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## ORIGINAL ARTICLE

# Ferrofluid convective heat transfer under the influence of external magnetic source

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## KEYWORDS

Nanofluid;  
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**Abstract** Ferrofluid convective heat transfer in a cavity with sinusoidal cold wall is examined under the influence of external magnetic source. The working fluid is Fe<sub>3</sub>O<sub>4</sub>-water nanofluid. Single phase model is used to estimate the behavior of nanofluid. Vorticity stream function formulation is utilized to eliminate pressure gradient source terms. New numerical method is chosen namely Control volume base finite element method. Influences of Rayleigh, Hartmann numbers, amplitude of the sinusoidal wall and volume fraction of Fe<sub>3</sub>O<sub>4</sub> on hydrothermal characteristics are presented. Results indicate that temperature gradient enhances as space between cold and hot walls reduces at low buoyancy force. Lorentz forces cause the nanofluid velocity to reduce and augment the thermal boundary layer thickness. Nusselt number augments with rise of buoyancy forces but it decreases with augment of Lorentz forces.

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## 1. Introduction

New kinds of fluid were required to reach more efficient performance in new days. Nanofluid was proposed as innovative way to enhance heat transfer. Teamah and Shehata [1] reported the Lorentz forces effect on free convection in trapezoidal enclosure. Hsiao [2] investigated electrical MHD nanofluid flow over a plate. He utilized FDM for simulation. Sheikholeslami and Ganji [3] presented various applications of nanofluid in their review paper. Sheremet et al. [4] simulated the unsteady

MHD flow in an enclosure. They used FDM to simulate that paper. Kandasamy et al. [5] examined the reaction of nanofluid versus chemical reaction. Ahmad and Mustafa [6] investigated the rotating nanofluid flow induced by an exponentially stretching. Their results revealed that temperature gradient reduces with augment of angular velocity. Awais et al. [7] simulated the slip effect on nanofluid motion in the presence of magnetic field. Hussein et al. [8] analyzed the natural convection of nanofluid in T-shaped cavity. They concluded that temperature gradient decreases with augment of heat source length. Radiation heat transfer over a sensor surface has been studied by Hamzah et al. [9]. They indicated the 30% augmentation in Nusselt number with use of nanofluid.

Selimefendigil and Oztop [10] examined nanofluid conjugate conduction-convection mechanism in a titled cavity. They proved that temperature gradient rises with augment of Grashof number. Sheikholeslami and Ellahi [11] selected

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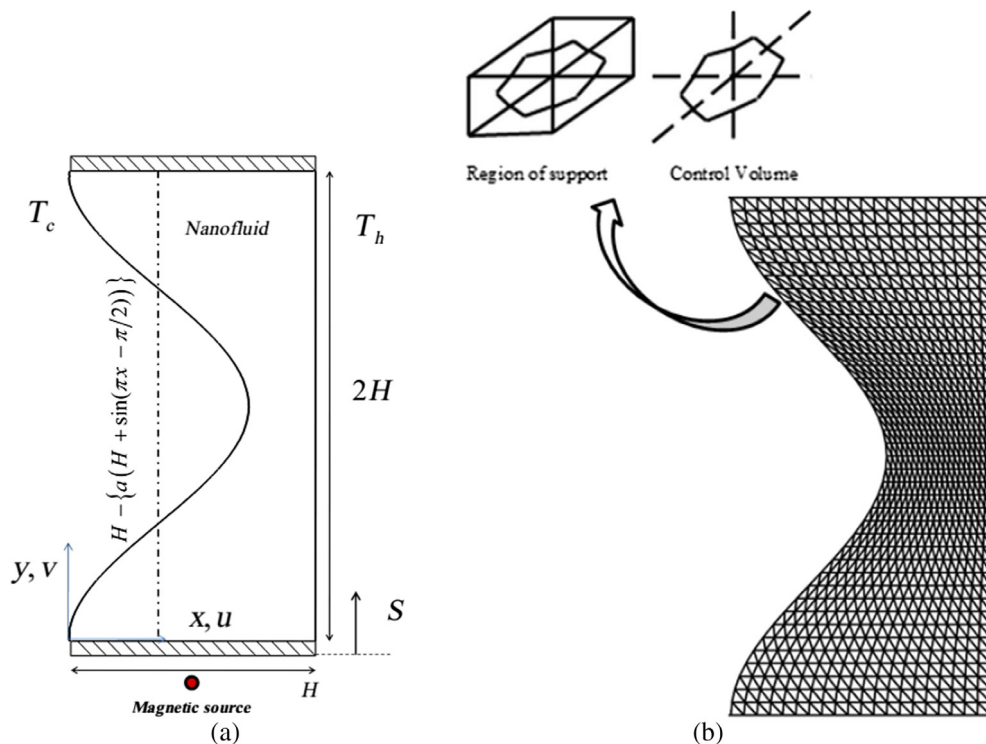
**Nomenclature**

$B$	magnetic induction	$\alpha$	thermal diffusivity
$Ec$	Eckert number	$\Omega \& \Psi$	dimensionless vorticity & stream function
$H$	the magnetic field strength	$\Theta$	dimensionless temperature
$\vec{g}$	gravitational acceleration vector	$\rho$	fluid density
$Nu$	Nusselt number	$\mu$	dynamic viscosity
$Ha$	Hartmann number	$\sigma$	electrical conductivity
$T$	fluid temperature		
$Ra$	Rayleigh number	<i>Subscripts</i>	
$V, U$	vertical and horizontal dimensionless velocity	<i>nf</i>	nanofluid
$Y, X$	vertical and horizontal space coordinates	<i>f</i>	base fluid
		<i>loc</i>	local
		<i>c</i>	cold
<i>Greek symbols</i>			
$\beta$	thermal expansion coefficient		
$\mu_0$	magnetic permeability of vacuum		

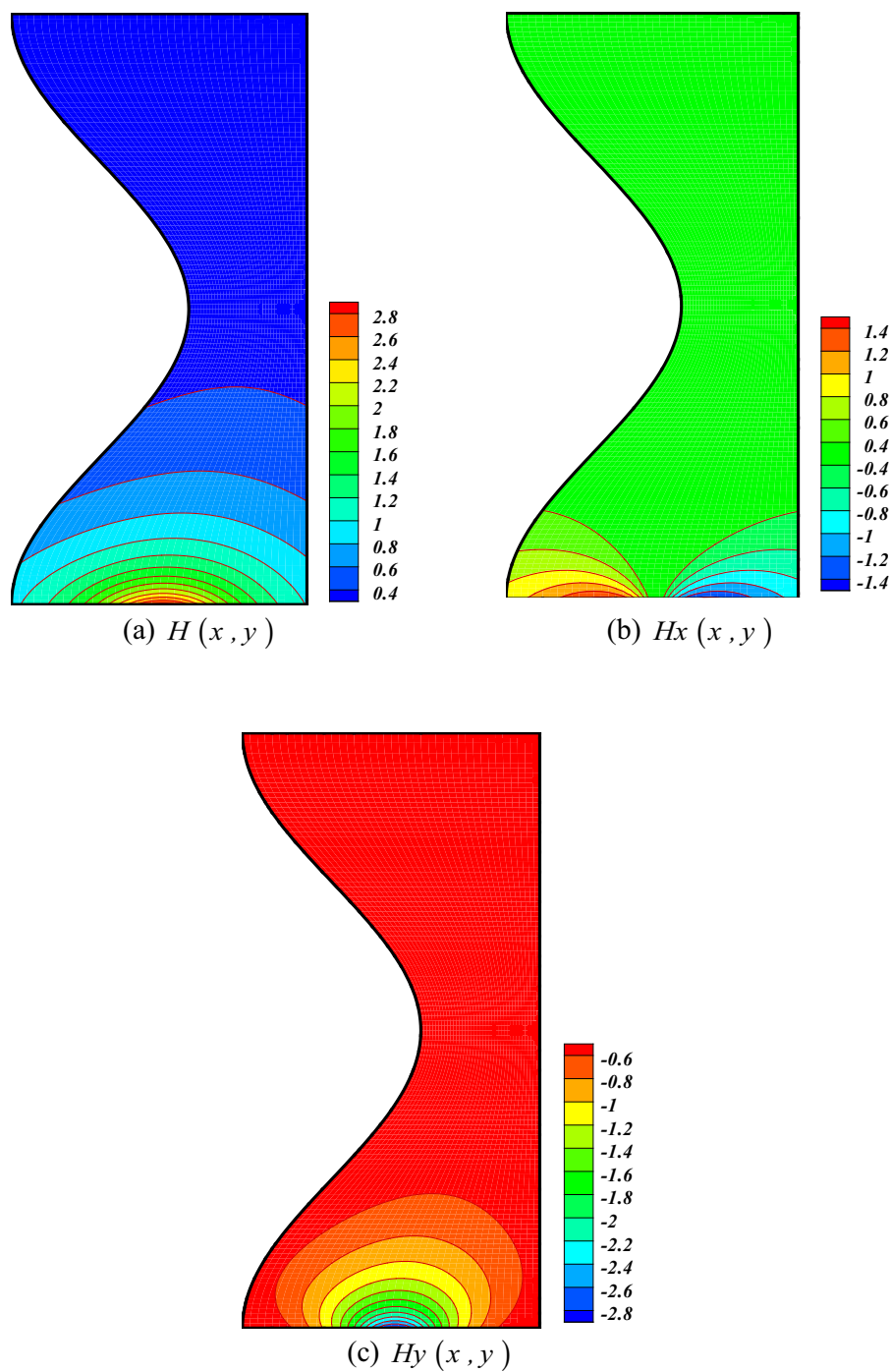
LBM to simulate Lorentz force influence on nanofluid temperature distribution. They depicted that temperature gradient reduces with augment of Hartmann number. Kefayati [12] considered second law analysis for nanofluid laminar natural convection in a permeable enclosure. He proved that irreversibilities augment as Rayleigh number enhances. MHD nanofluid free convective hydrothermal analysis in a tilted wavy enclosure was presented by Sheremet et al. [13]. Their results illustrated that change of titled angle causes convective heat transfer to enhance. Influence of non-uniform Lorentz forces on nanofluid flow style has been studied by Sheikholeslami Kandelousi [14]. He concluded that improvement in heat transfer reduces with rise of Kelvin forces. Sheik-

holeslami et al. [15] examined about the impact of radiation of nanofluid free convective heat transfer in existence of magnetic field. They showed that rate of heat transfer decreases with augment of Lorentz forces. Malvandi et al. [16] analyzed the fluid flow on a sheet. They presented the thermodynamic optimization of this problem. Several authors investigated about nanofluid heat transfer augmentation [17–33,1].

The purpose of this article was to investigate impact of magnetic source on hydrothermal behavior of nanofluid in a cavity with sinusoidal cold wall. CVFEM is chosen to simulate this paper. Effects of Rayleigh and Hartmann numbers, volume fraction of  $Fe_3O_4$  on hydrothermal treatment are considered.



**Figure 1** (a) Geometry and the boundary conditions; (b) the mesh of enclosure considered in this work.



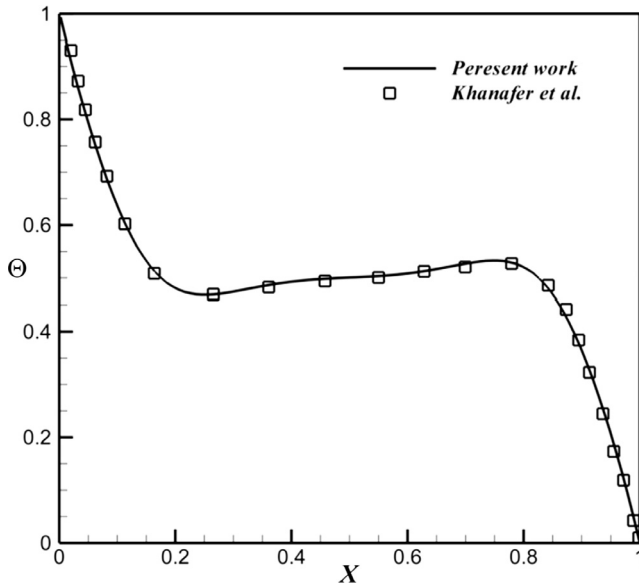
**Figure 2** Contours of the (a) magnetic field strength  $H$ ; (b) magnetic field intensity component in  $x$  direction  $H_x$ ; (c) magnetic field intensity component in  $y$  direction  $H_y$ .

**Table 1** Thermo-physical properties of water and nanoparticles.

	$\rho$ (kg/m <sup>3</sup> )	$C_p$ (J/kg K)	$k$ (W/m K)	$d_p$ (nm)	$\sigma$ ( $\Omega$ m) <sup>-1</sup>
Pure water	997.1	4179	0.613	–	0.05
Fe <sub>3</sub> O <sub>4</sub>	5200	670	6	47	25,000

**Table 2** Comparison of the average Nusselt number  $Nu_{ave}$  along lid wall for different grid resolutions at  $Ra = 10^5$ ,  $a = 0.3$ ,  $\phi = 0.04$ ,  $Ha = 20$ ,  $Ec = 10^{-5}$  and  $Pr = 6.8$ .

$51 \times 151$	$61 \times 181$	$71 \times 211$	$81 \times 241$	$91 \times 271$	$101 \times 301$
2.759311	2.766715	2.769861	2.775131	2.776103	2.779106



**Figure 3** Comparison of center line temperature between the present results and numerical results by Khanafer et al. [36]  $Gr = 10^4$ ,  $\phi = 0.1$  and  $Pr = 6.8$ (Cu-Water).

**2. Problem statement**

An enclosure with hot inner walls is considered. Fig. 1(a) depicts the boundary conditions. To obtain the shape of the left sinusoidal wall profile, the following equation should be used:

$$\frac{x}{H} = 1 - \left\{ a \left( 1 + \sin \left( \pi \frac{y}{H} - \pi/2 \right) \right) \right\} \tag{1}$$

Magnetic source has been considered as shown in Fig. 2.  $\overline{H}_x, \overline{H}_y, \overline{H}$  can be calculated as follows [34]:

$$\begin{aligned} \overline{H}_x &= (y - \bar{b})[(\bar{a} - x)^2 + (\bar{b} - y)^2]^{-\frac{1}{2}} \frac{\gamma}{2\pi}, \\ \overline{H}_y &= (\bar{a} - x)[(\bar{a} - x)^2 + (\bar{b} - y)^2]^{-\frac{1}{2}} \frac{\gamma}{2\pi}, \end{aligned} \tag{2}$$

$$\overline{H} = \sqrt{\overline{H}_x^2 + \overline{H}_y^2} \tag{3}$$

**3. Simulation method**

*3.1. Governing formulation*

2D laminar nanofluid flow and forced convective heat transfer are taken into account. The governing PDEs can be considered as follows:

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0, \tag{4}$$

$$\left( \frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} \right) = (\rho_{nf})^{-1} \left[ \left( \frac{\partial^2 u}{\partial y^2} + \frac{\partial^2 u}{\partial x^2} \right) \mu_{nf} - \frac{\partial P}{\partial x} - \sigma_{nf} B_y^2 u + \sigma_{nf} B_x B_y v \right] \tag{5}$$

$$\begin{aligned} \rho_{nf} \left( \frac{\partial v}{\partial x} + \frac{\partial v}{\partial y} \right) &= +\mu_{nf} \left( \frac{\partial^2 v}{\partial x^2} + \frac{\partial^2 v}{\partial y^2} \right) - \frac{\partial P}{\partial y} + B_y \sigma_{nf} B_x u \\ &\quad - B_x \sigma_{nf} B_x v + (T - T_c) \beta_{nf} g \rho_{nf} \end{aligned} \tag{6}$$

$$\begin{aligned} (\rho C_p)_{nf} \left( v \frac{\partial T}{\partial y} + u \frac{\partial T}{\partial x} \right) &= \sigma_{nf} (B_x v - B_y u)^2 + k_{nf} \left( \frac{\partial^2 T}{\partial x^2} + \frac{\partial^2 T}{\partial y^2} \right) \\ &\quad + \mu_{nf} \left\{ 2 \left( \frac{\partial u}{\partial x} \right)^2 + 2 \left( \frac{\partial v}{\partial x} \right)^2 + \left( \frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} \right)^2 \right\}, \end{aligned} \tag{7}$$

$$B = \mu_0 H.$$

$\rho_{nf}, (\rho C_p)_{nf}, \alpha_{nf}, \beta_{nf}, \mu_{nf}, k_{nf}$  and  $\sigma_{nf}$  are calculated as

$$\rho_{nf} = \rho_f (1 - \phi) + \rho_s \phi, \tag{8}$$

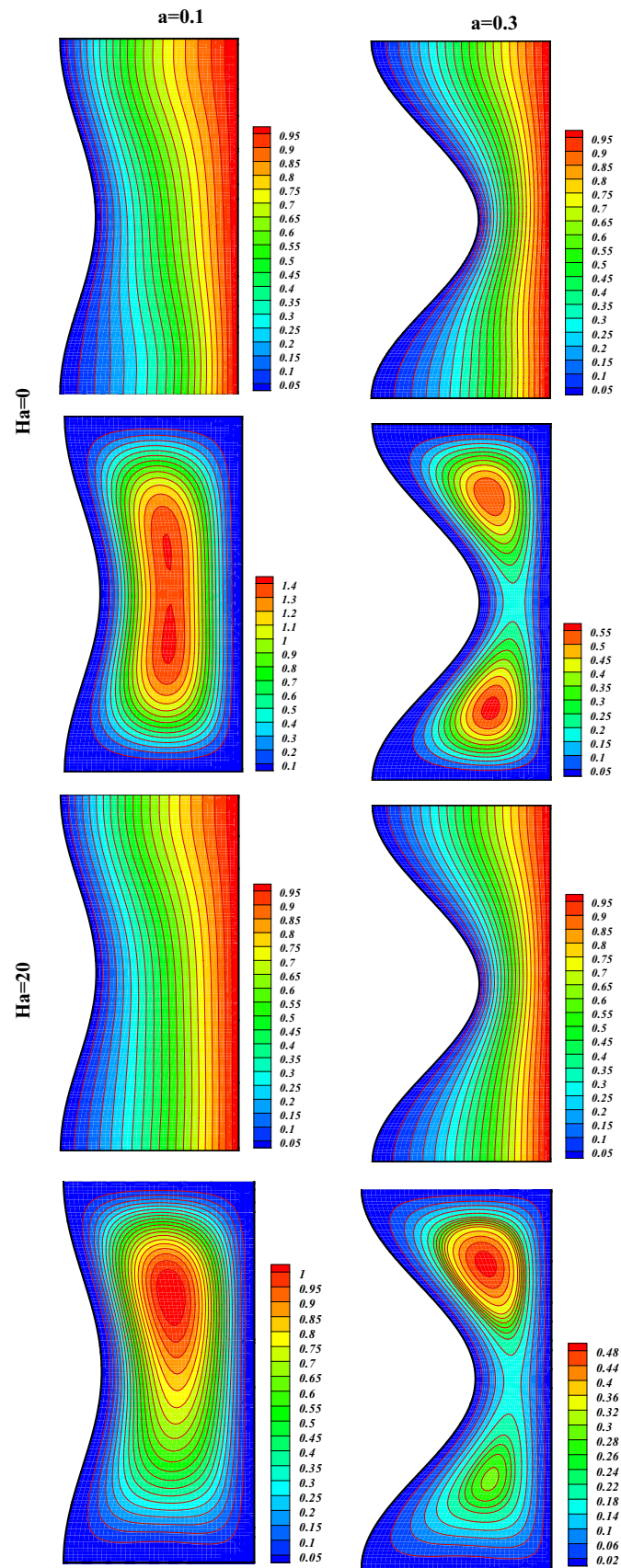
$$(\rho C_p)_{nf} = (\rho C_p)_f (1 - \phi) + (\rho C_p)_s \phi, \tag{9}$$

$$\alpha_{nf} = \frac{k_{nf}}{(\rho C_p)_{nf}}, \tag{10}$$

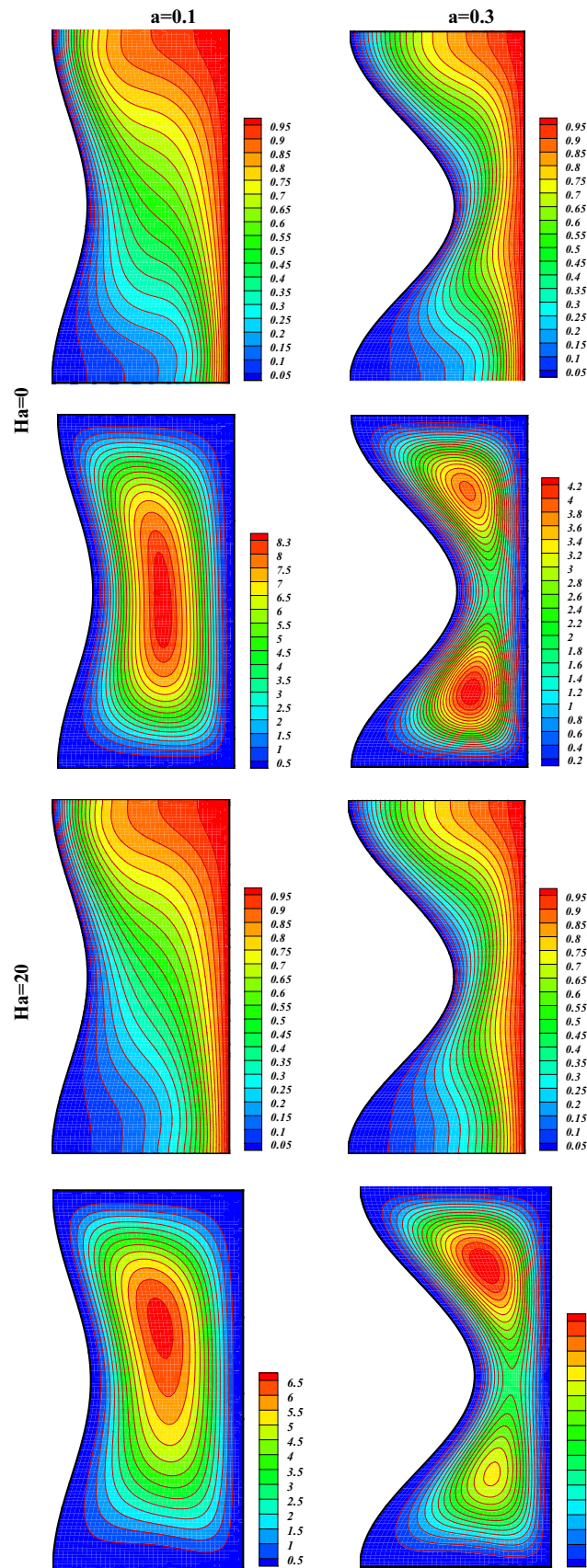
$$\beta_{nf} = \beta_f (1 - \phi) + \beta_s \phi. \tag{11}$$

**Table 3** Average Nusselt number versus at different Grashof number under various strengths of the magnetic field at  $Pr = 0.733$ .

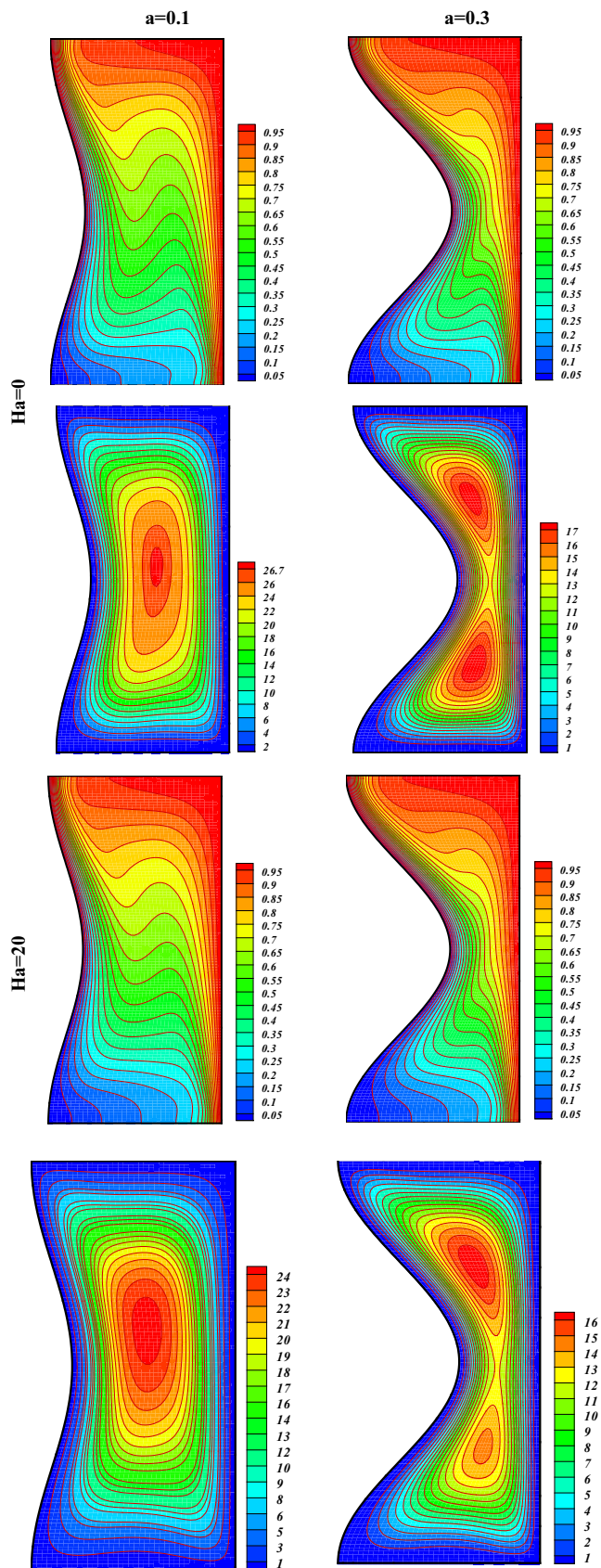
$Ha$	$Gr = 2 \times 10^4$		$Gr = 2 \times 10^5$	
	Present	Rudraiah et al. [37]	Present	Rudraiah et al. [37]
0	2.5665	2.5188	5.093205	4.9198
10	2.26626	2.2234	4.9047	4.8053
50	1.09954	1.0856	2.67911	2.8442
100	1.02218	1.011	1.46048	1.4317



**Figure 4** Isotherms (up) and streamlines (down) contours for different values of Hartmann number and amplitude of the sinusoidal wall when  $Ra = 10^3$ ,  $\phi = 0.04$ .



**Figure 5** Isotherms (up) and streamlines (down) contours for different values of Hartmann number and amplitude of the sinusoidal wall when  $Ra = 10^4$ ,  $\phi = 0.04$ .



**Figure 6** Isotherms (up) and streamlines (down) contours for different values of Hartmann number and amplitude of the sinusoidal wall when  $Ra = 10^5$ ,  $\phi = 0.04$ .

$$\mu_{nf} = \frac{\mu_f}{(1 - \phi)^{2.5}} \quad (12)$$

$$k_{nf} = \left( \frac{k_s + 2k_f - 2\phi(k_f - k_s)}{k_s + 2k_f + \phi(k_f - k_s)} \right) k_f \quad (13)$$

$$\frac{\sigma_{nf}}{\sigma_f} = 1 + \frac{3\left(\frac{\sigma_s}{\sigma_f} - 1\right)\phi}{\left(\frac{\sigma_s}{\sigma_f} + 2\right) - \left(\frac{\sigma_s}{\sigma_f} - 1\right)\phi} \quad (14)$$

Dimensionless parameters are defined as follows:

$$(a, b) = \left( \frac{\bar{a}, \bar{b}}{L} \right), (H, H_x, H_y) = \left( \frac{\bar{H}, \bar{H}_x, \bar{H}_y}{H_0} \right), (X, Y) = \left( \frac{x, y}{L} \right) \quad (15)$$

$$U = \frac{uL}{\alpha_f}, V = \frac{vL}{\alpha_f}, \Theta = \frac{T - T_c}{(q''L/k_f)}, P = \frac{p}{\rho_f(\alpha_f/L)^2}$$

So final equations are as follows:

$$\frac{\partial U}{\partial X} + \frac{\partial V}{\partial Y} = 0, \quad (16)$$

$$\frac{\partial U}{\partial X} U + \frac{\partial U}{\partial Y} V = Pr \left[ \frac{\mu_{nf}/\mu_f}{\rho_{nf}/\rho_f} \right] \left( \frac{\partial^2 U}{\partial Y^2} + \frac{\partial^2 U}{\partial X^2} \right) - Ha^2 Pr \left[ \frac{\sigma_{nf}/\sigma_f}{\rho_{nf}/\rho_f} \right] (H_y^2 U - H_x H_y V) - \frac{\partial P}{\partial X}, \quad (17)$$

$$V \frac{\partial V}{\partial Y} + U \frac{\partial V}{\partial X} = Pr \left( \frac{\partial^2 V}{\partial Y^2} + \frac{\partial^2 V}{\partial X^2} \right) \left[ \frac{\mu_{nf}/\mu_f}{\rho_{nf}/\rho_f} \right] - Ha^2 Pr \left[ \frac{\sigma_{nf}/\sigma_f}{\rho_{nf}/\rho_f} \right] (H_x^2 V - H_x H_y U) - \frac{\partial P}{\partial Y} + Ra Pr \left[ \frac{\beta_{nf}}{\beta_f} \right] \Theta, \quad (18)$$

$$V \frac{\partial \Theta}{\partial Y} + U \frac{\partial \Theta}{\partial X} = \left( \frac{\partial^2 \Theta}{\partial Y^2} + \frac{\partial^2 \Theta}{\partial X^2} \right) \left[ \frac{(\rho C_p)_{nf}}{k_{nf}} \right]^{-1} + Ha^2 Ec \left[ \frac{\sigma_{nf}}{\sigma_f} \right] \left[ \frac{(\rho C_p)_{nf}}{(\rho C_p)_f} \right] \{ V H_x - U H_y \}^2 + \left[ \frac{\mu_{nf}}{(\rho C_p)_{nf}} \right] Ec \left\{ 2 \left( \frac{\partial U}{\partial X} \right)^2 + 2 \left( \frac{\partial V}{\partial Y} \right)^2 + \left( \frac{\partial U}{\partial Y} + \frac{\partial V}{\partial X} \right)^2 \right\}, \quad (19)$$

and dimensionless parameters are

$$Ra_f = g\beta_f L^3 (q''L/k_f) / (\alpha_f \nu_f), Pr_f = \nu_f / \alpha_f, Ha = L\mu_0 H_0 \sqrt{\sigma_f / \mu_f}, Ec = (\mu_f \alpha_f) / [(\rho C_p)_f \Delta T L^2] \quad (20)$$

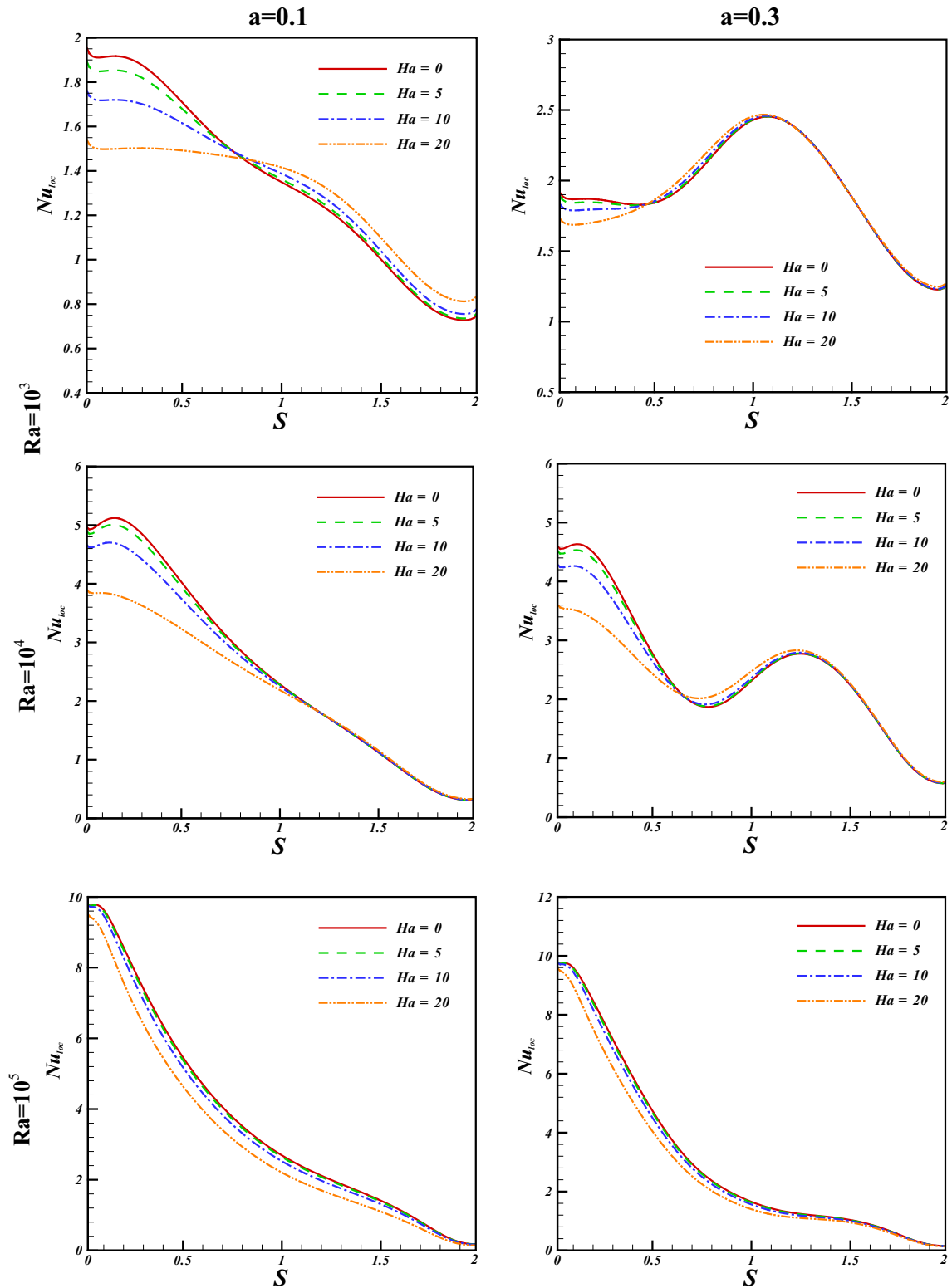
The thermo-physical properties of  $Fe_3O_4$  and water are presented in Table 1 [34]. Pressure gradient source terms discard by vorticity stream function.

$$\Omega = \frac{\omega L^2}{\alpha_f}, \Psi = \frac{\psi}{\alpha_f}, \omega = -\frac{\partial u}{\partial y} + \frac{\partial v}{\partial x}, (u, v) = \left( \frac{\partial \psi}{\partial y}, -\frac{\partial \psi}{\partial x} \right) \quad (21)$$

According to Fig. 1, boundary conditions are

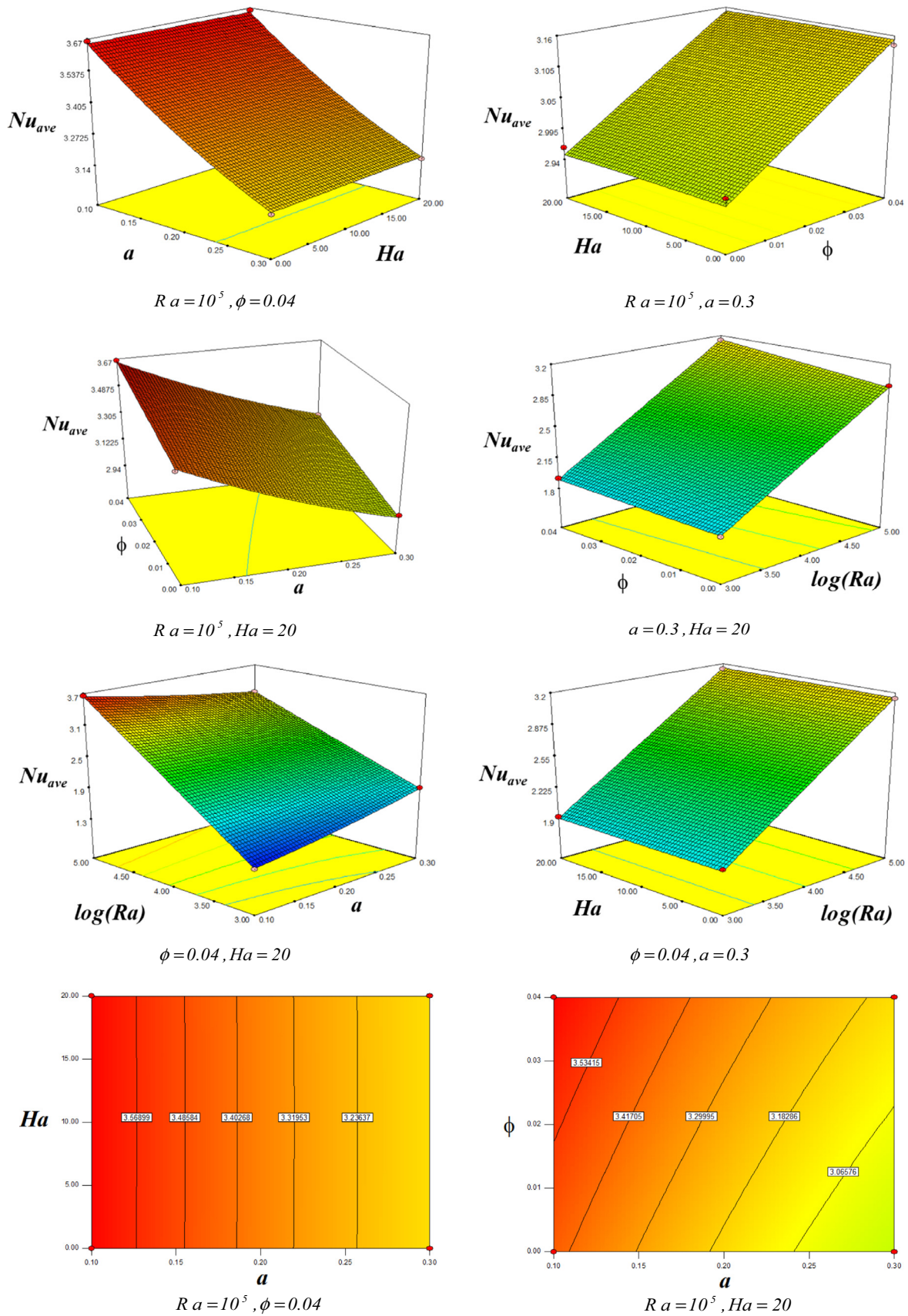
$$\begin{aligned} &\text{on left wall } \Theta = 0.0 \\ &\text{on all walls } \Psi = 0.0 \\ &\text{on right wall } \Theta = 1.0 \\ &\text{on other walls } \frac{\partial \Theta}{\partial n} = 0.0 \end{aligned} \quad (22)$$

$Nu_{local}$ ,  $Nu_{ave}$  along left wall are as follows:



**Figure 7** Effects of Hartmann number, amplitude of the sinusoidal wall and Rayleigh number on local Nusselt number  $Nu_{loc}$  along right wall.





**Figure 8** Effects of Hartmann number, amplitude of the sinusoidal wall and Rayleigh number on local Nusselt number  $Nu_{ave}$  along right wall.

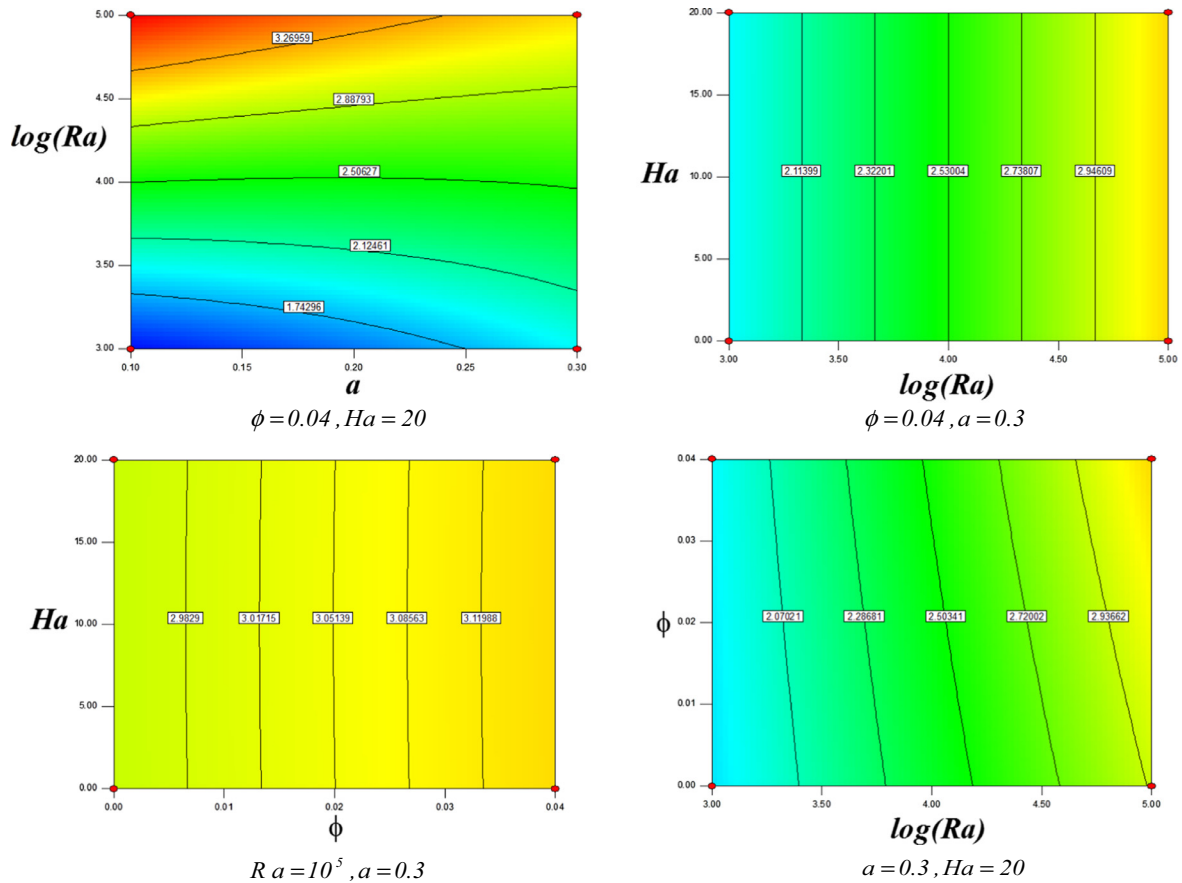


Fig. 8 (continued)

$$Nu_{loc} = \left(\frac{k_{nf}}{k_f}\right) \frac{\partial \Theta}{\partial n} \quad (23)$$

$$Nu_{ave} = \frac{1}{2} \int_0^2 Nu_{loc} dS \quad (24)$$

### 3.2. Numerical procedure

Linear interpolation is utilized for approximation of variables in the triangular element which is considered as building block (Fig. 1(b)). Algebraic equations are solved via Gauss-Seidel method. More details exist in reference book [35].

## 4. Mesh independency and validation

For selecting the mesh size to access mesh independency outputs, different grids are tested. As shown in Table 2, a grid size of  $71 \times 211$  should be chosen. Fig. 3 shows the accuracy of FORTRAN code for nanofluid heat transfer [36]. Table 3 illustrates that our code is validated for magnetohydrodynamic heat transfer [37].

## 5. Results and discussion

A cavity filled with nanofluid in existence of magnetic source is studied. Cold wall has sinusoidal shape. Various amounts of Hartmann number ( $Ha = 0-20$ ), Rayleigh number

( $Ra = 10^3, 10^4$  and  $10^5$ ), amplitude of the sinusoidal wall ( $a = 0.1-0.3$ ) and volume fraction of  $Fe_3O_4$  ( $\phi = 0$  and  $0.04$ ) have been considered.  $Ec$  and  $Pr$  are  $10^{-5}$  and  $6.8$ , respectively.

Figs. 4-6 depict the impacts of amplitude of the sinusoidal wall, Rayleigh and Hartmann numbers on streamlines and isotherms. At  $a = 0.1$ , one main eddy appears. As amplitude of the sinusoidal wall augments, the main eddy turn into two eddies. By applying magnetic field, Lorentz forces generate. This force causes the nanofluid flow to retard. Increasing Lorentz forces causes eddies to stretch vertically. In weak buoyancy forces, the domination mode is conduction. So reducing distance between cold and hot walls, rate of heat transfer augments but opposite behavior can be seen for stronger buoyancy forces. As buoyancy forces enhances, eddies become stronger and isotherms become nonparallel together. So temperature gradient near the right wall enhances. As Lorentz force augments, the velocity reduces and this force makes isotherms to become parallel together.

Figs. 7 and 8 depict the influence of the  $Fe_3O_4$  volume fraction, amplitude of the sinusoidal wall, Lorentz and buoyancy forces on  $Nu_{loc}$ ,  $Nu_{ave}$ . The correlation for  $Nu_{ave}$  corresponding to active parameters is as follows:

$$\begin{aligned} Nu_{ave} = & -2.85 + 9.12a + 1.33(\log(Ra)) - 4.34\phi + 1.79 \times 10^{-4}Ha \\ & - 2.6a(\log(Ra)) - 0.13a\phi - 1.6 \times 10^{-16}aHa \\ & + 1.9(\log(Ra))\phi - 1.15 \times 10^{-17}(\log(Ra))Ha \\ & - 1.18 \times 10^{-15}\phi Ha + 3.5a^2 - 7.5 \times 10^{-4}(\log(Ra))^2 \\ & - 0.45\phi^2 - 8.9 \times 10^{-6}Ha^2 \end{aligned} \quad (25)$$

As buoyancy forces augment, temperature gradient near the right wall enhances and in turn  $Nu_{ave}$  enhances with augmentation of Rayleigh number. Enhancing Lorentz forces leads to reduce  $Nu_{ave}$  due to domination of conduction mode. Adding  $Fe_3O_4$  nanoparticle into water makes Nusselt number to enhance.  $Nu_{ave}$  enhances with rise of  $a$  in low  $Ra$  but opposite changes occur for high  $Ra$ .

## 6. Conclusions

Ferrofluid heat transfer in a cavity with sinusoidal cold wall is presented under the impact of external magnetic field. CVFEM has been chosen as simulation method. Isotherms and streamlines are depicted for different values of volume fraction of  $Fe_3O_4$ , amplitude of the sinusoidal wall, Rayleigh and Hartmann numbers. Results indicate that impact of adding  $Fe_3O_4$  is more sensible for lower buoyancy forces. Temperature gradient enhances with rise of amplitude of the sinusoidal wall when buoyancy force is weak. Temperature augments with rise of Lorentz forces but it reduces with augment of volume fraction of  $Fe_3O_4$  and buoyancy forces.

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