Physics Letters B 732 (2014) 167–168

Contents lists available at ScienceDirect

Physics Letters B

www.elsevier.com/locate/physletb

Vanishing Higgs one-loop quadratic divergence in the scotogenic model and beyond

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ARTICLE INFO

Article history:
Received 16 January 2014
Received in revised form 21 March 2014
Accepted 23 March 2014
Available online 27 March 2014
Editor: J. Hisano

ABSTRACT

It is shown that the inherent one-loop quadratic divergence of the Higgs mass renormalization of the standard model may be avoided in the well-studied scotogenic model of radiative neutrino mass as well as other analogous extensions.

In quantum field theory, the additive renormalization of m^2 for a scalar field of mass m is a quadratic function of the cut-off scale Λ. The elegant removal of this quadratic divergence is a powerful theoretical argument for the existence of supersymmetric particles. However, given the recent discovery of the 126 GeV particle [1,2] at the Large Hadron Collider (LHC), presumably the long sought Higgs boson of the standard model, and the nonobservation of any hint of supersymmetry, it may be a good time to reexamine an alternative solution of the quadratic divergence problem.

It was suggested a long time ago [3] that in the standard model of quarks and leptons, the condition

$$\frac{3}{2}M_W^2 + \frac{3}{4}M_Z^2 + \frac{3}{4}m_H^2 = \sum_f N_f m_f^2, \tag{1}$$

where $N_f = 3$ for quarks and $N_f = 1$ for leptons, would make the coefficient of the Λ^2 contribution to m_H^2 vanish. This would predict $m_H = 316$ GeV, which we now know to be incorrect.

The same idea may be extended to the case of two Higgs doublets [4–7] where $\langle\phi_{1,2}^0\rangle = v_{1,2}$, with $v = \sqrt{v_1^2 + v_2^2} = 174$ GeV. In that case, the vanishing of quadratic divergences would also depend on how $\Phi_{1,2}$ couple to the quarks and leptons. In the scotogenic model of radiative neutrino mass [8], there are two scalar doublets (ϕ^+, ϕ^0) and (η^+, η^0), distinguished from each other by a discrete Z_2 symmetry, under which Φ is even and η odd. Thus only ϕ^0 acquires a nonzero vacuum expectation value v. This same discrete symmetry also prevents η from coupling to the usual quarks and leptons, except for the Yukawa terms

$$\mathcal{L}_Y = h_{ij}\left(\nu_i \eta^0 - l_i \eta^+\right) N_j + H.c., \tag{2}$$

where N_j are three neutral singlet Majorana fermions odd under Z_2. As a result, neutrinos obtain one-loop finite radiative Majorana masses as shown in Fig. 1. This is a well-studied model

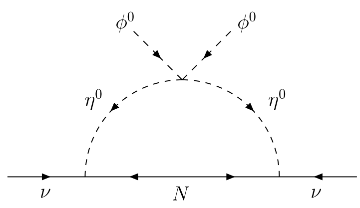

Fig. 1. One-loop generation of neutrino mass with Z_2 symmetry.

which also offers $\sqrt{2}Re(\eta^0)$ as a good dark-matter candidate [9]. The lightest N may also be a dark-matter candidate [10] but is more suitable if the dark-matter discrete symmetry Z_2 is extended to $U(1)_D$ as proposed recently [11].

The scalar potential of the scotogenic Z_2 model is given by [8]

$$\begin{aligned} V = {} & m_1^2 \Phi^\dagger \Phi + m_2^2 \eta^\dagger \eta + \frac{1}{2}\lambda_1 \left(\Phi^\dagger \Phi\right)^2 + \frac{1}{2}\lambda_2 \left(\eta^\dagger \eta\right)^2 \\ & + \lambda_3 \left(\Phi^\dagger \Phi\right)\left(\eta^\dagger \eta\right) + \lambda_4 \left(\Phi^\dagger \eta\right)\left(\eta^\dagger \Phi\right) \\ & + \frac{1}{2}\lambda_5 \left[\left(\Phi^\dagger \eta\right)^2 + \left(\eta^\dagger \Phi\right)^2\right]. \end{aligned} \tag{3}$$

Let $\phi^0 = v + H/\sqrt{2}$ and $\eta^0 = (\eta_R + i\eta_I)/\sqrt{2}$, then

$$m^2(H) = 2\lambda_1 v^2, \tag{4}$$

$$m^2\left(\eta^\pm\right) = m_2^2 + \lambda_3 v^2, \tag{5}$$

$$m^2(\eta_R) = m_2^2 + (\lambda_3 + \lambda_4 + \lambda_5) v^2, \tag{6}$$

$$m^2(\eta_I) = m_2^2 + (\lambda_3 + \lambda_4 - \lambda_5) v^2. \tag{7}$$

The corresponding two conditions for the vanishing of quadratic divergences are

$$\frac{3}{2}M_W^2 + \frac{3}{4}M_Z^2 + \frac{3}{4}m_H^2 + \left(\lambda_3 + \frac{1}{2}\lambda_4\right) v^2 = 3m_t^2, \tag{8}$$

http://dx.doi.org/10.1016/j.physletb.2014.03.047

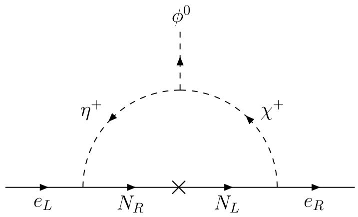

Fig. 2. One-loop generation of electron mass with soft Z_2 breaking.

$$\frac{3}{2}M_W^2 + \frac{3}{4}M_Z^2 + \left(\frac{3}{2}\lambda_2 + \lambda_3 + \frac{1}{2}\lambda_4\right)v^2 = \sum_{i,j} h_{ij}^2 v^2. \quad (9)$$

Consequently, the following two sum rules are obtained:

$$\lambda_3 + \frac{1}{2}\lambda_4 = \frac{3}{v^2}\left(m_t^2 - \frac{1}{2}M_W^2 - \frac{1}{4}M_Z^2 - \frac{1}{4}m_H^2\right) = 2.063, \quad (10)$$

$$h^2 - \frac{1}{2}\lambda_2 = \frac{1}{v^2}\left(m_t^2 - \frac{1}{4}m_H^2\right) = 0.863, \quad (11)$$

where $3h^2 = \sum_{i,j} h_{ij}^2$. Since λ_2 must be positive, Eq. (11) cannot be satisfied without the Yukawa couplings of Eq. (2). In other words, the existence of N, hence the radiative generation of neutrino mass, is necessary for this scenario. In a model with simply a second "inert" scalar doublet [12,13], vanishing quadratic divergence will not be possible. To test Eq. (10), Eqs. (5) to (7) may be used, i.e.

$$2\lambda_4 v^2 = m_R^2 + m_I^2 - 2m_+^2. \quad (12)$$

As for λ_3, it may be extracted [14,15] from $H \to \gamma\gamma$ using also m_+. However Eq. (11) is very difficult to test, because h^2 and λ_2 are not easily measurable.

Analogous extensions of the scotogenic model may also accommodate vanishing quadratic divergences. As an example, consider the addition of a charged scalar χ^+ odd under Z_2, then the electron may acquire a radiative mass by assigning e_R to be odd with the Yukawa couplings $f\bar{e}_R N_L \chi^-$ as shown in Fig. 2, where N_L is even under Z_2, but the soft Dirac mass term $\bar{N}_L N_R$ breaks Z_2 explicitly. With the addition of χ^+, the scalar potential has the extra terms

$$V' = m_3^2 \chi^+\chi^- + \frac{1}{2}\lambda_6(\chi^+\chi^-)^2 + \lambda_7(\chi^+\chi^-)(\Phi^\dagger\Phi) + \lambda_8(\chi^+\chi^-)(\eta^\dagger\eta) + [\mu(\eta^+\phi^0 - \eta^0\phi^+)\chi^- + H.c.]. \quad (13)$$

The conditions for vanishing quadratic divergence in this model are then:

$$\frac{3}{2}M_W^2 + \frac{3}{4}M_Z^2 + \frac{3}{4}m_H^2 + \left(\lambda_3 + \frac{1}{2}\lambda_4 + \frac{1}{2}\lambda_7\right)v^2 = 3m_t^2, \quad (14)$$

$$\frac{3}{2}M_W^2 + \frac{3}{4}M_Z^2 + \left(\frac{3}{2}\lambda_2 + \lambda_3 + \frac{1}{2}\lambda_4 + \frac{1}{2}\lambda_8\right)v^2 = \sum_{i,j} h_{ij}^2 v^2, \quad (15)$$

$$3\left(M_Z^2 - M_W^2\right) + (\lambda_6 + \lambda_7 + \lambda_8)v^2 = f^2 v^2. \quad (16)$$

Again, verification is possible, at least in principle. Other more involved scenarios such as the scotogenic $U(1)_D$ model [11] or that of a recent proposal [16], where all quark and lepton masses are radiative with either Z_2 or $U(1)_D$ dark matter, may also have similar viable solutions.

It is of course well-known that the one-loop vanishing of the Higgs quadratic divergence is not invariant under the renormalization-group running of the gauge, Yukawa, and quartic scalar couplings. Thus the two-loop SM contribution has also been studied [17,18]. Whereas it is impossible to have both set equal to zero, if the latter is viewed as a perturbation to the first for a physical cutoff [19], then the approximate validity of the Veltman condition remains a plausible solution. Other ideas regarding the inherent quadratic divergence of any scalar mass have also been discussed in the recent literature [20–22].

Acknowledgements

I thank Maria Krawczyk for discussions at Scalars 2013. This work is supported in part by the U.S. Department of Energy under Grant No. DE-SC0008541.

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