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Application of Adomian decomposition method to solve hybrid fuzzy differential equations

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Abstract

In this paper, we study the numerical solution of hybrid fuzzy differential equations by using Adomian decomposition method (ADM). This is powerful method which consider the approximate solution of a nonlinear equation as an infinite series usually converging to the accurate solution. Several numerical examples are given and by comparing the numerical results obtained from ADM and predictor corrector method (PCM), we have studied their accuracy.

Keywords: Hybrid systems; Fuzzy differential equations; Adomian decomposition method; Predictor corrector method; Approximate solution

1. Introduction

Fuzzy differential equations are important for studying and solving large proportions of problems in many topics in applied mathematics, particularly in relation to physics, geography, medicine, biology, etc. In many applications, some of our parameters are represented by fuzzy numbers rather than the crisp, and hence it is important to develop mathematical models and numerical procedures which would have appropriately treated general fuzzy differential equations and solved them. The knowledge about differential equations is often incomplete or vague. Fuzzy differential equations were first formulated by Kaleva [1] and Seikkala [2]. The fuzzy differential equations theory is well developed [3–12].

Hybrid systems are devoted to modeling, designing and validating of interactive systems of computer programs and continuous systems. That is control systems that are capable of controlling complex systems which have discrete event dynamics as well as continuous time dynamics which can be modeled by hybrid systems. The differential systems containing fuzzy valued functions and interaction with a discrete time controller are named as hybrid fuzzy differential systems (HFDEs).

Several numerical techniques have been applied to solve hybrid fuzzy differential equations. For example, Pederson and Sambandham have investigated the numerical solution of this equations by using the Euler and Runge-Kutta methods [13,14]. Prakash and Kalaiselvi have studied the predictor-corrector method for hybrid fuzzy differential equations [15]. Also, Fard and Bidgoli have solved HFDEs by Nystrom method [16].

In this study, we develop numerical methods for hybrid fuzzy differential equations by an application

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of the Adomian decomposition method. The decomposition method was first introduced by Adomian in the early 1980s. The Adomian decomposition method can be used to solve mathematical models or system of equation involving algebraic, differential integral and integro-differential [17–21].

The structure of paper is organized as follows: In section 2, we provide some basic definitions to fuzzy valued functions. In section 3 and section 4, contains a brief review of Adomian decomposition and the hybrid fuzzy differential equations, respectively. In section 5, hybrid fuzzy differential equations are solved by ADM and its convergence is expressed. Finally, in section 6, we present two examples to check the accuracy of the method and we compare ADM with predictor corrector method and exact solution.

2. Preliminaries

Definition 2.1. [22] A fuzzy number u is a fuzzy subset of the real line with a normal, convex and upper semicontinuous membership function of bounded support. The family of fuzzy numbers will be denoted by E. An arbitrary fuzzy number is represented by an ordered pair of functions $(\underline{u}(\alpha), \overline{u}(\alpha))$, $0 \le \alpha \le 1$, that satisfies the following requirements:

- $\underline{u}(\alpha)$ is a bounded left continuous nondecreasing function over [0, 1], with respect to any α,
- $\overline{u}(\alpha)$ is a bounded left continuous nonincreasing function over [0, 1], with respect to any α,
- $\underline{u}(\alpha) \le \overline{u}(\alpha)$, $0 \le \alpha \le 1$.

The α-level set

$$[u]^\alpha = \{x \in \mathbb{R} : u(x) \ge \alpha\},$$

is a closed bounded interval, denoted

$$[u]^\alpha = [\underline{u}^\alpha, \overline{u}^\alpha].$$

Definition 2.2. Triangular fuzzy number is a fuzzy set u in E that is characterized by an ordered triple $(u^l, u^c, u^r) \in \mathbb{R}^3$ with $u^l \le u^c \le u^r$ such that $[u]^0 = [u^l, u^r]$ and $[u]^1 = \{u^c\}$. the α-level set

$$[u]^\alpha = [u^c - (1-\alpha)(u^c - u^l), u^c + (1-\alpha)(u^r - u^c)], \tag{1}$$

for any $\alpha \in I = [0, 1]$.

Definition 2.3. The supremum metric, the space d_∞ on E is defined by

$$d_\infty(u, v) = \sup_{0 \le \alpha \le 1} \max\{|\underline{u}(\alpha) - \underline{v}(\alpha)|, |\overline{u}(\alpha) - \overline{v}(\alpha)|\},$$

and (E, d_∞) is a complete metric space.

Definition 2.4. A mapping $F : T \to E$ is Hukuhara differentiable at $t_0 \in T \subseteq \mathbb{R}$, if for some $h_0 > 0$, Hukuhara differences $F(t_0 + \Delta t) -_h F(t_0)$ and $F(t_0) -_h F(t_0 - \Delta t)$ exist in E, for

$$\lim_{\Delta t \to 0^+} d_\infty\left(\frac{F(t_0 + \Delta t) -_h F(t_0)}{\Delta t}, F'(t_0)\right) = 0,$$

and

$$\lim_{\Delta t \to 0^+} d_\infty\left(\frac{F(t_0) -_h F(t_0 - \Delta t)}{\Delta t}, F'(t_0)\right) = 0.$$

The fuzzy set $F'(t_0)$ is called the Hukuhara derivative of F at t_0.

Definition 2.5. [2] Let I be a real interval. A mapping $Y : I \to E$ is called a fuzzy process, and its α-level set is denoted by

$$[Y(t)]^\alpha = [\underline{Y}(t, \alpha), \overline{Y}(t, \alpha)], \quad t \in I, \ \alpha \in [0, 1],$$

The Seikkala derivative $Y'(t)$ of a fuzzy process y is defined by

$$SDY(t) = [Y'(t)]^\alpha = [\underline{Y'}(t, \alpha), \overline{Y'}(t, \alpha)],$$

$$t \in I, \ \alpha \in [0, 1],$$

provided that equation defines a fuzzy number $Y'(t) \in E$.

3. Adomian decomposition method (ADM)

Consider the differential equation

$$Lu + Ru + Nu = g,$$

where L is the highest order derivative which is assumed to be easily invertible, R is a linear differential operator of order less than L, N represents the nonlinear terms, and g is the source term.

The Adomian assumes that the unknown function u can be decomposed by a sum of components

$$u = \sum_{n=0}^{\infty} u_n,$$

also, the nonlinear operator Nu is usually represented by sum of series

$$Nu = \sum_{n=0}^{\infty} A_n(u_0, \ldots, u_n),$$

where $A_n(u_0, \ldots, u_n)$ are the appropriate Adomian's polynomials, which is defined as

$$A_k = \frac{1}{k!}\left(\frac{d^k}{d\lambda^k} G\left(\sum_{n=0}^{\infty} u_n \lambda^n\right)\right)\Bigg|_{\lambda=0}, \tag{2}$$

So to calculate terms of series $u = \sum_{n=0}^{\infty} u_n$, we use the following iterated scheme:

$$u_0 = L^{-1} y,$$

$$u_n = -L^{-1} R(u_n) - L^{-1}(A_{n-1}). \tag{3}$$

4. The hybrid fuzzy differential system

Consider the hybrid fuzzy differential system

$$\begin{cases} x'(t) = f(t, x(t), \lambda_k(x_k)), & t \in [t_k, t_{k+1}], \\ x(t_k) = x_k, \end{cases} \tag{4}$$

where $0 \le t_0 < t_1 < \cdots < t_k < \cdots$, $t_k \to \infty$, $f \in C[\mathbb{R}^+ \times E \times E, E]$, $\lambda_k \in C[E, E]$.

To be specific, the system would look like:

$$x'(t) = \begin{cases} x_0'(t) = f(t, x_0(t), \lambda_0(x_0)), & x_0(t_0) = x_0, & t_0 \le t \le t_1, \\ x_1'(t) = f(t, x_1(t), \lambda_1(x_1)), & x_1(t_1) = x_1, & t_1 \le t \le t_2, \\ \vdots \\ x_k'(t) = f(t, x_k(t), \lambda_k(x_k)), & x_k(t_k) = x_k, & t_k \le t \le t_{k+1}, \\ \vdots \end{cases}$$

Assuming that the existence and uniqueness of solutions of (4) hold for each $[t_k, t_{k+1}]$, by the solution of (4) we obtain the following function:

$$x(t) = x(t, t_0, x_0) = \begin{cases} x_0(t), & t_0 \le t \le t_1, \\ x_1(t), & t_1 \le t \le t_2, \\ \vdots \\ x_k(t), & t_k \le t \le t_{k+1}, \\ \vdots \end{cases} \tag{5}$$

We note that the solutions of (4) are piecewise differentiable in each interval for $t \in [t_k, t_{k+1}]$, for a fixed $x_k \in E$ and $k = 0, 1, 2, \ldots$.

5. Adomian decomposition method for a hybrid fuzzy differential system

In this section, for a hybrid fuzzy differential equation (4), we develop the ADM via an application of the ADM for fuzzy differential equations when f and λ_k in Eq. (4) can be obtained via the Zadeh extension principle from $f \in C[\mathbb{R}^+ \times \mathbb{R} \times \mathbb{R}, \mathbb{R}]$ and $\lambda_k \in C[\mathbb{R}, \mathbb{R}]$. We assume that the existence and uniqueness of solutions of Eq. (4) hold for each $[t_k, t_{k+1}]$.

Fix $k \in \mathbb{Z}^+$, so that the fuzzy initial value problem

$$\begin{cases} x_k'(t) = f(t, x(t), \lambda_k(x_k)), & t_k \le t \le t_{k+1}, \\ x(t_k) = x_k, \end{cases} \tag{6}$$

can be solved by ADM.

Consider $y(t)$ as approximation $x(t)$, in that

$$y(t) = (y^l(t), y^c(t), y^r(t)).$$

We decompose f to form below,

$$f = R + N,$$

and define $L^{-1}(\cdot) = \int_{t_0}^{t} (\cdot) dt$, so

$$y = \int_{t_0}^{t} g(t)dt - \int_{t_0}^{t} R(y)dt - \int_{t_0}^{t} N(y)dt,$$

and replace

$$y = \sum_{i=0}^{\infty} y_i, \quad Ny = \sum_{i=0}^{\infty} A_i(y_0, \ldots, y_n), \tag{7}$$

then relations (7) will be obtained as:

$$\sum_{i=0}^{\infty} y_i = \int_{t_0}^{t} g(t)dt - \int_{t_0}^{t} R\left(\sum_{i=0}^{\infty} y_i\right) dt - \int_{t_0}^{t} \left(\sum_{i=0}^{\infty} A_i\right) dt, \tag{8}$$

From the decomposition of relation (8), we will have the following assumptions to obtain y:

$$\begin{cases} y_0 = \int_{t_0}^{t} g(t)dt, \\ y_1 = -\int_{t_0}^{t} R(y_0)dt - \int_{t_0}^{t} A_0 dt, \\ \vdots \\ y_{i+1} = -\int_{t_0}^{t} R(y_i)dt - \int_{t_0}^{t} A_i dt, \quad i = 0, 1, \ldots. \end{cases} \tag{9}$$

Usually, all the terms of the series $\sum y_i$ not be computed. So, the exact solution $\sum_{i=0}^{\infty} y_i$ cannot be obtained. But an approximation, as close as we need, can be worked out using only the first few terms of the series, say $s_n = \sum_{i=0}^{n} y_i$.

Now, if series be convergence, then we have $\lim_{n\to\infty} s_n = x$.

6. Convergence of Adomian decomposition method

By Theorem 5.2 in [1], we may replace (6) by an equivalent system:

$$\begin{cases} \underline{x}'(t) = \underline{f}(t, x(t), \lambda_k(x_k)) = F_k(t, \underline{x}, \overline{x}), & \underline{x}(t_k) = \underline{x}_k, \\ \overline{x}'(t) = \overline{f}(t, x(t), \lambda_k(x_k)) = G_k(t, \underline{x}, \overline{x}), & \overline{x}(t_k) = \overline{x}_k, \end{cases} \tag{10}$$

Which possesses a unique solution $x = (\underline{x}, \overline{x})$, which is a fuzzy function. That is for each t, the pair $(\underline{x}(t, \alpha), \overline{x}(t, \alpha))$ is a fuzzy number, where $\underline{x}(t, \alpha)$ and $\overline{x}(t, \alpha)$ are respectively the solutions of the parametric form given by:

$$\begin{cases} \underline{x}'(t, \alpha) = \underline{f}(t, x(t, \alpha), \lambda_k(x_k(\alpha))) \\ = F_k(t, \underline{x}(t, \alpha), \overline{x}(t, \alpha)), \quad \underline{x}(t_k, \alpha) = \underline{x}_k(\alpha), \\ \overline{x}'(t, \alpha) = \overline{f}(t, x(t, \alpha), \lambda_k(x_k(\alpha))) \\ = G_k(t, \underline{x}(t, \alpha), \overline{x}(t, \alpha)), \quad \overline{x}(t_k, \alpha) = \overline{x}_k(\alpha), \end{cases} \tag{11}$$

for $\alpha \in [0, 1]$.

For a fixed α, we develop the ADM for each interval $[t_0, t_1], \ldots, [t_k, t_{k+1}], \ldots$.

Numerical solutions by ADM calculate to form below:

$$\begin{cases} \underline{s_n}(t, \alpha) = \underline{y_1}(t, \alpha) + \underline{y_2}(t, \alpha) + \cdots + \underline{y_n}(t, \alpha), \\ \overline{s_n}(t, \alpha) = \overline{y_1}(t, \alpha) + \overline{y_2}(t, \alpha) + \cdots + \overline{y_n}(t, \alpha). \end{cases}$$

Remark 5.1. The Adomian technique is equivalent to determining the sequence

$$s_n = y_1 + y_2 + \cdots + y_n,$$

by using the iterative scheme

$$\begin{aligned} s_0 &= 0, \\ s_{n+1} &= N(y_0 + s_n), \end{aligned} \tag{12}$$

if $n \to \infty$, then we have

$$N(y_0 + x) = N(\lim_{n\to\infty}(y_0 + s_n)) = \lim_{n\to\infty} N(y_0 + s_n) = \lim_{n\to\infty} s_{n+1} = x.$$

For the study of the numerical resolution of (12) Cherruault used fixed-point theorem [23].

Theorem 5.1. Let N be an operator from a Hilbert space H in to H, and x the exact solution of (6). $\sum_{i=0}^{\infty} y_i$ which is obtained by (9), converges to x when $\exists\, 0 \le \alpha < 1$, $\|y_{k+1}\| \le \alpha \|y_k\|$, $\forall\, k \in N \cup \{0\}$.

Proof. See [24].

Definition 5.1. For every $i \in N \cup \{0\}$, we define

$$\alpha_i = \begin{cases} \dfrac{\|y_{i+1}\|}{\|y_i\|}, & \|y_i\| \neq 0, \\ 0, & \|y_i\| = 0. \end{cases} \tag{13}$$

corollary 6.1. In Theorem 5.1, $\sum_{i=0}^{\infty} y_i$ converges to exact solution x, when $0 \le \alpha_i < 1$, $i = 1, 2, \ldots$.

From Theorem 5.1, we conclude that for a fixed α and $k \in \mathbb{Z}^+$, solutions obtained by (8) converge to exact solutions, therefore

$$\lim_{n\to\infty} \underline{s_n}(t, \alpha) = \underline{x}(t, \alpha),$$

$$\lim_{n\to\infty} \overline{s_n}(t, \alpha) = \overline{x}(t, \alpha).$$

7. Numerical examples

Here, we consider two examples to illustrate the ADM. Their computations have been carried out using MATLAB.

Example 6.1 Consider the initial value problem,

$$\begin{cases} x'(t) = x(t) + m(t)\lambda_k(x_{t_k}), \\ t \in [t_k, t_{k+1}], \quad t_k = k, \quad k = 0, 1, \ldots, \\ x(0) = [0.75, 1, 1.125], \end{cases} \tag{14}$$

where

$$m(t) = \begin{cases} 2(t(\mathrm{mod}\ 1)), & t(\mathrm{mod}\ 1) \le 0.5, \\ 2(1 - t(\mathrm{mod}\ 1)), & t(\mathrm{mod}\ 1) > 0.5, \end{cases} \tag{15}$$

and

$$\lambda_k(\mu) = \begin{cases} \widehat{0}, & k = 0, \\ \mu, & k \in \{1, 2, \ldots\}, \end{cases} \tag{16}$$

for which $\widehat{0} \in E^n$ is defined as $\widehat{0}(x) = 1$ if $x = 0$ and $\widehat{0}(x) = 0$ if $x \neq 0$.

The hybrid fuzzy initial problem (14) is equivalent to the following system of fuzzy initial value problems:

$$\begin{cases} x_0'(t) = x_0(t), \quad t \in [0, 1], \\ x(0) = [0.75, 1, 1.125], \\ x_i'(t) = x_i(t) + m(t)x_i(t_i), \quad t \in [t_i, t_{i+1}]. \end{cases}$$

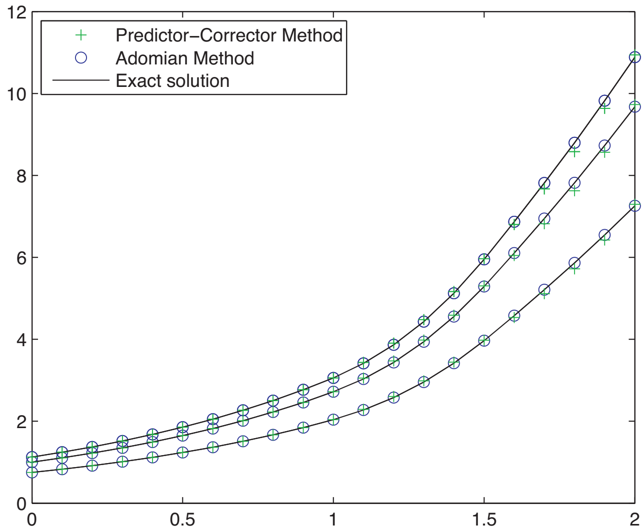

Fig. 1. Comparison between the exact solution and behavior of the solution obtained by ADM and PCM of Example 6.1.

In Eq. (14), $x(t) + m(t)\lambda_k(x_{t_k})$ is a continuous function of t, x and $\lambda_k(x_{t_k})$. Therefore, by Example 6.1 of Kaleva [1], for each $k = 0, 1, 2, \ldots$, the fuzzy initial value problem

$$\begin{cases} x'(t) = f(t, x(t), \lambda_k(x_k)), & t \in [t_k, t_{k+1}], \\ x(t_k) = x_k, \end{cases}$$

has a unique solution on $[t_k, t_{k+1}]$.

For [0, 1], the exact solution of Eq. (14) satisfies,

$x(t) = [0.75e^t, e^t, 1.125e^t]$.

For [1, 1.5] the exact solution of (14) satisfies,

$x(t) = x(1)(3e^{t-1} - 2t)$.

For [1.5, 2] the exact solution of (14) satisfies,

$x(t) = x(1)(2t - 2te^{t-1.5}(3\sqrt{e} - 4))$.

Table 1
Comparison of approximation solutions with exact solution for y^l of Example 6.1.

t	Exact	PCM	ADM
0.0	0.750000	0.750000	0.750000
0.1	0.828878	0.828878	0.828878
0.2	0.916052	0.916052	0.916052
0.3	1.012394	1.012396	1.012394
0.4	1.118868	1.118874	1.118868
0.5	1.236540	1.236550	1.236540
0.6	1.366589	1.366602	1.366589
0.7	1.510314	1.510333	1.510314
0.8	1.669155	1.669180	1.669155
0.9	1.844702	1.844734	1.844702
1.0	2.038711	2.038752	2.038711
1.1	2.274208	2.290615	2.274208
1.2	2.577355	2.604161	2.577355
1.3	2.955267	2.985790	2.955267
1.4	3.415808	3.442668	3.415808
1.5	3.967666	3.977376	3.967666
1.6	4.554410	4.533824	4.578278
1.7	5.210226	5.113589	5.210226
1.8	5.865754	5.719217	5.865754
1.9	6.547343	6.423019	6.547342
2.0	7.257731	7.295736	7.257731

Table 2
Comparison of approximation solutions with exact solution for y^c of Example 6.1.

t	Exact	PCM	ADM
0.0	1.000000	1.000000	1.000000
0.1	1.105170	1.105170	1.105170
0.2	1.221402	1.221402	1.221402
0.3	1.349858	1.349862	1.349858
0.4	1.491824	1.491832	1.491824
0.5	1.648721	1.648733	1.648721
0.6	1.822118	1.822137	1.822118
0.7	2.013752	2.013777	2.013752
0.8	2.225540	2.225574	2.225540
0.9	2.459603	2.459646	2.459603
1.0	2.718281	2.718336	2.718281
1.1	3.032278	3.054154	3.032278
1.2	3.436474	3.472214	3.436474
1.3	3.940357	3.981054	3.940357
1.4	4.554410	4.590224	4.554410
1.5	5.290221	5.303169	5.290221
1.6	6.104371	6.045098	6.104371
1.7	6.946969	6.818119	6.946968
1.8	7.821006	7.625623	7.821006
1.9	8.729790	8.564025	8.729790
2.0	9.676975	9.727648	9.676975

Table 3
Comparison of approximation solutions with exact solution for y^r of Example 6.1.

t	Exact	PCM	ADM
0.0	1.125000	1.125000	1.125000
0.1	1.243317	1.243317	1.243317
0.2	1.374078	1.374078	1.374078
0.3	1.518591	1.518594	1.518591
0.4	1.678302	1.678311	1.678302
0.5	1.854811	1.854825	1.854811
0.6	2.049883	2.049904	2.049883
0.7	2.265471	2.265500	2.265471
0.8	2.503733	2.503771	2.503733
0.9	2.767053	2.767102	2.767053
1.0	3.058067	3.058128	3.058067
1.1	3.411312	3.435923	3.411312
1.2	3.866033	3.906241	3.866033
1.3	4.432901	4.478686	4.432901
1.4	5.123712	5.164003	5.123712
1.5	5.951499	5.966065	5.951499
1.6	6.867417	6.800736	6.867417
1.7	7.815340	7.670384	7.815340
1.8	8.798632	8.578826	8.798632
1.9	9.821014	9.634529	9.821014
2.0	10.886597	10.943604	10.886597

Table 4
Comparison of approximation solutions with exact solution for y^l of Example 6.2.

t	Exact	PCM	ADM
0.0	0.750000	0.750000	0.750000
0.1	0.641065	0.641065	0.641065
0.2	0.538547	0.538547	0.538547
0.3	0.441418	0.435666	0.441418
0.4	0.348707	0.335927	0.348707
0.5	0.259487	0.238110	0.259487
0.6	0.172863	0.141043	0.172863
0.7	0.087970	0.043541	0.087970
0.8	0.003956	−0.055610	0.003956
0.9	−0.080016	−0.157676	−0.080016
1.0	−0.164790	−0.263982	−0.164790
1.1	−0.254229	−0.382524	−0.253783
1.2	−0.353721	−0.516839	−0.350204
1.3	−0.465896	−0.670146	−0.454287
1.4	−0.592617	−0.844844	−0.565938
1.5	−0.734925	−1.042712	−0.684844
1.6	−0.893071	−1.264997	− 0.810611
1.7	−1.066635	−1.512581	−0.942918
1.8	−1.254716	−1.786223	−1.081671
1.9	−1.456183	−2.183149	−1.227146
2.0	−1.669959	−2.653593	−1.380104

By using ADM:
For [0, 1] set,

$$L = \frac{d(\cdot)}{dt} \quad \rightarrow \quad L^{-1} = \int (\cdot)dt,$$

$$N = y(t),$$

and from relation (9), we have,

$$y^l = \frac{3}{4} + \frac{3}{4}t + \frac{3}{8}t^2 + \frac{1}{8}t^3 + \frac{1}{32}t^4 + \frac{1}{160}t^5 + \frac{1}{960}t^6 + \frac{1}{6726}t^7 + \cdots$$

$$y^c = 1 + t + \frac{1}{2}t^2 + \frac{1}{6}t^3 + \frac{1}{24}t^4 + \frac{1}{120}t^5 + \frac{1}{720}t^6 + \frac{1}{5040}t^7 + \frac{1}{40320}t^8 + \cdots$$

$$y^r = \frac{9}{8} + \frac{9}{8}t + \frac{9}{16}t^2 + \frac{3}{16}t^3 + \frac{3}{64}t^4 + \frac{3}{320}t^5 + \frac{1}{640}t^6 + \frac{1}{4480}t^7 + \cdots$$

which $y(t) = (y^l(t),\ y^c(t),\ y^r(t))$ is calculated by ADM convergence to exact solution $x(t)$.

For intervals $[t_k, t_{k+1}]$, $k = 1, 2, \ldots$, superpose, $N = y(t)$ and $g(t) = m(t)y_i(t_i)$.

The results of Example 6.1 on [0, 2], by using ADM, PCM and exact solution, are shown in Fig. 1 and the numerical results are shown in Table 1–3. The results show that this method is reliable and efficient technique to find analytic solutions for HFDEs.

Example 6.2 Consider the initial value problem,

$$\begin{cases} x'(t) = -x(t) + m(t)\lambda_k(x_{t_k}), \\ t \in [t_k, t_{k+1}], \ \ t_k = k, \ \ k = 0, 1, \ldots, \\ x(0) = [0.75,\ 1,\ 1.125], \end{cases} \tag{17}$$

where

$$m(t) = |\sin(\pi t)|, \quad k = 0, 1, \ldots, \tag{18}$$

$$\lambda_k(\mu) = \begin{cases} \widehat{0}, & k = 0, \\ \mu, & k \in \{1, 2, \ldots\}, \end{cases}$$

For [0, 1], the exact solution of (17) satisfies,

$$x(t) = [-0.1875e^t + 0.9375e^{-t},\ e^{-t},\ 0.1875e^t + 0.9375e^{-t}].$$

For [1, 2], the exact solution of (17) satisfies,

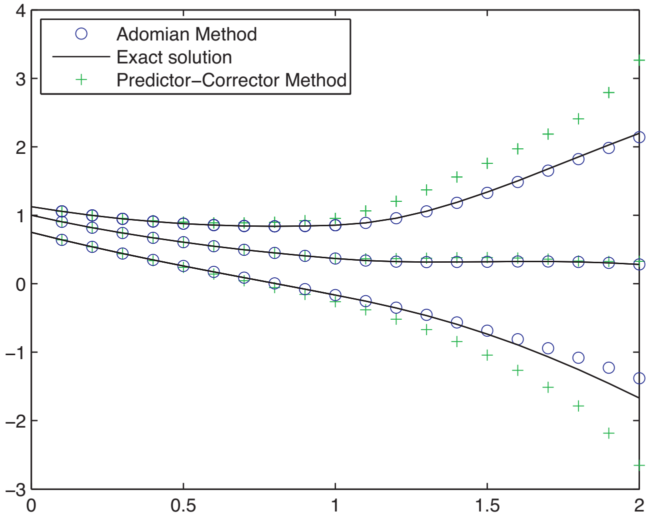

Fig. 2. Comparison between the exact solution and behavior of the solution obtained by ADM and PCM of Example 6.2.

$$x(t) = \left[-0.1875\left(e^t + \frac{1}{1+\pi^2}(e(\sin(\pi t) + \pi\cos(\pi t)) + \pi e^t)\right) + 0.9375\left(e^{-t} - \frac{1}{1+\pi^2}(e^{-1}(\sin(\pi t) - \pi\cos(\pi t)) - \pi e^{-t})\right),\ e^{-t} - \frac{1}{1+\pi^2}(e^{-1}(\sin(\pi t) - \pi\cos(\pi t)) - \pi e^{-t}), 0.1875\left(e^t + \frac{1}{1+\pi^2}(e(\sin(\pi t) + \pi\cos(\pi t)) + \pi e^t)\right) + 0.9375\left(e^{-t} - \frac{1}{1+\pi^2}(e^{-1}(\sin(\pi t) - \pi\cos(\pi t)) - \pi e^{-t})\right)\right].$$

Table 5
Comparison of approximation solutions with exact solution for y^c of Example 6.2.

t	Exact	PCM	ADM
0.0	1.000000	1.000000	1.000000
0.1	0.904837	0.904837	0.904837
0.2	0.818730	0.818730	0.818730
0.3	0.740818	0.740822	0.740818
0.4	0.670320	0.670328	0.670320
0.5	0.606530	0.606542	0.606530
0.6	0.548811	0.548825	0.548811
0.7	0.496585	0.496601	0.496585
0.8	0.449328	0.449346	0.449328
0.9	0.406569	0.406587	0.406569
1.0	0.367879	0.367898	0.367879
1.1	0.338415	0.363150	0.338415
1.2	0.322120	0.366600	0.322120
1.3	0.316184	0.373792	0.316184
1.4	0.317201	0.380288	0.317201
1.5	0.321465	0.382036	0.321465
1.6	0.325294	0.375775	0.325294
1.7	0.325361	0.359321	0.325361
1.8	0.318987	0.331754	0.318987
1.9	0.304378	0.325420	0.304378
2.0	0.280777	0.3262256	0.280777

Table 6
Comparison of approximation solutions with exact solution for y^r of Example 6.2.

t	Exact	PCM	ADM
0.0	1.125000	1.125000	1.125000
0.1	1.055504	1.055504	1.055504
0.2	0.996573	0.996573	0.996573
0.3	0.947615	0.953376	0.947615
0.4	0.908142	0.920938	0.908142
0.5	0.877757	0.899155	0.877757
0.6	0.856158	0.888004	0.856158
0.7	0.843127	0.887585	0.843127
0.8	0.838534	0.898134	0.838534
0.9	0.842334	0.920027	0.842334
1.0	0.854564	0.953791	0.854564
1.1	0.888758	1.063430	0.888672
1.2	0.957697	1.204215	0.957018
1.3	1.058741	1.371007	1.056503
1.4	1.187370	1.557884	1.182225
1.5	1.337673	1.759031	1.328014
1.6	1.502999	1.969576	1.487095
1.7	1.676689	2.186309	1.652824
1.8	1.852819	2.408264	1.819430
1.9	2.026894	2.793312	1.982687
2.0	2.196416	3.265266	2.140442

The hybrid fuzzy initial value problem (17) is equivalent to the following system:

$$\begin{cases} x_0^{l\,\prime}(t) = -x_0^{r\,\prime}(t), \quad t \in [0, 1], \\ x_0^{c\,\prime}(t) = -x_0^{c\,\prime}(t), \\ x_0^{r\,\prime}(t) = -x_0^{l\,\prime}(t), \\ x(0) = [0.75, 1, 1.125], \\ x_i^{l\,\prime}(t) = -x_i^{r\,\prime}(t) + m(t)x_i^{l\,\prime}(t_i), \quad t \in [t_i, t_{i+1}], \\ x_i^{c\,\prime}(t) = -x_i^{c\,\prime}(t) + m(t)x_i^{c\,\prime}(t_i), \\ x_i^{r\,\prime}(t) = -x_i^{l\,\prime}(t) + m(t)x_i^{r\,\prime}(t_i). \end{cases} \tag{19}$$

By using ADM:

$$\begin{cases} x_0^{l\,\prime\prime}(t) = x_0^{l}(t), \quad t \in [0, 1], \\ x_0^{c\,\prime}(t) = -x_0^{c}(t), \\ x_0^{r\,\prime\prime}(t) = x_0^{r}(t), \\ x(0) = [0.75, 1, 1.125], \\ x_i^{l\,\prime\prime}(t) = x_i^{l}(t) + m(t)x_i^{l}(t_i), \quad t \in [t_i, t_{i+1}], \\ x_i^{c\,\prime}(t) = -x_i^{c}(t) + m(t)x_i^{c}(t_i), \\ x_i^{r\,\prime\prime}(t) = x_i^{r}(t) + m(t)x_i^{r}(t_i). \end{cases} \tag{20}$$

for [0, 1] set, we consider $N = y(t)$, set

$$L = \frac{d^2(\cdot)}{dt^2} \quad \rightarrow \quad L^{-1} = \int\int(\cdot)dtdt.$$

The comparison between the exact and numerical solutions on [0, 2] is shown in Fig. 2 and the numerical results are shown in Tables 4–6. The approximate solution of this problem by ADM is nearly similar to those obtained with the exact solution, but the results of PCM were not in excellent agreement with the exact solution.

8. Conclusion

In this paper, we used the Adomian decomposition method (ADM) to obtain a numerical approximation for solution of hybrid fuzzy differential equations. This method is so powerful and efficient that it give approximations of higher accuracy. Two examples were tested by applying the ADM and predictor corrector method (PCM) and the results have shown remarkable performance. The results of our example tell us that successful method can be a way for the solutions of HFDEs and the numerical results show that ADM provides high accuracy compared to PCM.

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