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Thermosolutal Marangoni boundary layer magnetohydrodynamic flow with the Soret and Dufour effects past a vertical flat plate

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ABSTRACT

A numerical study of laminar magnetohydrodynamic thermosolutal Marangoni convection along a vertical surface in the presence of the Soret and Dufour effects has been performed. The diffusion-thermo implies that the heat transfer is induced by concentration gradient, and thermo-diffusion implies that the mass diffusion is induced by thermal gradient. In conformity to actuality, it is assumed that the surface tension varies linearly with both the temperature and concentration and that both interface temperature and concentration are quadratic functions of the interface arc length x. The general governing partial differential equations are converted into nonlinear ordinary differential equations using unique similarity transformations. The aim of this study is to investigate the effects of Hartmann number ($0 \le M \le 5$), thermosolutal surface tension ratio ($0 \le R \le 5$), Soret parameter ($0.1 \le Sr \le 2$), Dufour parameter ($0.03 \le Du \le 0.6$), Prandtl number ($0.72 \le Pr \le 10$) and Schmidt number ($0.3 \le Sc \le 3$) on the fluid velocity heat and mass transfer. It is found that, both of temperature and concentration gradient at the wall increases as the thermosolutal surface tension ratio increases. Also, the increase in Prandtl number results in an enhancement in the heat transfer at the wall.

1. Introduction

The heat and mass transfer has been considered by theoretical and experimental studies owing to their wide applications, such as geothermal systems, energy-storage units, heat insulation, and heat exchangers for the packed bed, drying technology, catalytic reactors, and nuclear waste repository. In addition, an energy flux can be generated not only by the temperature gradient but also by the concentration gradient. The energy flux caused by a concentration gradient is termed as the diffusion-thermo (Dufour) effect. Furthermore, mass fluxes can also be created by temperature gradients and it is termed as thermo-diffusion (Soret) effect and this effect might become significant when large density differences exist in the flow regime. Because of the importance of Soret (thermal-diffusion) and Dufour (diffusion-thermo) effects for the fluids with very light molecular weight as well as medium molecular weight many investigators have studied and reported results for these flows. Eckert and Drake [1] presented several cases when the Dufour effect cannot be neglected. Adrian [2] investigated numerically the heat and mass transfer characteristics of natural convection about a vertical surface embedded in a saturated porous medium subjected to a magnetic field by taking into account the diffusion-thermo (Dufour) and thermal-diffusion (Soret) effects. Alam et al. [3] studied theoretically the problem of steady two-dimensional free convection and mass transfer flow past a continuously moving semi-infinite vertical porous plate in a porous medium by including the Soret and Dufour effects. Weaver et al. [4] have pointed out that when the differences of the temperature and the concentration are large or when the difference of the molecular mass of the two elements in a binary mixture is great, the coupled interaction is significant. Anghel et al. [5] concluded that thermal-diffusion (Soret) and diffusion-thermo (Dufour) effects appreciably influence the flow field in free convection. Mahdy [6] examined the combined effect of spatially stationary surface waves and the presence of fluid inertia on the free convection along a heated vertical wavy surface embedded in an electrically conducting fluid saturated porous medium, subject to the diffusion-thermo (Dufour), thermo-diffusion (Soret) and magnetic field effects. Mahdy [7] investigated the effect of Soret and Dufour effects on non-Newtonian mixed convection in porous media. The effect of melting and/or thermo-diffusion on convective transport in a non-Newtonian fluid saturated non-Darcy porous medium is presented by Kairi and Murthy [8] and Srinivasacharya and RamReddy [9].

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Nomenclature

a, b	positive constant
B	applied magnetic field intensity
c_p	specific heat at constant pressure
c_s	specific heat at constant concentration
D	solutal diffusivity
f	dimensionless stream function
h	heat transfer coefficient
C	concentration
Du	Dufour parameter
j	mass transfer coefficient
k	reaction rate of solute
M	Hartmann number
Pr	Prandtl number
q_m	surface mass transfer
q_w	surface heat transfer
Nu	Nusselt number
R	thermosolutal surface tension ratio
Sc	Schmidt number
Sh	Sherwood number
Sr	Soret parameter
T	dimensional temperature
T_m	mean fluid temperature
(u, v)	velocity components of the fluid
(x, y)	coordinate axes

Greek symbols

α	thermal diffusivity
ϕ	dimensionless concentration
θ	dimensionless Temperature
υ	kinematic viscosity
η	similarity variable
ρ	density of the fluid
μ	dynamic viscosity
γ	surface tension on the first derivative of the temperature
$\breve{\gamma}$	surface tension on the first derivative of the concentration
δ	surface tension
δ_0	minimum value of the surface tension, a positive constant
σ	electrical conductivity
ψ	stream function

Subscripts

w	conditions at the surface
∞	conditions in the free stream

Kundu et al. [10] discussed the problem of combined effects of thermophoresis and chemical reaction magnetohydrodynamics mixed convection flow. The influence of thermophoresis and chemical reaction on MHD micropolar fluid flow with variable fluid properties was investigated by Das [11]. Das [12] presented a numerical solution for convective slip flow of rarefied fluids over a permeable wedge plate embedded in a Darcy–Forchheimer porous medium. Also, Das [13] discussed the effects of thermophoresis and thermal radiation on MHD mixed convective heat and mass transfer flow.

On the other hand, the dissipative layers which may occur along the liquid–gas or liquid–liquid interfaces were called as Marangoni boundary layers. Marangoni convection is induced by variations of surface tension along a liquid surface and appears in many nature and engineering. Marangoni effects can be divided into the thermal Marangoni effect (EMT) and the solute Marangoni effect (EMS). EMT is caused by the thermal imbalance of the interfacial region, and this imbalance is mainly caused by the heat source and the temperature gradient. EMS is caused by the imbalance of the interfacial adsorption, and this imbalance is mainly caused by the chemical reaction and the concentration gradient. The importance of Marangoni convection, carrying out single crystal growth under the microgravity environment is recognized. The surface tension gradients that are responsible for Marangoni convection can be both temperature and/or concentration gradients (Magyari and Chamkha [14]). It seems that the basic research work in this field was first promoted by Napolitano [15,16]. Marangoni flow induced by surface tension variations along the liquid–fluid interface causes undesirable effects in crystal growth melts in the same manner as buoyancy induced natural convection [17,18]. As pointed out by Napolitano [19], the field equations in the bulk fluids do not depend explicitly on the geometry of the interface when using as coordinates the arc length (x). This, together with the other surface balance equations, introduces kinematic, thermal and pressure couplings for the flow fields in the two fluids. Napolitano and Golia [20] have shown that the fields are uncoupled when the momentum and energy resistivity ratios of the two layers and the viscosity ratio of the two fluids are much less than one. Furthermore, as shown by Napolitano and Russo [21], similarity solutions for Marangoni boundary layers exist when the interface temperature gradient varies as a power of the interface arc length (x). The power laws for all other variables, including the mean curvature, were determined. Numerical solutions were found, analyzed and discussed on Marangoni boundary layers in subsequent papers by Golia and Viviani [22,23], Pop et al. [24], Chamkha et al. [25], and Magyari and Chamkha [14], Yan and Liancun [26]. An excellent review paper on Marangoni effects has been recently published by Tadmor [27]. The aim of this paper is to study the effect of Soret and Dufour on thermosolutal Marangoni boundary layer in the presence of uniform magnetic field of an electrically conducting fluid.

2. Flow analysis

A two-dimensional steady laminar, boundary layer flow of an incompressible, viscous, electrically conducting fluid over a plate

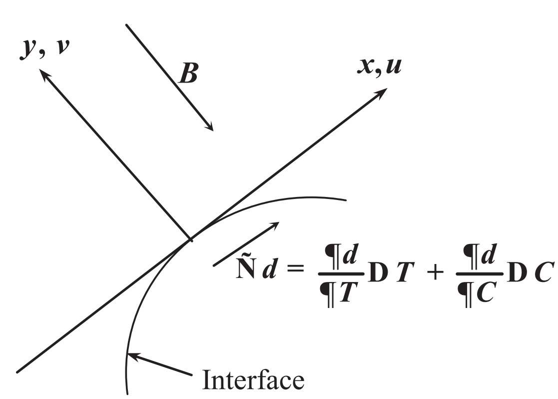

Fig. 1. Physical model and coordinate system.

Table 1
Comparison values of $f'(0)$ for various values of M.

$M = 0$		$M = 1$		$M = 2$	
[26]	Present	[26]	Present	[26]	Present
2.4569	2.519945	2.1572	2.226772	1.624	1.6785735

Table 2
Comparison of $-\theta'(1)$ with Wang [29] and Ishak et al. [30] for $\gamma = 0$ and $Re = 10$.

Pr	Wang [29]	Ishak et al. [30]	Present
0.7	1.568	1.5683	1.58679
2	3.035	3.0360	3.03553
7	6.160	6.1592	6.15776
10	10.77	7.4668	7.46419

surface in the presence of surface tension due to temperature and concentration gradients at the wall is investigated. A uniform magnetic field is applied in the horizontal direction normal to the plane, we consider a Cartesian coordinate system (x, y), where x and y are the coordinates measured along the interface and normal to it as it is shown in Fig. 1. The Soret effect, for instance, has been utilized for isotope separation and in a mixture between gases and with very light molecular weight (H_2, He), and of medium molecular weight (H_2, air) the Dufour effect was found to be of considerable magnitude such that it cannot be neglected. The fluid properties are assumed to be constant in a limited temperature range. It is assumed that the induced magnetic field, the external electric field and the electric field due to the polarization of charges are negligible. Unlike the Boussineq effect in buoyancy-induced flow, Marangoni effect acts as a boundary condition on the governing equations for the flow. Taking the above assumptions into consideration, the laminar boundary layer equations of a viscous and incompressible fluid describing mass, linear momentum, energy and concentration can be written in the usual dimensional form as

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0, \tag{1}$$

$$u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y} = \upsilon\frac{\partial^2 u}{\partial y^2} - \frac{\sigma B^2}{\rho}u, \tag{2}$$

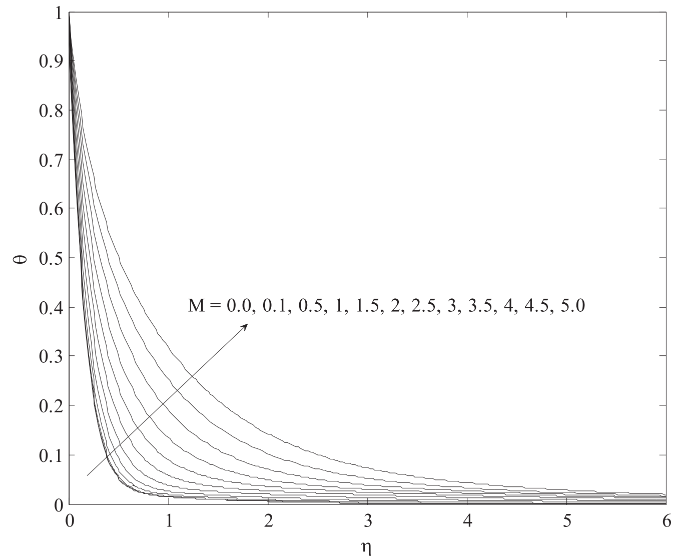

Fig. 3. Effect of Hartmann number on temperature distribution.

$$u\frac{\partial T}{\partial x} + v\frac{\partial T}{\partial y} = \alpha\frac{\partial^2 T}{\partial y^2} + \frac{Dk}{c_s c_p}\frac{\partial^2 C}{\partial y^2}, \tag{3}$$

$$u\frac{\partial C}{\partial x} + v\frac{\partial C}{\partial y} = D\frac{\partial^2 C}{\partial y^2} + \frac{Dk}{T_m}\frac{\partial^2 T}{\partial y^2}, \tag{4}$$

In the above equations x and y represent the Cartesian coordinates measured along the plate and normal to it, respectively, u and v are the velocity components along the x and y axes, υ is the kinematic viscosity, α is the thermal diffusivity, ρ is the density of the fluid, T_∞, C_∞ are the constant, denote the temperature and concentration of species far from the wall, respectively, D is the diffusion coefficient, c_s, c_p are the specific heat at constant pressure and concentration susceptibility, k is the thermal-diffusion ratio, T, C are the temperature of the fluid and concentration. T_m is the mean fluid temperature. σ, B are the electrical conductivity and the

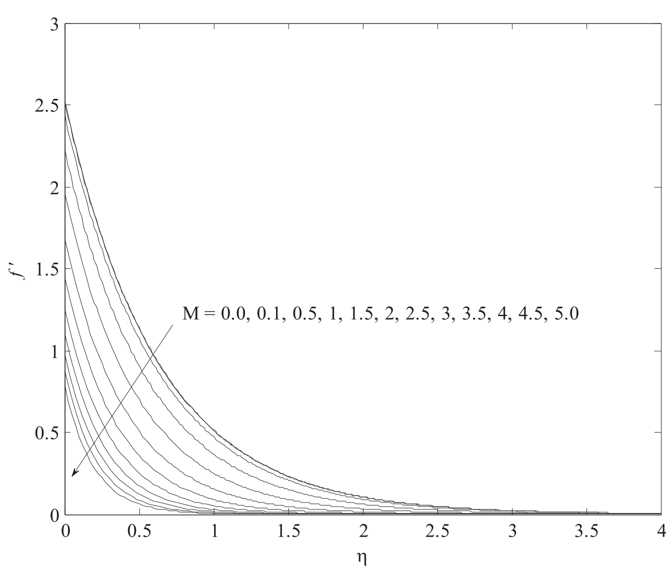

Fig. 2. Effect of Hartmann number on velocity distribution.

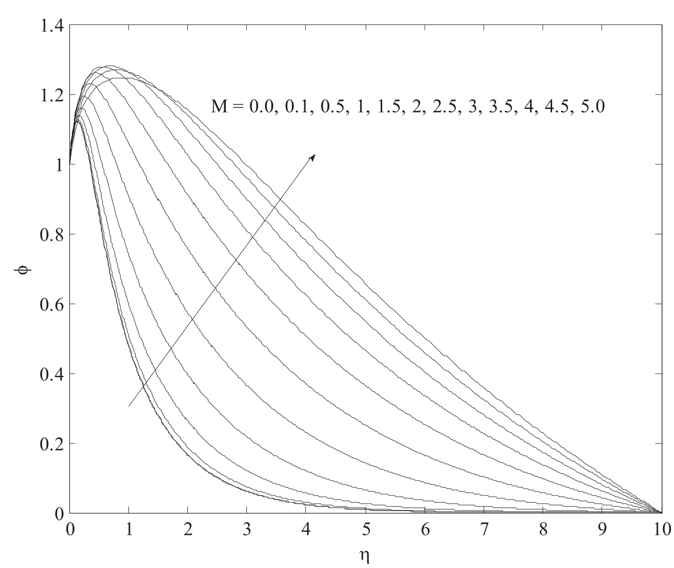

Fig. 4. Effect of Hartmann number on concentration distribution.

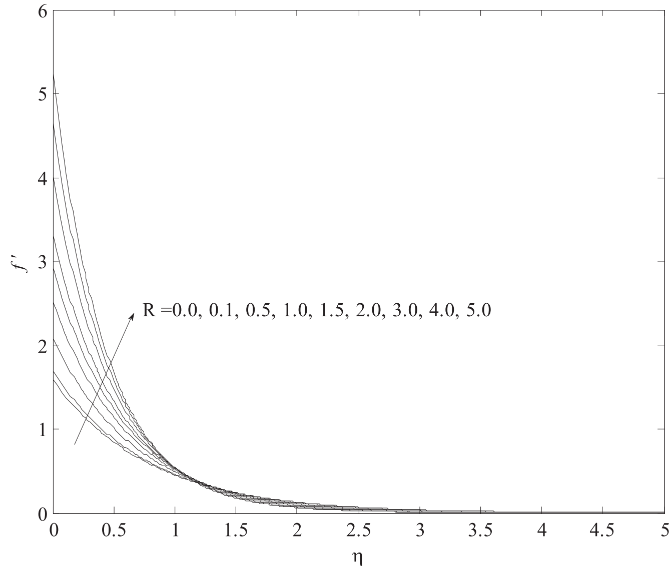

Fig. 5. Effect of thermosolutal surface tension ratio on velocity distribution.

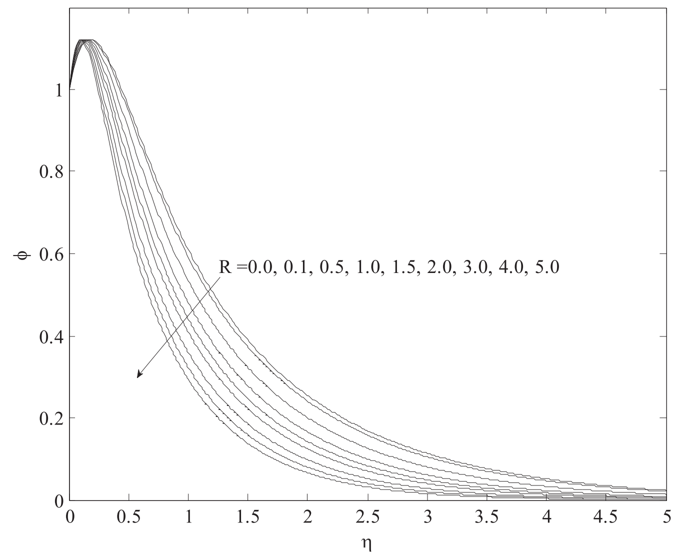

Fig. 7. Effect of thermosolutal surface tension ratio on concentration distribution.

applied magnetic flux density. The dependence of surface tension on temperature and concentration can be expressed as

$$\delta = \delta_0 - \gamma(T - T_\infty) - \breve{\gamma}\left(C - C_\infty\right) \tag{5}$$

with $\gamma = -\partial\delta/\partial T|_C$, $\breve{\gamma} = -\partial\delta/\partial C|_T$.

Considering the following dimensional boundary conditions for the governing equations

$$\mu\frac{\partial u}{\partial y}\bigg|_{y=0} = -\frac{\partial\delta}{\partial T}\bigg|_C\frac{\partial T}{\partial y}\bigg|_{y=0} - \frac{\partial\delta}{\partial C}\bigg|_T\frac{\partial C}{\partial y}\bigg|_{y=0}$$

$$v(x,0) = 0, \quad T(x,0) = T_\infty + ax^2, \quad C(x,0) = C_\infty + bx^2$$

$$u(x,\infty)\to 0, \quad T(x,\infty)\to T_\infty, \quad C(x,\infty)\to C_\infty \tag{6}$$

A majority of the existing exact solutions in fluid mechanics are similarity solutions which reduce the number of independent variables by one or more. The methods for generating similarity transformations for equations of physical interest are discussed by Ames [28]. Similarity solutions are often asymptotic solutions to a given problem and may have utility in this area of limiting solutions. Similarity solutions may be used to gain physical insight into these details of complex fluid flows and these solutions exhibit most of the characteristic as well as the influence of the physical and thermal parameters of the actual problem. In order to get a similarity solution of the problem we define the following transformations:

$$\eta = \left(\frac{a\gamma}{\mu v}\right)^{1/3} y, \quad \psi = \left(\frac{a\gamma\nu}{\rho}\right)^{\frac{1}{3}} x f(\eta), \quad \theta(\eta) = \frac{T - T_\infty}{ax^2}, \quad \phi(\eta) = \frac{C - C_\infty}{bx^2} \tag{7}$$

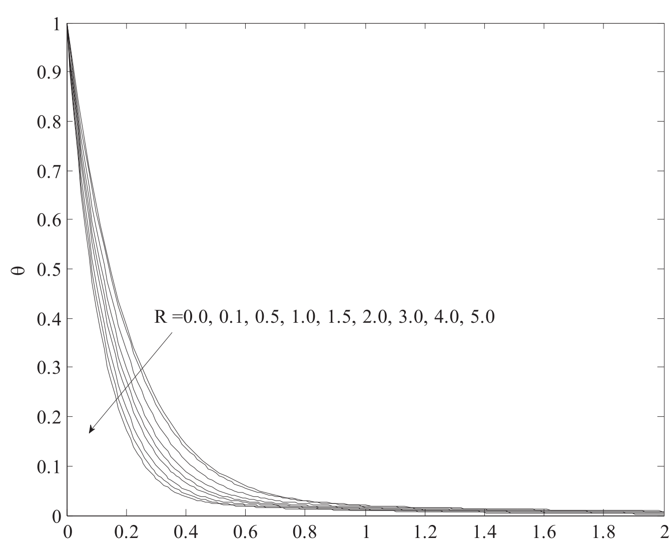

Fig. 6. Effect of thermosolutal surface tension ratio on temperature distribution.

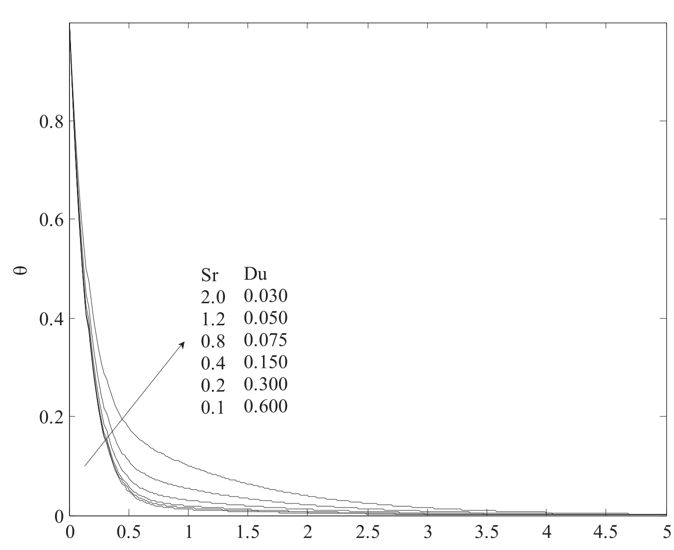

Fig. 8. Effect of Soret and Dufour parameters on temperature distribution.

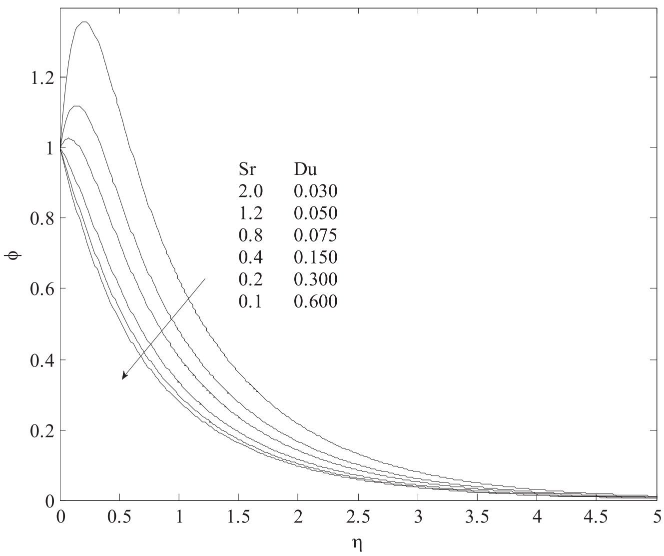

Fig. 9. Effect of Soret and Dufour parameters on concentration distribution.

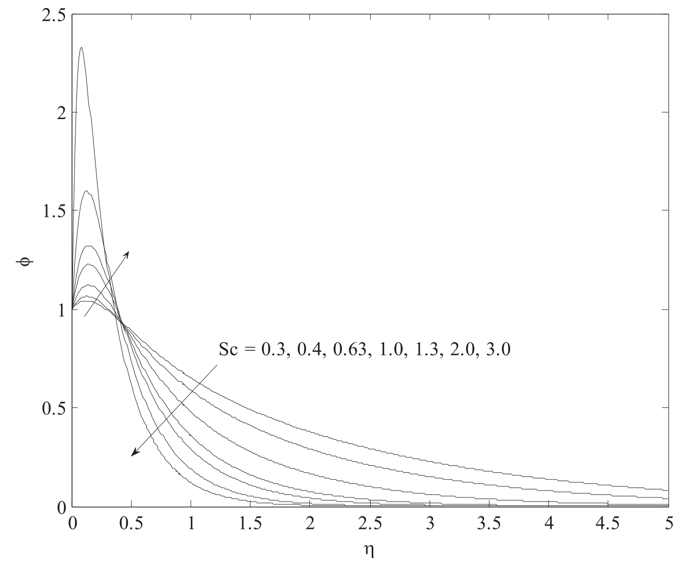

Fig. 11. Effect of Schmidt number on concentration distribution.

Substituting Eq. (7) into Eqs. (2)–(4) we obtain the following non-linear ordinary governing differential equations

$$f''' + ff'' - f'^2 - M^2 f' = 0 \tag{8}$$

$$\frac{1}{Pr}\theta'' + f\theta' - 2f'\theta + Du\phi'' = 0 \tag{9}$$

$$\frac{1}{Sc}\phi'' + f\phi' - 2f'\phi + Sr\theta'' = 0 \tag{10}$$

The boundary conditions (6) then turn into

$$f(0) = 0, f''(0) = -2(1+R), \theta(0) = 1, \phi(0) = 1$$

$$f(\infty) \to 0, \theta(\infty) \to 0, \phi(\infty) \to 0 \tag{11}$$

here, the prime denotes ordinary differentiation with respect to the similarity variable η. Furthermore, $M = \sqrt{\sigma}B\sqrt[6]{\mu}/\sqrt[3]{\rho a \gamma}$ represents the Hartmann number, $Sr = Dka/\upsilon T_{mb}$ denotes Soret parameter, $Du = Dka/\upsilon c_s c_{pb}$ is the Dufour parameter, $Pr = \upsilon/\alpha$ is the Prandtl number, $Sc = \upsilon/D$ is the Schmidt number and $R = b\breve{\gamma}/a\gamma$ is the ratio of the solutal and thermal Marangoni numbers.

The local heat and mass flux may be written by as

$$q_w = -k\frac{\partial T}{\partial y}\bigg|_{y=0} = -kax^2\sqrt[3]{\frac{a\gamma}{\mu\nu}}\theta'(0), q_m = -k\frac{\partial C}{\partial y}\bigg|_{y=0}$$
$$= -Dbx^2\sqrt[3]{\frac{a\gamma}{\mu\nu}}\phi'(0)$$

The local heat and mass transfer coefficients are given by

$$h = \frac{q_w}{ax^2}, j = \frac{q_m}{bx^2}$$

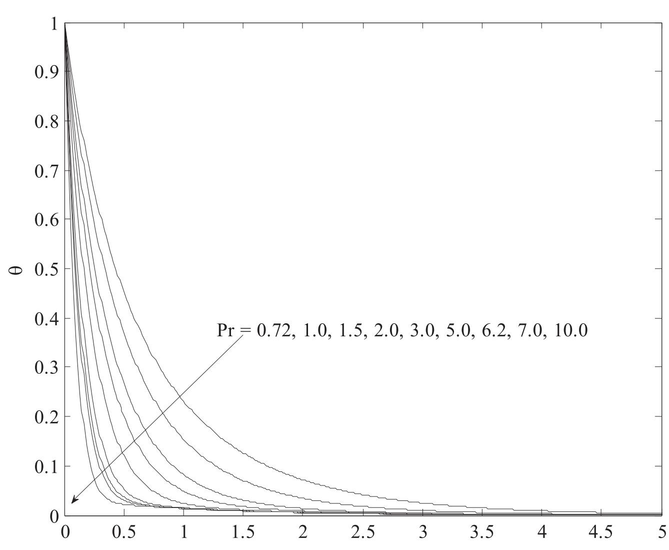

Fig. 10. Effect of Prandtl number on temperature distribution.

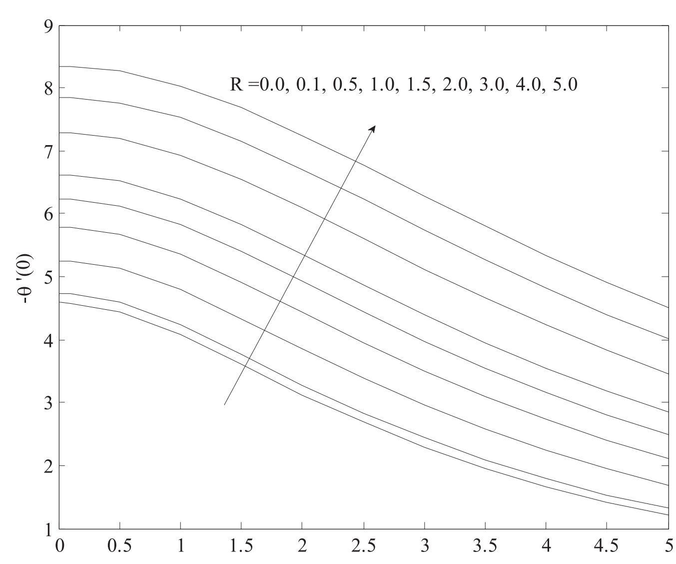

Fig. 12. Effect of thermosolutal surface tension ratio on temperature gradient vs. Hartmann number.

In practical applications, the quantity of physical interest in our case are the local Nusselt *Nu* and Sherwood numbers *Sh*, which may be written in non-dimensional form as:

$$Nu = \frac{hx}{k} = -\sqrt[3]{\frac{a\gamma}{\mu\nu}}x\theta'(0),\; Sh = \frac{jx}{D} = -\sqrt[3]{\frac{a\gamma}{\mu\nu}}x\phi'(0) \tag{12}$$

3. Numerical method

In order to obtain numerical solutions for Eqs. (8)–(11), we transfer the problem Eqs. (8)–(11) to a system of first-order equations by denoting the f, f' and f'' using variables f, P and g respectively.

$$\begin{aligned} &f' = P, P' = g, g' = P^2 + M^2P - fg \\ &\text{with } f(0) = 0, P(\infty) = 0, g(0) - 2(1+R) \end{aligned} \tag{13}$$

In order to obtain numerical solutions for the Eq. (13), we introduced the shooting parameter t as

$$P(t) = 0 \tag{14}$$

According to the assumption of the stream function, we transfer the Eq. (13) to the following form

$$\left[\frac{\partial f}{\partial t}\right]' = \frac{\partial(f')}{\partial t} = \frac{\partial P}{\partial t}$$

$$\left[\frac{\partial P}{\partial t}\right]' = \frac{\partial(P')}{\partial t} = \frac{\partial g}{\partial t}$$

$$\left[\frac{\partial g}{\partial t}\right]' = \frac{\partial(g')}{\partial t} = \left(2P + M^2\right)\frac{\partial P}{\partial t} - g\frac{\partial f}{\partial t} - f\frac{\partial g}{\partial t}$$

$$\left.\frac{\partial f}{\partial t}\right|_\eta = 0, \left.\frac{\partial P}{\partial t}\right|_\eta = 1, \left.\frac{\partial g}{\partial t}\right|_\eta = 0$$

In the process, an initial value is given to the shooting parameters, and then the classical fourth-order Runge–Kutta method is used to get the results. The iteration condition is $|P(\infty) - \beta| > 10^{-5}$. Iteration accuracy is 10^{-5}. Then, the Newton method is used to fix the shooting parameter

$$t_{i+1} = t_i - \frac{P(t_i) - \beta}{\partial P(t_i)/\partial t_i} \tag{15}$$

where, β is the shooting target ($\beta = P(\infty) = 0$). Eqs. (13) and (14) are used to obtain the item $\partial P(t_i)/\partial t_i$ in the fixed Eq. (15).

4. Results and discussion

To obtain a clear insight of the behavior of velocity temperature and concentration fields, a comprehensive numerical computation is carried out using the method described in the previous section for various values of governing parameters. Here, Soret number ranging from 0.1 to 2.0, Dufour number ranging from 0.03 to 0.6, the Hartmann number ranging from 0.0 to 5.0, Schmidt number ranging from 0.3 to 3.0, Prandtl number ranging from 0.72 to 10.0 and the ratio of the solutal and thermal Marangoni numbers ranging from 0.0 to 5.0. In addition, to validate the method used in this study and to judge the accuracy of the present analysis, comparison with available results of Yan and Liancun [26] corresponding to the $f'(0)$ is made, Table 1, and found in a very good agreement. Other comparison was performed with Wang [29] and Ishak et al. [30] to check the accuracy of our method. From Table 2, it is found that, the present method compare very well with the results obtained by Wang [29] and Ishak et al. [30].

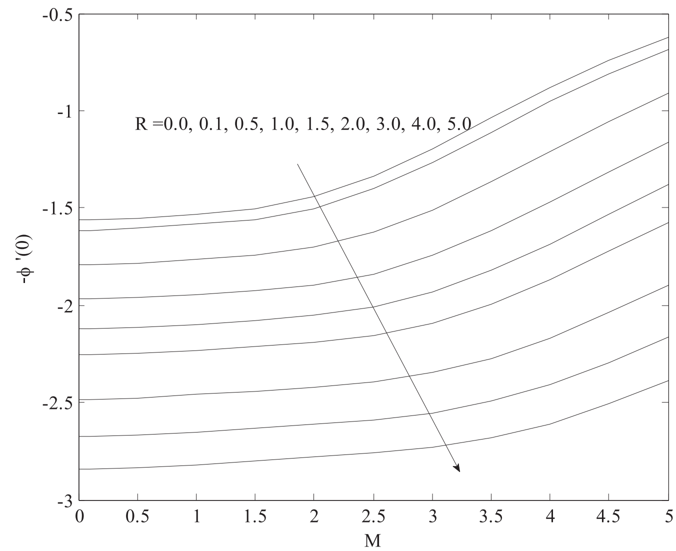

Fig. 13. Effect of thermosolutal surface tension ratio on concentration gradient vs. Hartmann number.

Figs. 2–4, respectively, depict the effects of the Hartmann number on the fluid velocity, temperature and concentration distributions. Application of a magnetic field normal to an electrically-conducting fluid has the tendency to produce a drag-like force called the Lorentz force which acts in the direction opposite to that of the flow, causing a flow retardation effect. This causes the fluid velocity to decrease. However, this decrease in flow speed is accompanied by corresponding increases in the fluid thermal state level. These behaviors are clearly depicted in the decrease in the fluid velocity and increase in the fluid temperature and concentration distributions as observed in Figs. 2–4. Furthermore, the magnetic parameter tends to decrease temperature gradient and concentration gradient (in absolute sense) as seen in Figs. 12–17.

The influence of the thermosolutal surface tension ratio R on the fluid velocity, temperature and concentration distributions are

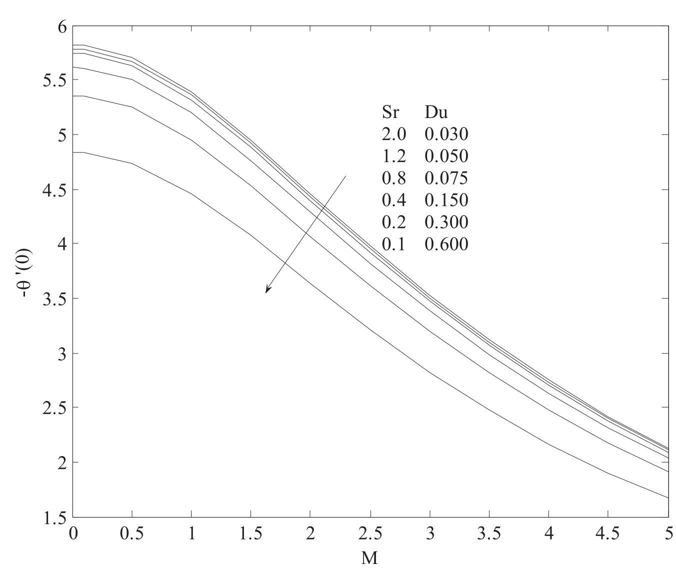

Fig. 14. Effect of Soret and Dufour parameters on temperature gradient vs. Hartmann number.

illustrated in Figs. 5–7. It is clear that as thermosolutal surface tension ratio increases, the velocity distribution increases whereas the fluid temperature and concentration decrease. This can be explained from Equation (11). This equation shows that the increase in R leads to increase the velocity gradients which in turn decrease the velocity distributions. In addition, the effect of Soret and Dufour parameters on temperature and concentration are presented in Figs. 8 and 9. It is observed from these figures that as the Soret number decreases (and the Dufour number increases) the fluid temperature profile increases. This behavior can be explained as follows: It is well known that, if two chemically different non-reacting gases or liquids, which were initially at the same temperature, are allowed to diffuse into each other, and then there arises a difference of temperatures in the system. In fact, this is, exactly, the definition of the inverse phenomenon of thermal diffusion (Dufour effects). The increase in this temperature difference leads to increase the buoyancy force which brings strong natural convection. Fig. 10 shows the behavior of the temperature distributions for the variation of Prandtl number Pr, Prandtl number signifies the ratio of momentum diffusivity to thermal diffusivity. It is seen that the temperature decreases with increasing Pr. Moreover, the thermal boundary layer thickness decreases by increasing Prandtl numbers. Wall temperature gradient $\theta'(0)$ is negative for all values of Prandtl number as seen from Fig. 17 which means that the heat is always transferred from the surface to the ambient fluid. An increase in Prandtl number reduces the thermal boundary layer thickness. Fluids with lower Prandtl number will possess higher thermal conductivities (and thicker thermal boundary layer structures), so that heat can diffuse from the sheet faster than for higher Pr fluids (thinner boundary layers). Fig. 11 displays the effects of the Schmidt number Sc on the concentration profiles. As the Schmidt number increases, first the peak increases then at a point the concentration decreases. The reductions in concentration profiles are accompanied by simultaneous reductions in the concentration boundary layers thicken.

The variation of temperature gradient and concentration gradient vs. Hartmann number for various values of other parameters are plotted in Figs. 12–17. As it is clear from Figs. 12 and 13, as the thermosolutal surface tension ratio R increases, both of temperature and concentration gradients at the wall increase (in absolute sense), the same effect occurs for Schmidt number on concentration gradient Fig. 16 and Prandtl number on temperature gradient Fig. 17. The effect of Soret and Dufour parameters on temperature and concentration gradient at the wall are depicted in Figs. 14 and 15. Increasing Dufour parameter (and decreasing Soret parameter) tends to decrease both of temperature and concentration gradients at the wall (in absolute sense).

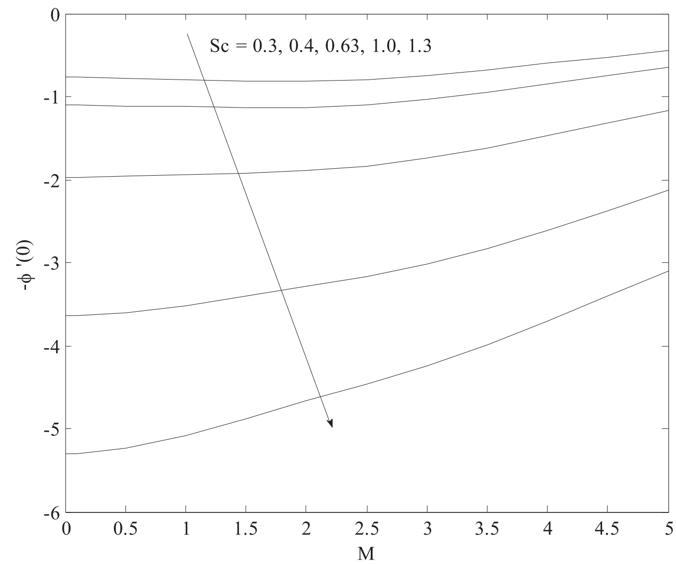

Fig. 16. Effect of Schmidt number on concentration gradient vs. Hartmann number.

5. Conclusions

In this contribution, the mechanical and thermal properties of steady MHD thermosolutal Marangoni boundary layer past a vertical flat plate taking into account the Soret and Dufour effect have been investigated systemically. With the help of appropriate similarity transformation, the governing boundary layer equations for momentum thermal energy and concentration are reduced to coupled non-linear ordinary differential equations which are then solved numerically. Results for the velocity, temperature and concentration distributions as well as temperature and concentration

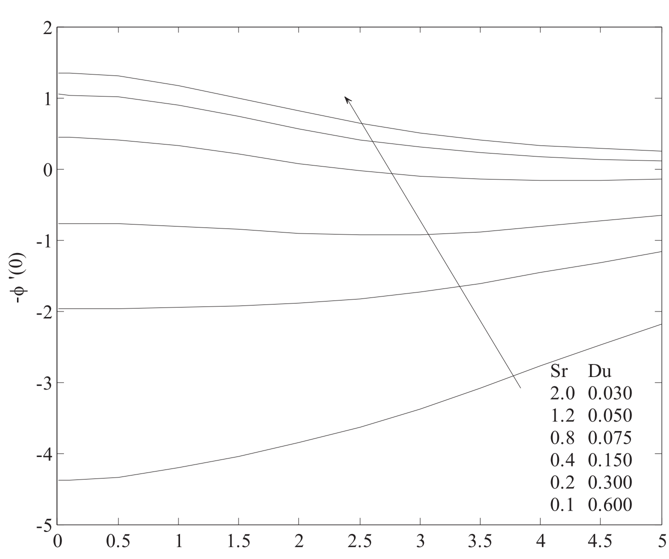

Fig. 15. Effect of Soret and Dufour parameters on concentration gradient vs. Hartmann number.

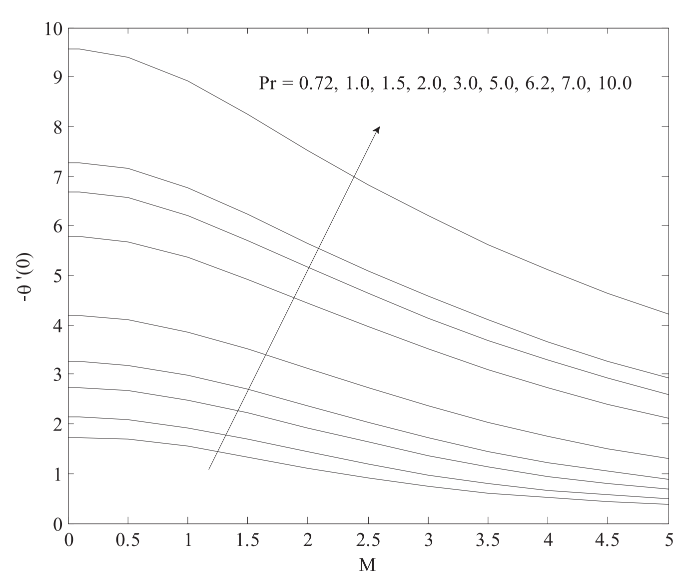

Fig. 17. Effect of Prandtl number on temperature gradient vs. Hartmann number.

gradient are presented for representative governing parameters. From this investigation, it can be concluded that,

1. An increase in the Hartmann number leads to increase both of the temperature and concentration distributions, whereas, it decreases the velocity features.
2. An increases in thermosolutal surface tension ratio results in a reduction in the distributions of the temperature and concentration, however, it supports the fluid flow.
3. The increase in the Dufour number with the decrease in the Soret number leads to an increase in the temperature profiles and a decrease in the concentration profiles.
4. The rate of heat transfer increases by increase the thermosolutal surface tension ratio however the opposite behavior was observed for mass transfer.
5. The local Nusselt number can be enhanced by increases the Prandtl number whearas it decreases by increase the Hartmann number.
6. The concentration gradient can be reduced by increase the Schmidt number but the increase in the Hartmann number leads to the inverse behavior.
7. The specific application for these kinds of problems can be found in many practical projects, such as aerospace, materials science and crystal growth.
8. This problem can be generalized by including a porous medium. Also, it can be studied using internal flow model.

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