

$Z'$ bosons are known to naturally exist in well-motivated extensions of the SM [19]. The $Z'$ mass is constrained by direct searches at Fermilab, weak neutral current data, and precision studies at LEP and the SLC [20–22], which give a model-dependent lower bound around 500 GeV. The latter also severely limits the $Z-Z'$ mixing angle $|\theta| < c < 10^{-3}$. A $Z'$ could be relevant to the NuTeV experiment [23] and, if the couplings are not family universal [21,24], to the anomalous value of the forward–backward asymmetry $A_{FB}$ [22]. (Earlier hints of a discrepancy in atomic parity violation have largely disappeared due to improved calculations of radiative corrections [25].) We therefore study the $Z'$ boson in the mass range of a few hundred GeV to 1 TeV, assuming no mixing between $Z$ and $Z'$. Interesting phenomena arise when the $Z'$ couplings to physical fermion eigenstates are nondiagonal. This is possible if there exist additional exotic fermions that have different $U(1)'$ charges from the ordinary fermions, as found in $E_6$ models [26–28]. However, in these models left-handed fermion mixings may induce undesirable flavor changing neutral currents (FCNC) mediated by the $Z$ boson even in the absence of $Z-Z'$ mixing or nonuniversal family couplings. One can avoid this consequence by confining the mixing to be between right-handed fermions and the exotic quarks [29]. Alternatively, other models give family nonuniversal $Z'$ couplings as a result of different ways of constructing families in some string models [30–33]. FCNC and possibly new $C\bar{P}$-violating phenomena will also occur in these models after fermion mixings are taken into account. These can occur for both left and right-handed fermions.

Although experiments on FCNC processes (such as the mass difference between $K_L$ and $K_S$ and the $K_L \to \mu^+\mu^-$ decay) have significantly constrained the $Z'$ couplings of the first and second generation quarks to be almost the same and diagonal, the couplings to the third generation are not well constrained. Similar statements apply to the charged leptons. It has been shown in Refs. [30–33] that indeed the third generation fermions can have different $Z'$ couplings from the other two generations.

We use all of the above-mentioned features to study the imprints of the $Z'$ boson on certain processes that involve $b \to s$ transitions. In Section 2, we present the model and framework to be studied. In Section 3, we show the constraints on the model parameters from the current data of $S_{\phi K_S}$, $A_{\phi K_S}$, and the branching ratio $B(B_d \to \phi K)$. In Section 4, we study a related process $B_d \to \eta' K_S$, also including its $C\bar{P}$ asymmetries and branching ratio. We conclude in Section 5.

### 2. Formalism

In this Letter, we concentrate on models in which the interactions between the $Z'$ boson and fermions are flavor nonuniversal for left-handed couplings and flavor diagonal for right-handed couplings. The analysis can be straightforwardly extended to general cases in which the right-handed couplings are also nonuniversal across generations. The basic formalism of flavor changing effects in the $Z'$ model with family nonuniversal and/or nondiagonal couplings has been laid out in Ref. [24], to which we refer readers for detail. Here

\(^1\) See Ref. [13] for the definition of the scale factor.
we just briefly review the ingredients needed in this Letter.

We write the $Z'$ term of the neutral-current Lagrangian in the gauge basis as

$$\mathcal{L}^{Z'} = -g' J'_{\mu} Z'^{\mu},$$

where $g'$ is the gauge coupling associated with the $U(1)'$ group at the $M_W$ scale. We neglect its renormalization group (RG) running between $M_W$ and $M_{Z'}$.

The $Z'$ boson is assumed to have no mixing with the SM $Z$ boson. The chiral current is

$$J'_\mu = \sum_{i,j} \bar{\psi}_I' \gamma_\mu [(\epsilon_{dL})_{ij} P_L + (\epsilon_{dR})_{ij} P_R] \psi'_I,$$

where the sum extends over the flavors of fermion fields, the chirality projection operators are $P_{L,R} \equiv (1 \mp \gamma_5)/2$, the superscript $I$ refers to the gauge interaction eigenstates, and $\epsilon_{dL}$ ($\epsilon_{dR}$) denote the left-handed (right-handed) chiral couplings. $\epsilon_{dL}$ and $\epsilon_{dR}$ are Hermitian under the requirement of a real Lagrangian. The fermion Yukawa coupling matrices $Y_d$ in the weak basis can be diagonalized as

$$Y_d = V_{dL,R} Y_{dL,R} V_{dL,R}^\dagger,$$

using the bi-unitary matrices $V_{dL,R}$ in $\psi_{dL,R} = V_{dL,R} \times \psi_{dL,R}^I$, where $\psi_{dL,R}^I \equiv P_{L,R} \psi_{dL,R}^I$ and $\psi_{dL,R}$ are the mass eigenstate fields. The usual CKM matrix is then given by $V_{\text{CKM}} = V_{dL} V_{sL}^\dagger$. The chiral $Z'$ coupling matrices in the physical basis of down-type quarks thus read

$$B_{dL}^{(I)} = V_{dL} \epsilon_{dL} V_{dL}^\dagger,$$

$$B_{dR}^{(I)} = V_{dR} \epsilon_{dR} V_{dR}^\dagger,$$

where the $B_{dL,R}^{(I)}$ are Hermitian. We do not need the corresponding couplings for up-type quarks or charged leptons in our discussions.

As long as $\epsilon_{dL,R}$ is not proportional to the identity matrix, $B_{dL,R}^{(I)}$ will have nonzero off-diagonal elements that induce FCNC interactions. To see this, consider as an example the simplified $\epsilon_{dL}$ matrix for the down-type quarks of the form

$$\epsilon_{dL} = Q_d \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & X \end{pmatrix},$$

where both $d$ and $s$ quarks have the same $Z'$ charge $Q_d$ and $X$ is the ratio of the $Z'$ charge of $b$ to $Q_d$. If we assume the mixing is among the down-type quarks only, $V_{dL} = V_{\text{CKM}}$ and $V_{uL} = 1$. Any redefinition of the quark fields by pure phase shifts would have no effect on the resultant $B_{dL}^{(I)}$. All the off-diagonal couplings are proportional to the $Z'$ charge difference between the $b$ quark and the $d, s$ quarks, as expected. Using the standard parametrization [34], the explicit form of the off-diagonal $Z'$ coupling between $b$ and $s$ quarks, for example, is

$$B_{dL}^{(I)} = (1 - X) Q_d \cos \theta_{13} \cos \theta_{23} + \cos \theta_{23} \sin \theta_{12} \sin \theta_{13} e^{-i \delta_{13}}.$$

More generally, one can always pick a basis for the weak eigenstates in which the $\epsilon_{dL,R}$ matrices are diagonal and of the form (6), though with different $Q_d$ and $X$ for the $\epsilon_{dL}$ and $\epsilon_{dR}$. However, the Yukawa matrices $Y_d$ and $Y_u$ will in general not be diagonal in that basis, so that $V_{uL} \neq 1$ and $V_{dL} \neq V_{\text{CKM}}$. In that case $B_{dL}^{(I)} = V_{dL} \epsilon_{dL} V_{dL}^\dagger$ will in general be nondiagonal and complex, with new mixing angles and $CP$-violating phases not directly related to $V_{\text{CKM}}$.

Instead of restricting ourselves to models with particular parameter choices in the couplings and mixings, we will take the effective theory point of view and constrain the effective couplings relevant to the decay modes of interest in the following analysis. However, to be more definite, we assume that the right-handed coupling matrix $B_{dR}^{(I)}$ is flavor diagonal. If $B_{dR}^{(I)}$ is nondiagonal, new operators involving different chirality structures will be induced in $B$ decays.

3. $B_d \rightarrow \phi K_S$

Within the SM, the $B^0 \rightarrow \phi K^0$ decay proceeds through the loop-induced $b \rightarrow s \bar{s} s$ transition, which

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2 Such mixings would modify the expressions for the phenomenological $\xi^{LL}$ and $\xi^{LR}$ parameters defined in Section 3.1, but would not alter the discussion of the implications.

3 One could instead always work in the $V_{uL} = 1$ basis, in which case $\epsilon_{dL,R}$ would in general be off-diagonal and complex.
involves dominantly the QCD penguin but also some electroweak (EW) and chromomagnetic penguin contributions. To illustrate possible modifications due to the existence of an extra $U(1)'$ gauge boson, we will neglect the smaller contributions from weak annihilation diagram in the following analysis although they can play some role in enhancing the branching ratios [35]. This two-body hadronic $B$ meson decay can be conveniently analyzed in the framework of the effective weak Hamiltonian and factorization formalism [36,37].

Since the penguin diagrams receive dominant contributions from the top quark running in the loop, the effective Hamiltonian relevant for the charmless $|\Delta S| = 1$ decays can be written as

$$H_{\text{eff}}^{\text{SM}} = \frac{G_F}{\sqrt{2}} \left[ V_{ub}V_{ts}^* c_1 O_1 + c_2 O_2 \right]$$

$$- V_{tb}V_{ts}^* \left[ \sum_{i=3}^{10} c_i O_i + c_8 O_8 \right] + \text{h.c.}$$

(8)

Here

$$O_1 = (\bar{u}_a b_a)_{V-A} (\bar{s}_\beta u_\beta)_{V-A},$$

$$O_2 = (\bar{u}_a b_\beta)_{V-A} (\bar{s}_\beta u_a)_{V-A}$$

are tree-level color-favored and color-suppressed operators,

$$O_{3(5)} = (\bar{s}_a b_a)_{V-A} (\bar{s}_\beta s_\beta)_{V-A(V+A)},$$

$$O_{4(6)} = (\bar{s}_a b_\beta)_{V-A} (\bar{s}_\beta s_a)_{V-A(V+A)}$$

are the QCD penguin operators,

$$O_{7(9)} = \frac{3}{2} \bar{s}_a (\bar{s}_\beta b_\beta)_{V-A(V+A)},$$

$$O_{8(10)} = \bar{s}_a (\bar{s}_\beta b_\beta)_{V-A(V+A)}$$

are the EW penguin operators ($\epsilon_s = -1/3$ is the electric charge of the strange quark), and

$$O_9 = \frac{G_F}{8\pi^2} m_b \bar{s}_a a^{\mu\nu} T_{a\beta}^{\mu\nu} (1 + \gamma_5)b_\beta G_{\mu\nu}$$

(12)

is the chromomagnetic operator, where $(\tilde{q}_1 q_2)_{V+A} = \tilde{q}_1 q_2 (1 + \gamma_5)q_2$ and $a, \beta$ refer to color indices.

We mention in passing that the $Z'$ boson will also modify the $|\Delta B| = 2$ effective Hamiltonian relevant to $B_d \rightarrow B_d$ mixing, but in an unnoticeable way. This is because the additional contribution is proportional to the square of the $Z'$ couplings between the first and third generations, $|B_{dB}^{\ell, R}|^2$, which is much more suppressed than the SM contribution. Although the $Z'$ also contributes to $b \rightarrow (c\bar{c})s$ transitions at the tree level and gains a color factor relative to the SM tree process, it is nevertheless suppressed by the $B_{sR}^{\ell, R}$ couplings and the $Z'$ mass in comparison with $V_{cb}$ and the $W$ mass [38]. Consequently, we do not study its effect on $\Delta M_{B_d}$ and $\sin 2\beta$ in charmed modes. Nevertheless, it can have significant effects on the $B_s - \bar{B}_s$ system if the couplings $B_{sR}^{\ell, R}$ are not too small, as we assume in the current analysis.

3.1. Decay amplitude and branching ratio

In the generalized factorization approach [37], the $B_d \rightarrow \phi K^0$ decay amplitude is

$$A(\overline{B}_d \rightarrow \phi K^0) = \frac{G_F}{\sqrt{2}} V_{tb} V_{ts}^* \left[ a_3 + a_4 + a_5 - \frac{1}{2}(a_7 + a_9 + a_{10}) \right]$$

(13)

where

$$X^{(BK, \phi)} \equiv \langle \phi | (\bar{s}_a s_a)_{V-A} | 0 \rangle \langle K | (\bar{s}_\beta b_\beta)_{V-A} | \overline{B} \rangle$$

(14)

is a factorizable hadronic matrix element. The coefficients $a_i$ are given by

$$a_{2i-1} = c_{2i-1}^{\text{eff}} + \frac{1}{N_C^{\text{eff}}} c_{2i-1}^{\text{eff}},$$

$$a_{2i} = c_{2i}^{\text{eff}} + \frac{1}{N_C^{\text{eff}}} c_{2i-1}^{\text{eff}},$$

(15)

where $c_i^{\text{eff}}$ are effective Wilson coefficients that should be used when one replaces the one-loop hadronic matrix elements in the effective Hamiltonian with the corresponding tree-level ones [37]. Nonfactorizable effects are encoded in the effective number of colors $N_C^{\text{eff}}$. Throughout this Letter, we take the naive choice $N_C^{\text{eff}} = 3$ for illustration.

For the input parameters $\alpha_s(M_Z) = 0.118$, $\alpha_{\text{EM}} = 1/128$; the Wolfenstein parameters [39] $\lambda = 0.2240$, $\Lambda = 0.825$, $\rho = 0.21$ and $\eta = 0.34$ [40]; $\sin^2 \theta_W = 0.23$, $M_W = 80.42$ GeV; and the running quark masses...
should be multiplied by $10^{-5}$.

Table 1
The SM Wilson coefficients used in the present analysis. We assume the naive factorization for $a_i$ (i.e., $N_C^{\text{eff}} = 3$), and ignore small differences between the $b \to s$ and $b \to \bar{s} s$ decays, expecting more significant effects from new physics. $c_i^{\text{eff}}$ and $a_i$ ($i = 3, \ldots, 10$) should be multiplied by $10^{-5}$.

<table>
<thead>
<tr>
<th>Operator</th>
<th>$c_i^{\text{eff}}$</th>
<th>$a_i$ ($N_C^{\text{eff}} = 3$)</th>
</tr>
</thead>
<tbody>
<tr>
<td>$O_1$</td>
<td>1.198</td>
<td>1.064</td>
</tr>
<tr>
<td>$O_2$</td>
<td>$-0.403$</td>
<td>$-0.004$</td>
</tr>
<tr>
<td>$O_3$</td>
<td>$2817 + 301i$</td>
<td>815</td>
</tr>
<tr>
<td>$O_4$</td>
<td>$-6006 - 903i$</td>
<td>$-5067 - 803i$</td>
</tr>
<tr>
<td>$O_5$</td>
<td>$2036 + 301i$</td>
<td>$-425$</td>
</tr>
<tr>
<td>$O_6$</td>
<td>$-7384 - 903i$</td>
<td>$-6705 - 803i$</td>
</tr>
<tr>
<td>$O_7$</td>
<td>$-28 - 12i$</td>
<td>$-5 - 12i$</td>
</tr>
<tr>
<td>$O_8$</td>
<td>70</td>
<td>$60 - 4i$</td>
</tr>
<tr>
<td>$O_9$</td>
<td>$-1079 - 12i$</td>
<td>$-957 - 12i$</td>
</tr>
<tr>
<td>$O_{10}$</td>
<td>366</td>
<td>$6 - 4i$</td>
</tr>
</tbody>
</table>

$m_t = 168$ GeV, $m_b = 4.88$ GeV, $m_c = 1.5$ GeV, $m_s = 122$ MeV, $m_\mu = 4.2$ MeV, and $m_d = 7.6$ MeV [41], the next-to-leading order (NLO) effective Wilson coefficients [36,37] for the $|\Delta S| = 1$ weak Hamiltonian at the scale $\mu = 2.5$ GeV within the SM are given in the second and third columns of Table 1.

The $B_d \to \phi K^0$ decay width is given by

$$\Gamma(B_d \to \phi K^0) = \frac{\alpha^3}{8\pi m_\phi^2} \left| A(B_d \to \phi K^0) \right|^2,$$

(16)

where

$$p_c = \sqrt{m_B^2 - (m_\phi + m_K)^2} \sqrt{m_B^2 - (m_\phi - m_K)^2}}$$

(17)

is the momentum of the decay particles in the center-of-mass frame. With $\tau_{B_d} = 1.534$ ps [42], $f_\phi = 237$ MeV, $F_1^{\phi K}(m_\phi^2) = 0.407$ [43] and meson masses given in Ref. [13], the $C P$-averaged branching ratio in the SM is

$$\mathcal{B}_{\text{SM}}(B^0 \to \phi K^0) \approx 11 \times 10^{-6}.$$

(18)

This result is slightly above the 95% CL range of the current world average value $(8.3 \pm 1.1) \times 10^{-6}$ given in Table 2, but is close to the previous calculation [17]. (We ignore theoretical uncertainties in the SM here and in illustrating the consequences of $Z'$ physics in the following sections.)

With FCNC, the $Z'$ boson contributes at tree level, and its contribution will interfere with the standard model contributions. In particular, the flavor-changing couplings of the $Z'$ with the left-handed fermions will contribute to the $O_5$ and $O_7$ operators for left (right)-handed couplings at the flavor-conserving vertex, i.e., $c_0\gamma(M_W)$ receive new contributions from $Z'$. On the other hand, the right-handed flavor changing couplings yield new operators with coefficients that contain another weak phase associated with $B_d^\pm$. We will ignore these contributions in this Letter.

The effective Hamiltonian of the $b \to s \bar{s} s$ transition mediated by the $Z'$ is

$$\mathcal{H}_{\text{eff}}^{Z'} = \frac{4G_F}{\sqrt{2}} \left( \frac{g' M_Z}{g_Y M_{Z'}} \right)^2 B_{1i}^L (B_{1i}^L O_0 + B_{1j}^R O_7) + \text{h.c.},$$

(19)

where $g_Y = e/(\sin \theta_W \cos \beta_W)$, and $B_{1j}^L$ and $B_{1j}^R$ refer to the left- and right-handed effective $Z'$ couplings of the quarks $i$ and $j$ at the weak scale, respectively. The diagonal elements are real due to the hermiticity of the effective Hamiltonian, but the off-diagonal elements may contain weak phases. Only one new weak phase associated with $B_{1i}^L$ can be introduced into the theory under our assumption of neglecting $B_{1j}^R$. We denote

Table 2
Experimental results of the $C P$-averaged branching ratios (quoted in units of $10^{-6}$) and $C P$ asymmetries of the $B \to \phi K$ and $B \to \eta' K$ decays. References are given in square brackets. The scale factor $S$ (defined in Ref. [13]) is displayed in parentheses when it is larger than 1.

<table>
<thead>
<tr>
<th>Mode</th>
<th>Belle</th>
<th>CLEO</th>
<th>Avg.</th>
</tr>
</thead>
<tbody>
<tr>
<td>$B^0 \to \phi K^0$</td>
<td>8.4^{+1.5}_{-1.3} \pm 0.5 [44]</td>
<td>9.0^{+2.2}_{-1.8} \pm 0.7 [45]</td>
<td>5.4^{+3.7}_{-2.7} \pm 0.7 [46]</td>
</tr>
<tr>
<td>$S_{\phi K_S}$</td>
<td>0.45 \pm 0.43 \pm 0.07 [4]</td>
<td>$-0.96 \pm 0.50^{+0.10}_{-0.11}$ [4]</td>
<td>\text{---}</td>
</tr>
<tr>
<td>$A_{\phi K_S}$</td>
<td>0.38 \pm 0.37 \pm 0.12 [4]</td>
<td>$-0.15 \pm 0.29 \pm 0.07$ [4]</td>
<td>\text{---}</td>
</tr>
<tr>
<td>$B^0 \to \eta' K^0$</td>
<td>60.6 \pm 5.6 \pm 4.6 [47]</td>
<td>68 \pm 10^{+16}_{-6}$ [48]</td>
<td>89^{+18}_{-16} \pm 9 [49]</td>
</tr>
<tr>
<td>$S_{\eta' K_S}$</td>
<td>0.02 \pm 0.34 \pm 0.03 [4]</td>
<td>0.43 \pm 0.27 \pm 0.05 [4]</td>
<td>\text{---}</td>
</tr>
<tr>
<td>$A_{\eta' K_S}$</td>
<td>$-0.10 \pm 0.22 \pm 0.03$ [4]</td>
<td>$-0.01 \pm 0.16 \pm 0.04$ [4]</td>
<td>\text{---}</td>
</tr>
</tbody>
</table>
this by $\phi_L$ and write $B_{sb}^L = |B_{sb}^L|^e^{i\phi_L}$. As $H_{\text{eff}}^{Z'}$ has the same operators $O_9$ and $O_7$ as in the SM effective Hamiltonian, the strong phases from long-distance physics should be the same.

Since heavy degrees of freedom in the theory have already been integrated out at the scale of $M_W$, the RG evolution of the Wilson coefficients after including the new contributions from $Z'$ is exactly the same as in the SM. We obtain the branching ratio

$$B^{\text{SM}+Z'}\left(\bar{B} \to \phi K^0\right) \simeq B^{\text{SM}}\left(\bar{B} \to \phi K^0\right) \times \left|1 - \left[(41.8 - 7.1i)\xi^{LL} + (46.2 - 8.6i)\xi^{LR}\right]e^{i\phi_L}\right|^2,$$

where

$$\xi^{LL} = \left|\frac{g'M_Z}{g'g_MZ'}\right|^2 \frac{B_{ss}^L B_{ss}^L}{V_{tb}V_{ts}^*},$$

$$\xi^{LR} = \left|\frac{g'M_Z}{g'g_MZ'}\right|^2 \frac{B_{sb}^L B_{sb}^R}{V_{tb}V_{ts}^*},$$

and $|V_{tb}V_{ts}^*| \simeq 0.04$. The second and third terms in Eq. (20) represent the $Z'$ contributions from left- and right-handed couplings with the $s\bar{s}$ in the final state, respectively. We have assumed for definiteness that $B_{ss}^L$ and $B_{ss}^R$ have the same sign, so that the $\xi^{LL}$ and $\xi^{LR}$ terms interfere constructively. The branching ratio predicted by our model depends on the absolute ratios $\xi^{LL}$ and $\xi^{LR}$ and the weak phase $\phi_L$.

We show the branching ratios as a function of $\phi_L$ in Fig. 1. Generally, one expects a ratio $g'/g_Y \sim O(1)$ and $M_{Z'}$ to be a few to around 10 times $M_Z$. We assume that the product $|B_{ss}^L B_{ss}^L|$ is numerically about the same as $|V_{sb}V_{ts}^*|$, and take $\xi^{LL} = 0.02$ and 0.005 as representative values for numerical analyses in this and the following sections. It is straightforward to scale the results to other $\xi^{LL}$ values.

To quantify the effects of right-handed couplings, we consider $\xi^{LR} = 0.02$ and 0.005 and show the corresponding curves in Fig. 1. The branching ratio curves are almost symmetric about $\phi_L = 0$, with the slight asymmetry set by the small strong phases in the Wilson coefficients. This echoes the fact that the contributing amplitudes in Eq. (20) have the largest constructive interference when $\phi_L \simeq 0$. To be consistent with the measured branching ratio of $B^0 \to \phi K^0$, our weak phase $\phi_L$ in the region $(-80^\circ)$–$(60^\circ)$ is favored, with the exact range depending upon $\xi^{LL}$ and $\xi^{LR}$ in the model. For some parameter choices, it can leave us a two-fold ambiguity, which can be resolved using further information to be discussed in the following subsection.

3.2. Time-dependent $CP$ asymmetries

The time-dependent $CP$ asymmetry for $B \to \phi K_S$ is

$$a_{\phi K_S}(t) = \frac{\Gamma(B^0(t) \to \phi K_S) - \Gamma(B^0(t) \to \phi K_S^*)}{\Gamma(B^0(t) \to \phi K_S) + \Gamma(B^0(t) \to \phi K_S^*)} = A_{\phi K_S}\cos(\Delta M_{B_S}t) + S_{\phi K_S}\sin(\Delta M_{B_S}t),$$

where
Fig. 2. The time-dependent CP asymmetries, $S_{\phi K_S}$ and $A_{\phi K_S}$, versus $\phi_L$. The current experimental ranges at 1$\sigma$ level are shown by the thin horizontal solid and dotted lines, respectively. The SM predicts $(S_{\phi K_S}, A_{\phi K_S}) \simeq (0.73, 0)$ (not shown). The thick solid and dashed curves are $S_{\phi K_S}$ for $\xi_{LL} = 0.02$ and 0.005, respectively. The corresponding curves for $A_{\phi K_S}$ are displayed using single-dotted and double-dotted dashes. Plot (a) has $\xi_{LR} = 0$; plot (b) has $\xi_{LR} = \xi_{LL} = 0.02$ and 0.005.

where the direct and the indirect CP asymmetry parameters are given respectively by

$$A_{\phi K_S} = \frac{\lambda_{\phi K_S}}{\lambda_{\phi K_S}|^2 + 1}, \quad S_{\phi K_S} = \frac{2 \text{Im} \lambda_{\phi K_S}}{\lambda_{\phi K_S}|^2 + 1}.$$  \hspace{1cm} (23)

The parameter $\lambda_{\phi K_S}$ is defined by

$$\lambda_{\phi K_S} = \eta_{\phi K_S} \left( \frac{q}{p} \right)_B \left( \frac{p}{q} \right)_K \frac{A(\phi K^0)}{A(\phi K^0)},$$  \hspace{1cm} (24)

where $\eta_{\phi K_S} = -1$ is the CP eigenvalue of the $\phi K_S$ state, and

$$\left( \frac{q}{p} \right)_B = \frac{V_{tb} V_{td}^*}{V_{tb} V_{td}} \quad \text{and} \quad \left( \frac{p}{q} \right)_K = \frac{V_{cs} V_{cd}^*}{V_{cs} V_{cd}}$$ \hspace{1cm} (25)

are factors that account for the mixing effects in neutral $B$ and $K$ meson systems, respectively.

We show our estimates of $S_{\phi K_S}$ and $A_{\phi K_S}$ as a function of the new weak phase $\phi_L$ in Fig. 2. In Fig. 2(a), we have choices $\xi^{LL} = 0.02$ and 0.005, but set $\xi^{LR} = 0$. The SM prediction of $S_{\phi K_S}$ and $A_{\phi K_S}$ are 0.73 and 0, respectively. We see that the measured $A_{\phi K_S}$ does not give much constraint on the weak phase $\phi_L$, except for the regions between $-55^\circ$–$80^\circ$ when $\xi^{LL} = 0.02$. The $S_{\phi K_S}$ data can be readily fitted within 1$\sigma$ for values of $\xi^{LL}$ chosen here.

In Fig. 2(b), we turn on the right-handed couplings and set $\xi^{LR} = \xi^{LL}$. We notice that the variation of $A_{\phi K_S}$ is within the experimental 1$\sigma$ limits for the most range of $\phi_L$. As illustrated by the thick solid curve in Fig. 2(b), there are four possible ranges of $\phi_L$ that can fit the averaged $S_{\phi K_S}$ if both $\xi^{LL}$ and $\xi^{LR}$ are large enough. For smaller $\xi^{LL}$ and $\xi^{LR}$, however, only a region of negative $\phi_L$ is favored.

In order to satisfy both CP asymmetry constraints, $\phi_L$ should have negative value in most cases. Only $\xi^{LL} = \xi^{LR} = 0.02$ can have some positive $\phi_L$ range. Combining the constraints from $B(B \rightarrow \phi K)$ at 95% CL and both $S_{\phi K_S}$ and $A_{\phi K_S}$ at 1$\sigma$ level, we find the following allowed regions of $\phi_L$. If we take $\xi^{LR} = \xi^{LL} = 0.02$, $5^\circ \lesssim \phi_L \lesssim 15^\circ$ is favored. If we take $\xi^{LR} = \xi^{LL} = 0.005$, $-80^\circ \lesssim \phi_L \lesssim -55^\circ$ is favored. If we take $\xi^{LL} = 0.02$ and ignore $\xi^{LR}$, then $-70^\circ \lesssim \phi_L \lesssim -55^\circ$ is favored. For $\xi^{LL} = 0.005$ and $\xi^{LR} = 0$, $-80^\circ \lesssim \phi_L \lesssim -30^\circ$ is favored.

4. $B_d \rightarrow \eta' K_S$

The $B_d \rightarrow \eta' K_S$ is another decay mode whose time-dependent CP asymmetry $S_{\eta' K_S}$ is expected to
The perturbative calculations of the $B \to \eta' K$ branching ratios are significantly smaller than the observed values. This discrepancy can be explained by adding a singlet-penguin amplitude, where $\eta'$ is produced through a flavor-singlet neutral current, to interfere constructively with the QCD penguin contributions [50,51]. Another analysis [52] found that it is hard to obtain a sizable flavor-singlet amplitude from perturbative calculations, but QCD penguin amplitudes can be enhanced by an asymmetric treatment of the $s \bar{s}$ component of the $\eta'$ wavefunction. Since this matter is debatable, we will follow the usual effective Hamiltonian approach [36,37] and put the emphasis on what kind of effects the $Z'$ boson may provide.

Following the notation in Ref. [17], the decay amplitude of $\bar{B}^0 \to \eta' \bar{K}^0$ can be written as

$$
\bar{A}(\bar{B}^0 \to \eta' \bar{K}^0) = \frac{i G_F}{\sqrt{2}} \times \left[ V_{ub} V_{us}^* \{ a_4 - \frac{a_{10}}{2} + \left( a_6 - \frac{a_8}{2} \right) R_1 \} X_1 
- V_{tb} V_{ts}^* \left\{ \left( a_4 - \frac{a_{10}}{2} \right) + \left( a_6 - \frac{a_8}{2} \right) \right\} X_2 
+ \left[ 2(a_3 - a_5) - \frac{1}{2}(a_7 - a_9) \right] X_3 \right].
$$

where

$$
R_1 = \frac{2m_{K^0}^2}{(m_b - m_d)(m_s + m_d)},
$$

$$
R_2 = \frac{2(m_{K^0}^2 - m_{\eta'}^2)}{2m_s(m_b - m_s)}. \quad \text{and} \quad X_1 = i \left( m_B^2 - m_{\eta'}^2 \right) \frac{X_{\eta'}}{\sqrt{2}} f_K F_{0B}^K (m_{\eta'}^2),
$$

$$
X_2 = i \left( m_B^2 - m_{K^0}^2 \right) \frac{X_{\eta'}}{\sqrt{2}} f_K F_{0B}^K (m_{\eta'}^2),
$$

$$
X_3 = i \frac{2m_{K^0}^2}{m_B^2} Y_{\eta'} \sqrt{2} - \frac{f_{K^0}^2}{f_K^2} F_{0B}^K (m_{\eta'}^2),
$$

(27)

and $X_{\eta'} = 0.57$ and $Y_{\eta'} = 0.82$ are mixing parameters for the choice of the $\eta'$ meson wavefunction to be $(2s\bar{s} + u\bar{u} + d\bar{d})/\sqrt{6}$.

Since the $B \to \eta' K$ has two pseudoscalar mesons in the final state, the decay width is

$$
\Gamma(B_d \to \eta' K^0) = \frac{p_c}{8\pi m_B^2} |A(B_d \to \eta' K^0)|^2,
$$

(28)

where $p_c$ is defined in a similar way to Eq. (17). With $f_\pi = 131$ MeV and $f_K = 159.8$ MeV [13], $F_{0B}^K (m_{\eta'}^2) = 0.335$ and $F_{0B}^K (m_{\eta}^2) = 0.391$ [43], we have $\mathcal{B}_{SM}^0 (B \to \eta' K^0) \approx 38 \times 10^{-6}$, which is much lower than the experimental average of $(65.18 \pm 6.18) \times 10^{-6}$ (see Table 2).

As in the case of $B \to \phi K$ decays, our model makes extra contributions to $O_9$ and $O_7$ at the weak scale. The branching ratio is

$$
\mathcal{B}_{SM+Z'}(\bar{B} \to \eta' \bar{K}^0) \approx \mathcal{B}_{SM} (\bar{B} \to \eta' \bar{K}^0)
\times |1 - [(7.0 - 0.5i)\xi^{LL} + (2.9 - 0.4i)\xi^{LR}] e^{i\phi_L}|^2.
$$

(29)

We notice that the coefficient of $\xi^{LL}$ and that of $\xi^{LR}$ also tend to have constructive interference between themselves according to our assumption that $B_{15}^L$ and $B_{15}^R$ have the same sign. The magnitudes of these coefficients, however, are much smaller than those in Eq. (20). This is simply because the terms that receive contributions from the $Z'$ boson (mostly $a_9$) have some cancellation between the $X_2$ and $X_3$ terms in Eq. (26). These observations qualitatively tell us why the $\eta' K$ decays are not affected quite as much by the $Z'$ effects.

We see in Fig. 3 that the $Z'$ boson can explain the gap between the observed branching ratio and the SM prediction only around $\phi_L = \pm 180^\circ$ even with large couplings in both $\xi^{LL}$ and $\xi^{LR}$. As we will see, however, this region is not favored by the CP
Fig. 3. The branching ratio \( \mathcal{B}^{SM+Z}(B \rightarrow \eta' K^0) \) in units of \( 10^{-6} \) versus \( \phi_L \). The current experimental range at 95% CL is shown by the two horizontal dotted lines. The SM prediction is the thin horizontal line. The thick solid and dashed curves include both left-handed and right-handed couplings with \( \xi^{LL} = \xi^{LR} = 0.02 \) and 0.005, respectively. The single-dot-dashed and double-dot-dashed curves involve only the left-handed couplings with \( \xi^{LL} = 0.02 \) and 0.005, respectively.

Asymmetry constraints. Therefore, we must attribute this anomaly to some other unknown source.

The asymmetry curves for \( B_d \rightarrow \eta' K_S \) are shown in Fig. 4. We do not get useful constraints from current data on \( A_{\eta' K_S} \). The value of \( A_{\eta' K_S} \) does not vary much from its SM prediction throughout the whole range of \( \phi_L \). The averaged value of \( S_{\eta' K_S} \) can be explained at 1\( \sigma \) level by simultaneously taking large values of both left- and right-handed couplings (the solid curve in Fig. 4(b)). In this case, however, only negative \( \phi_L \) around \((-120^\circ)\)–\((-40^\circ)\) is favored from the \( S_{\eta' K_S} \) constraint. Other cases do not explain the \( S_{\eta' K_S} \) anomaly though all of them favor negative value of \( \phi_L \) to approach the 1\( \sigma \) limit.

Leaving 95% CL of branching ratio constraints, we have only \( \xi^{LL} = \xi^{LR} = 0.02 \) case that can satisfy both \( B \rightarrow \eta' K \) and \( B \rightarrow \eta' K_S \) asymmetry with a two-fold range of \( \phi_L \) \((-120^\circ)\)–\((-100^\circ)\) and \((-60^\circ)\)–\((-40^\circ)\). Attributing the branching ratio of \( B \rightarrow \eta' K \) to some unknown effects, the latter is favored by the \( B \rightarrow \phi K \) branching ratio.

Fig. 4. The time-dependent \( CP \) asymmetries, \( S_{\eta' K_S} \) and \( A_{\eta' K_S} \), versus \( \phi_L \). The current experimental ranges at 1\( \sigma \) level are shown by the thin horizontal solid and dotted lines, respectively. The SM predicts \( (S_{\eta' K_S}, A_{\eta' K_S}) \simeq (0.73, 0) \) (not shown). The thick solid and dash-dotted curves are \( S_{\eta' K_S} \) for \( \xi^{LL} = 0.02 \) and 0.005, respectively. The corresponding curves for \( A_{\eta' K_S} \) are displayed using single-dotted and double-dotted dashes. Plot (a) has \( \xi^{LR} = 0 \); plot (b) has \( \xi^{LR} = \xi^{LL} = 0.02 \) and 0.005.
5. Conclusions

In this Letter we have considered models with an extra $Z'$ in the mass range of a few hundred GeV to around 1 TeV. With a family nonuniversal structure in the $Z'$ couplings, flavor changing neutral currents are induced via the fermion mixing, thereby producing interesting effects. Currently, constraints on the $Z'$ coupling between the second and third generations are not restrictive. With non-diagonal left-handed and diagonal right-handed $Z'$ couplings in the down-type quarks, we studied the impact of such $Z'$ models on rare $B$ meson decay processes that are sensitive to new physics.

In the present analysis, we have assumed that the left- and right-chiral couplings $B^L_{s \bar{s}}$ and $B^R_{s \bar{s}}$ have the same sign, rendering constructive interference in the $Z'$ contributions. We do not include the right-handed flavor changing couplings, which will give rise to new operators not existent in the SM. Involving these or choosing different values for the effective number of colors $N^\text{eff}$, for which the branching ratios change sensitively, would change the results.

We have found that with the inclusion of the $Z'$ contributions, $S_{\phi K_S}$ can be appreciably different from the SM prediction, while the branching ratio of $B^0 \to \phi K^0$ and $A_{\phi K_S}$ are still within the experimental ranges. We find that a sizable weak phase associated with the $B^0_{s \bar{b}}$ coupling is favored in the ranges of $(-80^\circ)$–$(-30^\circ)$, depending upon the $\xi^{LL}$ and $\xi^{LR}$ parameter choices.

We have also studied the influence of the new $Z'$ on the $B^0 \to \eta' K^0$ decay. The $S_{\phi K_S}$ data do not restrict the choice of $\phi_L$. The $S_{\phi K_S}$ constraint from the data can be satisfied if large couplings are taken. Though the discrepancy between the observed branching ratio and the SM prediction can be explained with this $Z'$ effect, we cannot explain both branching ratio and $CP$ asymmetries constraints with a common weak angle. Combining with the constraints from the $B^0 \to \phi K^0$ decays, $S_{\phi K_S}$ and $A_{\phi K_S}$, we find that a value of $\phi_L$ around $(-60^\circ)$–$(-40^\circ)$ is favored.

We have observed that the $CP$ asymmetries of the $\phi K_S$ mode are more sensitive to the $Z'$ effects than the $\eta' K_S$ decay. This is because of a cancellation between different parts ($s\bar{s}$ versus $u\bar{u}$ and $d\bar{d}$) in the $\eta'$ wavefunction.

We are currently investigating $Z'$ effects on $b \to s\ell^+\ell^-$ decays, which have been recently measured to good precision and should provide a tight bound on $B_s \to \mu^+\mu^-$ decay via $Z'$ exchange [53,54].

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References


For a recent discussion, see P. Langacker, hep-ph/0308033.


