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Assessing misconceptions in reasoning about variability among high school students

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Abstract

Previous studies demonstrated that students encountered difficulties in understanding the concept of variability. Hence, the purpose of this study is to identify the misconceptions in reasoning about variability among tenth grade students in Malaysia. In this study, a statistical reasoning test consisting of five main questions with 16 items was employed. A total of 412 students from nine secondary schools participated in solving problems. The results showed that 41.5% of students answered a question concerning standard deviation incorrectly. Two new misconceptions about variability have been discovered in this study. Several recommendations are proposed in order to overcome students' incorrect reasoning. This study helps instructors and researchers to design appropriate assessment and learning activities in a statistics classroom.

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1. Introduction

Most of the students face difficulties in understanding the concept of variability as it is a complicated domain to teach and learn (Ben-Zvi & Garfield, 2004). Earlier studies indicated that students held misconceptions or incorrect reasoning in variability (Lee & Meletiou-Mavrothesis, 2003; Reading & Shaugnessy, 2004; Garfield & Ben-Zvi, 2008b). For instance, students tended to compare the variability by looking at the vertical axes instead of horizontal axes of the histograms. Not only that, students gave incorrect justification for variability even though they selected the correct answer (Lee & Meletiou-Mavrothesis, 2003). Moreover, students also faced problems when concepts of center and variability were combined in the task (Sharma, 2003).

Another difficulty is that students know what standard deviation is and how to compute it, but do not know how to interpret it (Garfield & Ben-Zvi, 2008b). Study of Matthew and Clark (2007) also showed that students were unable to explain the algorithm of standard deviation and could not memorize concept of standard deviation as the formula of standard deviation and computation thereof was complex and messy (Shaughnessy, 1997; Reading & Shaughnessy, 2004). In addition, students are taught what the formula is and how to carry out the

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calculation. Such instruction cannot foster conceptual understanding of standard deviation for the students (delMas & Liu, 2005). The difficulties encountered by the students will consequently lead to the misconceptions in statistical reasoning (Garfield, 2003; Tempelaar, Gijselaers, & van der Loeff, 2006; Garfield & Ben-Zvi, 2007). Statistical reasoning involves the approach that person reason with statistical concepts and grasp the statistical information. It is including interpreting data, connecting one concept to another, and explaining statistical procedures (Garfield & Chance, 2000).

2. Reasoning about variability

Variability consists of range, interquartile, variance, and standard deviation. It is synonym for dispersion and spread. The term of variability is always utilized interchangeably with the term of variation. However, Reading and Shaughnessy (2004) deemed these two terms are different as variability is regarded as the attribute of the entity that is apparent and variation involves portraying or assessing that attribute. Distribution is related to the conceptual understanding of variability (Makar & Confrey, 2003; Bakker, 2004; Ben-Zvi, 2004; delMas & Liu, 2005; Makar & Confrey, 2005; Reading & Reid, 2006; Pfannkuch & Reading, 2006) and variability of the data can be inspected via distribution which operates as lens (Wild, 2006). According to the statistical thinking framework of Wild and Pfannkuch (1999), there are four features of consideration of variation, i.e. measuring and modeling, noticing and acknowledging, explaining and dealing with, and investigative strategies. Two supplementary features were proposed by Reading and Shaughnessy (2004), namely representing and describing. In fact, variability is the primary constituent of statistical thinking (Pfannkuch, 1997; Shaughnessy, 1997; Pfannkuch & Wild, 2004).

Torok and Watson (2000) developed four levels of variation including weak appreciation of variation, isolated appreciation of aspects of variation and clustering, inconsistent appreciation of variation and clustering, and good, consistent appreciation of variation and clustering. Meanwhile, Reading and Reid (2005) developed a hierarchy of levels of consideration of variation, namely no consideration of variation, weak consideration of variation, developing consideration of variation, and strong consideration of variation. Besides, Garfield and Ben-Zvi (2005) put forth seven elements of understanding of variability, i.e. promoting insightful concepts of variability, expressing and characterizing variability, utilizing variability to make comparisons, distinguishing variability in unique kinds of distributions, recognizing patterns of variability in appropriate models, employing variability to guess outcomes or random samples, and regarding variability as component of statistical thinking.

There were some earlier studies that utilize technological tools to develop students' reasoning about variability. Ben-Zvi (2004) conducted a study on two seventh-grade students to identify reasoning about variability of students in comparing groups using spreadsheet. Seven development stages of reasoning about variability were recognized, namely start from narrow and unrelated information, how to explain variability in raw data in informal way, how to make a statistical hypothesis that justify variability, how to clarify variability when comparing groups using frequency tables, how to utilize center and spread to compare groups, how to model variability informally via governing distant values, and how to discern and differentiate the variability within and between the distributions in a graph. A few factors assisted students to enhance their statistical reasoning. One of the factors is that students had prior experiences with the data. Besides, they also conduct experiment constantly using different methods and tools. Furthermore, the utilization of information technology allows students investigate data directly and deal with data representations. In addition, the interaction between the instructors and students also provide guidance to the students.

An exploratory study had been done by delMas and Liu (2005) on 13 university students who took introductory statistics course. There are two objectives in the study, i.e. to foster students' understanding on how deviation from the mean and frequency to identify the standard deviation value as well as enhance students' understanding on how the shape is related to the size of standard deviation. In the study, students used the conceptually enhanced software when they are interviewed by the researcher. The findings showed that eleven types of justifications were discovered including Mean in the Middle, Big Mean, Balance, Bell-shaped, Equally Spread Out, Contiguous, Mirror Image, Far Away, Location, Guess and Check, and More Bars in the Middle.

The justifications are classified into three categories, which are largest standard deviation, smallest standard deviation, and the same standard deviation.

A few studies were implemented on students' understanding of variability, but little is known about the real situation in Malaysian high schools. Thus, this study was carried out to diagnose tenth grade students' misconceptions in reasoning about variability, particularly in standard deviation. The results attained from the data analysis were utilized to answer the research question, i.e. 'What are the misconceptions held by tenth grade students about variability?

3. Methodology

3.1 Participants

There were 412 tenth grade students from nine Malaysian secondary schools participating in this study. 172 of them were male while 240 of them were female, that is 41.74% and 58.25% respectively. Among them, there were 229 Malay students (55.58%), 159 Chinese (38.59%), 22 Indian (5.34%), 1 Kadazan Dusun (0.24%), and 1 Iban (0.24%). The researcher conducted this study at the end of November 2011, so students already studied descriptive statistics. The statistical reasoning test was administered to the students during the mathematics or additional mathematics period. The students had to complete the test within one hour. Then, the students' solutions were used as data in this study. Owing to the ethical issues, each student was given a code: C in C008MI refers to school, 008 represents participants, M is for male, and I is for Indian.

3.2 Instrumentation

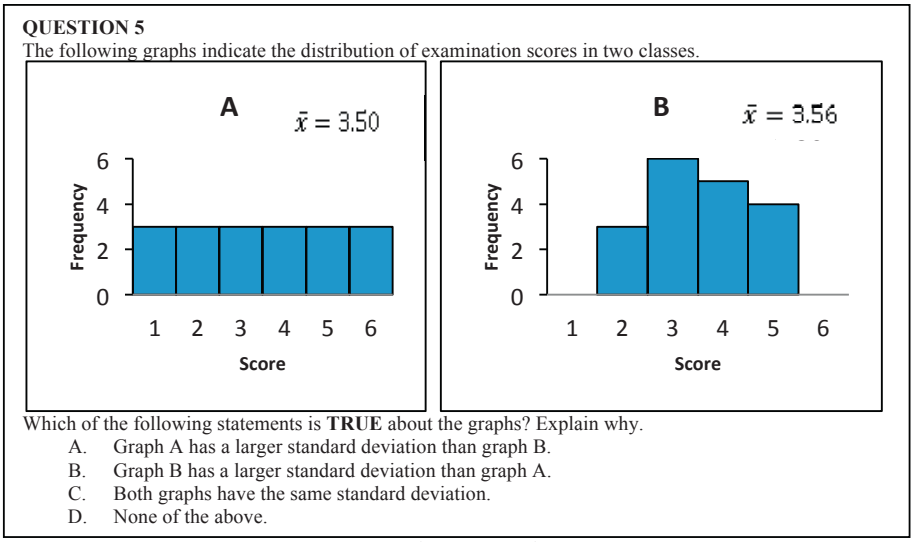

Figure 1. Question 5

A statistical reasoning test was utilized as instrument in this study to assess students' misconceptions in descriptive statistics. There are five questions with 16 items altogether. Question 5 in Figure 1 is about comparing standard deviation for two different data sets. It was modified from the item in Garfield and Ben-Zvi (2008). In the first part, students had to compare the standard deviation on the histogram, while in the second part, they were required to provide their reasoning. 'A' is the correct solution for Question 5a. For Question 5b,

the correct answer is graph A has a higher standard deviation than graph B because graph B has a higher proportion of its value clustered closer to the mean and graph A is more spread out than graph B.

4. Findings and discussions

Table 1. Correct and incorrect reasoning of students in Question 5a and 5b

Question 5a	Question 5b	Number of students	Percentage (%)
Correct	Correct	10	2.43
Incorrect	Incorrect	171	41.50
Incorrect	Correct	0	0
Correct	Incorrect	21	5.10

From Table 1, we can see that only 10 students out of 412 (2.42%) answered Question 5a and 5b correctly. One hundred and seventy one of them solve Question 5 wrongly, i.e. 41.5%. There are 21 students (5.10%) who were able to solve Question 5a, but could not provide their reason in Question 5b, for instance students with code F132, F152, H200, K283, and L303.

Table 2. Misconceptions in question 5

No	Misconceptions	Number of Students	Percentage (%)
1	Value of standard deviation is equivalent to value of mean	221	53.64
2	Same frequency equals to same standard deviation	40	9.71
3	Irrelevant	2	0.49
4	No attempt	77	18.69

Two new misconceptions are illustrated in Table 2, i.e. “value of standard deviation is equivalent to value of mean”, and “same frequency equals to same standard deviation”. 9.17% of students chose ‘C’ as their answer because they thought that frequencies of score for graph A and B were same, that is 18, so the standard deviation must be the same for both graphs. In addition, high percentage of students (53.64%) selected ‘B’ as they recognize that value of mean is equivalent to value of standard deviation. It means that standard deviation increases when mean increases, and standard deviation decreases when mean decreases. Besides, 0.49% of students solved this question incorrectly, and 18.69% of them did not try to answer the question. These results confirmed the statement of delMas and Liu (2005) that students tend to misunderstand the concept of variability when it is represented graphically.

5. Recommendations

Some recommendations are suggested to address students’ misconceptions in reasoning about variability. Firstly, instructors can incorporate information technology into statistics classroom as it can promote statistical reasoning of students (delMas & Liu, 2005; Chan & Ismail, 2012). The existence of technology incurs the changes in pedagogy of statistics education from traditional teacher-centred instruction to student-centred instruction. Previously, students just received and absorbed the knowledge given by the instructors. But now students participate actively in the classroom and construct their own knowledge through group work, discussion, interaction with each other (Moore, 1997; Forsyth, 2003). Instructors can guide the students to analyze data sets, generate graphs, carry out simulations, and solve open-ended questions using technological tools in statistics class or course. Besides, technological tools enable students to perform many computations precisely in a short time. Hence, students will have adequate time to focus on interpreting the findings and understanding the underlying concepts rather than on calculation mechanics (Garfield & Ben-Zvi, 2008a).

Secondly, remedial instruction can be conducted to reduce students’ incorrect reasoning about variability. Instructors should employ different types of strategies in the classroom, such as mastery learning, cooperative learning, and collaborative learning. Besides, the learning materials or assessments ought to be diversified as well, for instance project works, open-ended problems, real world data and so forth. In the classroom, instructors

have to emphasize on the development of conceptual understanding of statistics in order to enhance students' statistical reasoning. Thirdly, instructors can promote classroom discourse by encouraging students react to the questions, learn to ask questions and defend their solutions and arguments. The examples of good questions that can lead to discourse are 'what do you think' and 'what would happen if'. It is imperative to create an effective classroom climate where students have the opportunity to articulate and express their opinions.

6. Conclusion

To conclude, Malaysian tenth grade students still harboured misconceptions in reasoning about variability. Thus, as far as this problem is concerned, instructors ought to correct the students' misconceptions before they get worse. This study provides a guideline for the instructors and researchers to plan for the suitable assessments and instructional activities in the statistics classroom.

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References

Bakker, A. (2004). Reasoning about shape as a pattern in variability. *Statistics Education Research Journal*, *3*(2), 64-83.

Ben-Zvi, D. (2004). Reasoning about Variability in Comparing Distributions. *Statistics Education Research Journal*, *3*(2), 42-63.

Ben-Zvi, D., & Garfield, J. (2004). Research on Reasoning about Variability: A Forward. *Statistical Education Research Journal, 3*(2), 4-6.

Chan, S.W., & Zaleha Ismail (2012). The Role of Information Technology in Developing Students' Statistical Reasoning. *Procedia-Social and Behavioral Sciences*, *46*, 3660-3664.

delMas, R.C., & Liu, Y. (2005). Exploring Students' Conceptions of the Standard Deviation. *Statistics Education Research Journal, 4*(1), 55-82.

Forsyth, D.R. (2003). *The Professor's Guide to Teaching: Psychological Principles and Practices*. Washington: American Psychological Association.

Garfield, J. (2003). Assessing Statistical Reasoning. *Statistics Education Research Journal*, *2*(1), 22-38.

Garfield, J., & Chance, B. (2000). Assessment in Statistics Education: Issues and Challenges. *Mathematical Thinking and Learning*, *2*(1&2), 99-125.

Garfield, J., & Ben-Zvi, D. (2005). A Framework for Teaching and Assessing Reasoning about Variability. *Statistics Education Research Journal*, *4*(1), 92-99.

Garfield, J., & Ben-Zvi, D. (2007). How Students Learn Statistics Revisited: A Current Review of Research on Teaching and Learning Statistics. *International Statistical Review*, *75*(3), 372-396.

Garfield, J.B., & Ben-Zvi, D. (2008a). Using Technology to Improve Student Learning of Statistics. In J.B. Garfield, & D. Ben-Zvi (Eds.), *Developing Students' Statistical Reasoning: Connecting Research and Teaching Practice* (pp. 91-114). Springer.

Garfield, J.B., & Ben-Zvi, D. (2008b). Learning to reason about variability. In J.B. Garfield, & D. Ben-Zvi (Eds.), *Developing Students' Statistical Reasoning: Connecting Research and Teaching Practice* (pp. 201-214). Springer.

Lee, C., & Meletiou-Mavrotheris, M. (2003). Some Difficulties of Learning Histograms in Introductory Statistics. *Joint Statistical Meetings – Section on Statistical Education*. Retrieved October 17, 2011 from http://www.statlit.org/PDF/2003LeeASA.pdf

Makar, K.,& Confrey, J. (2003). Chunks, Clumps and Spread Out: Secondary Pre-service Teachers' Notions of Variation and Distribution. In C. Lee (Ed.), *Proceedings of the Third International Research Forum on Statistical Reasoning, Thinking and Literacy (SRTL-3)* (pp.). Mount Pleasant, Michigan: Central Michigan University.

Makar, K., & Confrey, J. (2005). "Variation-talks": Articulating Meaning in Statistics. *Statistics Education Research Journal*, *4*(1), 27-54.

Matthews, D., & Clark, J. (2007). *Successful Students' Conceptions of Mean, Standard Deviation and the Central Limit Theorem*. Retrieved April 6, 2012 from http://www1.hollins.edu/faculty/clarkjm/stats1.pdf

Moore, D.S. (1997). New Pedagogy and New Content: The Case of Statistics. *International Statistical Review*, *65*(2), 123-165.

Pfannkuch, M. (1997). Statistical thinking: One statistician's perspective. In F. Biddulph & K. Carr (Eds.), *People in mathematics education (Proceedings of the 20th annual conference of the Mathematics Education Research Group of Australasia)* (pp. 406-413). Rotorua, New Zealand: MERGA.

Pfannkuch, M., & Reading, C. (2006). Reasoning about Distribution: A Complex Process. *Statistics Education Research Journal*, *5*(2), 4-9.

Pfannkuch, M., & Wild, C. (2004). Towards an understanding of statistical thinking. In D. Ben-Zvi & J. Garfield (Eds.), *The challenge of developing statistical literacy, reasoning, and thinking* (pp.17-46). Dordrecht, the Netherlands: Kluwer Academic Publishers.

Reading, C., & Shaughnessy, M. (2004). Reason about variation. In D. Ben-Zvi, & J. Garfield (Eds.), *The Challenge of Developing Statistical Literacy, Reasoning, and Thinking* (pp. 201-226). The Netherlands: Kluwer Academic Publishers.

Reading, C., & Reid, J. (2005). Consideration of Variation: A Model for Curriculum Development. In G. Burrill, & M. Camden (Eds.), *Curriular Development in Statistics Education: International Association for Statistical Education 2004 Roundtable* (pp. 36-53). Voorburg, The Netherlands: International Statistical Institute.

Reading, C., & Reid, J. (2006). A Emerging Hierarchy of Reasoning about Distributions: From a Variation Perspective. *Statistics Education Research Journal*, *5*(2), 46-68.

Sharma, S. (2003). An Exploration of High School Students' Understanding of Sample Size and Sampling Variability: Implications for Research. *Journal of Educational Studies*, *25*(1&2), 68-83.

Shaughnessy, J.M. (1997). Missed Opportunities in Research on the Teaching and Learning of Data and Chance. In F. Bidulph and K. Carr (Eds.). *Proceedings of the Twentieth Annual Conference of the Mathematics Education Research Group of Australasia* (pp. 6-22). Rotorua, N.Z.: University of Waikata.

Tempelaar, D.T., Gijselaers, W.H., & van der Loeff, S.S. (2006). Puzzles in Statistical Reasoning. *Journal of Statistics Education*, *14*(1). Retrieved March 15, 2012, from http://www.amstat.org/publications/jse/v14n1/tempelaar.html

Torok, R., & Watson, J. (2000). Development of the Concept of Statistical Variation: An Exploratory Study. *Mathematics Education Research Journal*, *12*(2), 147-169.

Wild, C. (2006). The Concept of Distribution. *Statistics Education Research Journal*, *5*(2), 10-26.

Wild, C.J., & Pfannkuch, M. (1999). Statistical Thinking in Empirical Enquiry. *International Statistical Review, 67*(3), 223-265.