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Angular analysis of $B \to J/\psi K_1$: Towards a model independent determination of the photon polarization with $B \to K_1\gamma$

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ABSTRACT

We propose a model independent extraction of the hadronic information needed to determine the photon polarization of the $b \to s\gamma$ process by the method utilizing the $B \to K_1\gamma \to K\pi\pi\gamma$ angular distribution. We show that exactly the same hadronic information can be obtained by using the $B \to J/\psi K_1 \to J/\psi K\pi\pi$ channel, which leads to a much higher precision.

1. Introduction

The circular polarization of the photon in the $b \to s\gamma$ process has a unique sensitivity to new physics, namely to the right-handed charged current (see e.g. [1–3]). While it is a very fundamental observable, the experimental determination of the photon polarization was not achieved at a high precision in the previous B factory experiments. Therefore, this is a very important challenge for LHCb as well as for the upgrade of B factory, Belle II experiment. Various theoretical ideas to measure the photon polarization have been proposed (pioneered by [4–8] and followed by [9–12]) and many experimental efforts are currently on-going [13,14]. Since the photon polarization measurement determines the Wilson coefficient $C_7^{(\prime)}$, it will have an important consequence to the global fit as well [15].

Recently the LHCb Collaboration has presented an interesting result [16] on the so-called up-down asymmetry of the $B \to K\pi\pi\gamma$ decay, originally proposed in [7,8]. The up-down asymmetry, which is the difference of the number of events with photon emitted above and below the $K\pi\pi$ decay plane in the $K\pi\pi$ reference frame, can indeed provide the information on the photon polarization. The basic idea is to determine the photon polarization by measuring the K_1 polarization, which is correlated with the photon polarization, through its angular distribution in the $B \to K\pi\pi\gamma$ decay.

To determine the photon polarization from the LHCb result, we need the detailed prediction of the $K_1 \to K\pi\pi$ strong decay. In our previous works [9,17], we have obtained this information by using the other experimental results, mainly the isobar model description from the ACCMOR Collaboration [18], complemented by the theoretical model computation using the ${}^3P_0$ model [19]. The $B \to K_1(1270)\gamma \to K\pi\pi\gamma$ channel, different from the $K_1(1400)$ channel, requires various unconventional treatments and unfortunately, our conclusion is that there are certain uncertainties remaining to describe this channel. The main difficulties are (see [17] for the detailed discussions):

- the existence of two intermediate processes, $K_1(1270) \to K^*\pi$ and $K_1(1270) \to K\rho$, with the latter being just on the edge of the $K\rho$ phase space and having however a large branching ratio. Quasi-threshold effects must be taken into account;
- furthermore, as we found, the final estimation of photon polarization is also sensitive to the contribution of the $K_1(1270)$ decay channels with scalar isobars, $K_1(1270) \to K(\pi\pi)_{S-wave}$ or $K_1(1270) \to (K\pi)_{S-wave}\pi$, which are not well determined, neither by experiment nor by theory.

These problems must be solved in the future with more detailed analysis of K_1 resonances, which are produced from B, τ or J/ψ decays.

In this article, we rather propose a model independent approach to circumvent the problem. In all the previous works, only a partial angular distribution was considered, i.e. taking into account only one θ angle. We show in this article that with a more complete angular description, the information on the K_1 decay needed for photon polarization determination can be extracted directly from $B \to K\pi\pi + \gamma$ decay. That is, using the angles involving not

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only the $\cos\theta$ like distribution which yields the up-down asymmetry, but also the azimuthal angle ϕ dependence, we can obtain the full hadronic information without the isobar model description of the resonances.

In fact, with the limited statistics available for $B \to K\pi\pi + \gamma$, this method is currently difficult. On the other hand, it turns out that we can obtain the same hadronic information from another channel $B \to K\pi\pi + J/\psi$ where two orders of magnitudes higher statistics, with respect to the photon channel, is available [20]. We show that the full angular distribution measurement allows us to separate the B decay and K_1 decay parts so that we can extract the same hadronic information from the $B \to K\pi\pi + J/\psi$ decay.

For the moment, for a simpler illustration of the approach, we consider the case of only one K_1 resonance, which may be practically supported by the fact that $B \to K_1(1270)\gamma$ seems largely dominant over $B \to K_1(1400)\gamma$ [14,16,21,22].

The rest of the article is organized as follows: in section 2, we introduce the kinematical variables including the θ and ϕ angles which are crucial for our work. In section 3, we write down the decay amplitudes of $B \to K_1 J/\psi$ and $B \to K_1\gamma$ with K_1 decaying to $K\pi\pi$. In section 4, we derive the angular distributions for these decays. Then, we demonstrate in section 5 that the hadronic information we need to determine the photon polarization in $B \to K_1\gamma$ can be obtained directly from the measurement of angular coefficients in $B \to K_1 J/\psi$ and/or $B \to K_1\gamma$, and we conclude in section 6.

2. Kinematics of $B^+ \to V K_1^+ \to V K^+\pi^+\pi^-$ decay ($V = J/\psi, \gamma$)

In this section, we describe all the definitions of the kinematical variables (see Fig. 1). We use $B^+ \to V K_1^+ \to V K^+\pi^+\pi^-$ decay as an example but one can obtain the similar formulae for other charge combinations. Throughout this article, we work in the K_1 rest frame. We can move to the conventional B rest frame or any other frame simply by a Lorentz transformation. First, we assign the three momenta as

$$\pi^+(\vec{p}_1), \quad \pi^-(\vec{p}_2), \quad K^+(\vec{p}_3)\,. \tag{1}$$

Now, we define a standard orthogonal frame, with respect to the spin direction of K_1, or $V = J/\psi, \gamma$. First, the Oz is defined as the V direction

$$\vec{e}_z = \frac{\vec{p}_V}{|\vec{p}_V|} = \frac{-\vec{p}_B}{|\vec{p}_B|}\,. \tag{2}$$

We define the axis perpendicular to the $K\pi\pi$ decay plane by $\vec{n}$:

$$\vec{n} = \frac{\vec{p}_1 \times \vec{p}_2}{|\vec{p}_1 \times \vec{p}_2|}\,. \tag{3}$$

Then, the Oy is chosen as normal to the Oz and $V = J/\psi, \gamma$ direction by

$$\vec{e}_y = \frac{\vec{p}_V \times \vec{n}}{|\vec{p}_V \times \vec{n}|}\,. \tag{4}$$

Finally, Ox is then chosen as the normal to Oy and Oz: $\vec{e}_x = \vec{e}_y \times \vec{e}_z$.

One also defines a polar angle θ, of $\vec{n}$ with respect to the $\vec{e}_z$:

$$\cos\theta = \vec{e}_z \cdot \vec{n} \tag{5}$$

Let us here set a condition for θ as

$$\vec{e}_x \cdot \vec{n} = \sin\theta > 0, \quad 0 < \theta < \pi\,. \tag{6}$$

Now we rotate $\vec{e}_x$ onto the $K\pi\pi$ decay plane and define the result as $\vec{e}_x'$ which can be written as

$$\vec{e}_x' = \vec{e}_y \times \vec{n} \tag{7}$$

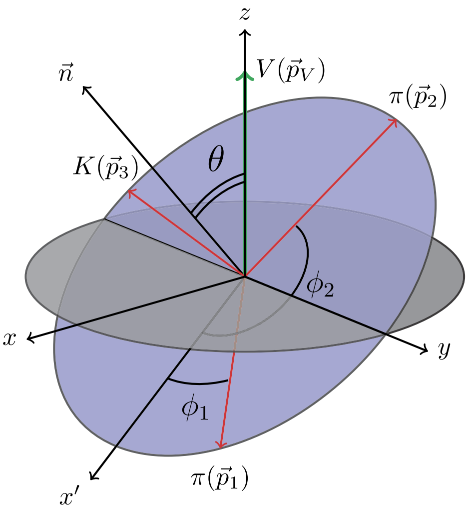

Fig. 1. Kinematics of the $B \to K_1(\to K\pi\pi)V$ decay.

We can then define a second orthogonal frame, which is based on the K_1 decay plane, $\vec{e}\,'$, $\vec{e}_y$, $\vec{n}$. Defining $\phi_{1,2}$ to be the azimuthal angle from the $\vec{e}_x'$ axis in this (x', y) decay plane, the components of the pions three momenta,

$$\vec{p}_{1,2} = |\vec{p}_{1,2}|(\cos\phi_{1,2}\,\vec{e}_x' + \sin\phi_{1,2}\,\vec{e}_y)\,, \tag{8}$$

can be expressed in terms of θ, $\phi_{1,2}$ in the standard frame as:

$$\begin{aligned}
(\vec{p}_{1,2})_x &= |\vec{p}_{1,2}|\cos\theta\cos\phi_{1,2}\,,\\
(\vec{p}_{1,2})_y &= |\vec{p}_{1,2}|\sin\phi_{1,2}\,,\\
(\vec{p}_{1,2})_z &= -|\vec{p}_{1,2}|\sin\theta\cos\phi_{1,2}\,.
\end{aligned} \tag{9}$$

The advantage is that the angles θ, $\phi_{1,2}$ are connected directly with the decay plane. We note that the linear combination of the $\phi_{1,2}$ angles,

$$\delta \equiv \phi_2 - \phi_1\,, \tag{10}$$

is a function the Dalitz variables defined by

$$\begin{aligned}
s &= (p_{K_1})^2\\
s_{13} &= (p_1+p_3)^2 = (p_{K_1}-p_2)^2\,,\\
s_{23} &= (p_2+p_3)^2 = (p_{K_1}-p_1)^2\,,\\
s_{12} &= (p_1+p_2)^2 = (p_{K_1}-p_3)^2\,.
\end{aligned} \tag{11}$$

In the K_1 rest frame, $\vec{p}_{K_1} = 0$ and $|\vec{p}_{1,2,3}|$ can be expressed in terms of s_{23}, s_{13}, s_{12} respectively. Since only two of them are independent, we choose s_{23}, s_{13} for symmetry. Then the relative angle between the three momenta of the two pions

$$\cos\delta = \frac{\vec{p}_1\cdot\vec{p}_2}{|\vec{p}_1||\vec{p}_2|} = \frac{|\vec{p}_3|^2 - |\vec{p}_1|^2 - |\vec{p}_2|^2}{2|\vec{p}_1||\vec{p}_2|}\,, \tag{12}$$

is expressible in terms of s, s_{13}, s_{23}. The same holds for the other relative angles between the three momenta.[1] This means that the $K\pi\pi$ system is rigid once the masses of the two $K\pi$ subsystems have been chosen. It is still allowed to rotate however: if the normal is fixed by a definite θ, there remains a free rotation of the rigid $K\pi\pi$ system around $\vec{n}$ in the decay plane. We choose the angle defining this rotation as:

[1] We have furthermore $0 < \delta < \pi$, $(\sin\delta > 0)$, because the angles ϕ_1, ϕ_2 are measured in the plane oriented by the normal $\vec{n} = \vec{p}_1 \times \vec{p}_2/|\vec{p}_1 \times \vec{p}_2|$.

$$\phi \equiv \frac{\phi_1 + \phi_2}{2} \,. \tag{13}$$

In this way, the angle ϕ in the reference [7] is now fixed, which allows to perform definite calculations. Note that our definition is just one possible among many others while we have found it convenient because it simplifies the calculations.

Then, re-expressing $\phi_{1,2}$ as

$$\phi_{1,2} = \phi \mp \frac{\delta}{2} \,,$$

one can get the components of $\vec{p}_{1,2}$ in Eq. (9), expressed in terms of ϕ and the Dalitz variables.

3. The decay amplitudes and rates

The four body decay rate can be written as the product of the decay rates of $B \to K_{1s_z} V_{s_z}$ and $K_{1s_z} \to K\pi\pi$ summed over the different V polarizations:

$$d\Gamma_4^V(s) \equiv d\Gamma(B \to K_1 V \to (K\pi\pi)V)_s \tag{14}$$
$$= \sum_{s_z} \frac{(2\pi)^4}{2M_B} \left| \mathcal{M}^V_{s_z}(B \to K_{1s_z} V \to (K\pi\pi)V)_s \right|^2 (2\pi)^3 ds d\Phi_2 d\Phi_3 \,,$$

where s_z is the polarization of $V = J/\psi, \gamma$:

$$s_z = 0, \pm 1 \quad (\text{for } V = J/\psi), \qquad s_z = \pm 1 \quad (\text{for } V = \gamma) \,. \tag{15}$$

We follow the PDG convention, i.e. $\int_\Omega d\Phi_2 = \frac{1}{(2\pi)^5} \frac{|\vec{p}^*_V|}{2M_B}$, $\int_\psi d\Phi_3 = \frac{1}{32(2\pi)^8} \frac{1}{s} ds_{13} ds_{23} d\phi d(\cos\theta)$, where the angles are those defined in the previous section. Here, B can be $B^\pm$, B^0 or $\overline{B}^0$. Denoting the amplitude of $B \to K_1(s)V$ as $\mathcal{A}_{s_z}(s)$ and of $K_1(s) \to K\pi\pi$ as $\epsilon^\mu_{K_{1s_z}} \mathcal{J}_\mu$, one can write:

$$\mathcal{M}^V_{s_z}(B \to K_{1s_z} V \to (K\pi\pi)V)_s = \frac{\mathcal{A}^V_{s_z}(s) \times (\epsilon^\mu_{K_{1s_z}} \mathcal{J}_\mu(s_{13}, s_{23})_s)}{(s - m^2_{K_1}) + i m_{K_1} \Gamma_{K_1}(s)} \,. \tag{16}$$

In the following, we consider only the dominant $K_1 = K_1(1270)$ for simplicity, though it can be readily extended to include $K_1(1400)$. The propagator of the K_1, which is parametrized here as Breit–Wigner function, is introduced in order to use the $K\pi\pi$ invariant mass $m_{K\pi\pi} \equiv \sqrt{s}$ as the varying K_1 mass. The K_1 rest frame is meant as the actual $K\pi\pi$ system. This is not a convention, but an assumption on the off-shell extrapolation of amplitudes, partially justified by unitarity. Note that this implies that the Dalitz plot (s_{13}, s_{23}) depends on s as well.

In Eq. (16), the full kinematical variable dependence of $\mathcal{J}$ is left implicit but it can be displayed with help of two form factors as $\mathcal{C}_{1,2}$ [9]:

$$\mathcal{J}_\mu(s_{13}, s_{23})_s \equiv \mathcal{C}_1(s, s_{13}, s_{23}) p_{1\mu} - \mathcal{C}_2(s, s_{13}, s_{23}) p_{2\mu} \,. \tag{17}$$

These form factors could be made explicit in a quasi-two-body approach to the K_1 decay [17]. Here, on the contrary, we want to determine them in a model independent way by using the experimental data to avoid the ambiguities described in the introduction.

4. Angular distribution

Now, we define the probability density function (PDF) for a given value of s. First, the different transverse ($s_z = \pm$) and the longitudinal ($s_z = 0$) polarizations of V state do not interfere, thus the decay rate is written as[2]:

$$\frac{d\Gamma(B \to K_1 V \to (K\pi\pi)V)_s}{ds_{13} ds_{23} d(\cos\theta) d\phi}$$
$$= \frac{(2\pi)^4}{2M_B} (2\pi)^3 ds \frac{1}{(2\pi)^5} \frac{|\vec{p}^*_V|}{2M_B}$$
$$\times \frac{1}{32(2\pi)^8 s} \left| \frac{1}{(s - m^2_{K_1}) + i m_{K_1} \Gamma_{K_1}(s)} \right|^2 \tag{18}$$
$$\times \sum_{s_z} |\mathcal{A}^V_{s_z}(s)|^2 \left| \vec{\epsilon}_{K_{1s_z}} \cdot \vec{\mathcal{J}}_{K_1}(s_{13}, s_{23})_s \right|^2 \,,$$

where $\vec{p}^*_V$ is the three momentum of V in the B reference frame, while the K_1 polarization vector $\vec{\epsilon}_{K_1}$ and $\vec{\mathcal{J}}_{K_1}$ are defined in the K_1 reference frame. Therefore, the θ and ϕ dependence is contained in the factor $\left| \vec{\epsilon}_{K_{1s_z}} \cdot \vec{\mathcal{J}}_{K_1}(s_{13}, s_{23})_s \right|^2$. Note that in Eq. (18), the *width* in the denominator could also be related to $\vec{\mathcal{J}}_{K_1}$, except, we have to add all charge combinations, $K_1^+ \to K^+\pi^+\pi^-$ and $K_1^+ \to K^0\pi^+\pi^0$ for K_1^+ and $K_1^0 \to K^+\pi^0\pi^-$ and $K_1^0 \to K^0\pi^+\pi^-$ for K_1^0 (and similar for the charge conjugations).

The PDF $\mathcal{W}^V(s_{13}, s_{23}, \cos\theta, \phi)_s$ is obtained from Eq. (18) and is normalized as:

$$\int ds_{13} \int ds_{23} \int d(\cos\theta) \int d\phi \, \mathcal{W}^V(s_{13}, s_{23}, \cos\theta, \phi)_s = 1 \,. \tag{19}$$

Thus, the PDF can be written in terms of the squared decay amplitudes, which are the functions of the kinematical variables we are interested in, without the irrelevant pre-factors:

$$\mathcal{W}^V(s_{13}, s_{23}, \cos\theta, \phi)_s =$$
$$\frac{\sum_{s_z} |\mathcal{A}^V_{s_z}(s)|^2 \left| \vec{\epsilon}_{K_{1s_z}} \cdot \vec{\mathcal{J}}_{K_1}(s_{13}, s_{23})_s \right|^2}{\int ds_{13} \int ds_{23} \int d(\cos\theta) \int d\phi \sum_{s_z} |\mathcal{A}^V_{s_z}(s)|^2 \left| \vec{\epsilon}_{K_{1s_z}} \cdot \vec{\mathcal{J}}_{K_1}(s_{13}, s_{23})_s \right|^2} \tag{20}$$

Next we make explicit the angular distribution of $\mathcal{W}^V$ using the definition of the coordinate system and angles given in section 2:

$$\mathcal{W}^V(s_{13}, s_{23}, \cos\theta, \phi)_s$$
$$\equiv a^V + (a^V_1 + a^V_2 \cos 2\phi + a^V_3 \sin 2\phi) \sin^2\theta + b^V \cos\theta \,, \tag{21}$$

where the angular coefficients depend on the Dalitz variables and fixed value of s. They can be written as:

$$a^V(s, s_{13}, s_{23}) = N^V_s \xi^V_a \left[|c_1|^2 + |c_2|^2 - 2\text{Re}(c_1 c_2^*) \cos\delta \right] , \tag{22}$$
$$a^V_1(s, s_{13}, s_{23}) = N^V_s \xi^V_{a_i} \left[|c_1|^2 + |c_2|^2 - 2\text{Re}(c_1 c_2^*) \cos\delta \right] , \tag{23}$$
$$a^V_2(s, s_{13}, s_{23}) = N^V_s \xi^V_{a_i} \left[(|c_1|^2 + |c_2|^2) \cos\delta - 2\text{Re}(c_1 c_2^*) \right] , \tag{24}$$
$$a^V_3(s, s_{13}, s_{23}) = N^V_s \xi^V_{a_i} \left[(|c_1|^2 - |c_2|^2) \sin\delta \right] , \tag{25}$$
$$b^V(s, s_{13}, s_{23}) = -N^V_s \xi^V_b \left[2\text{Im}(c_1 c_2^*) \sin\delta \right] , \tag{26}$$

where the factor $N^V_s > 0$ is the normalization factor, which is equal to the inverse of the denominator of Eq. (20).

The ξ's represent the $B \to K_1 V$ decay, and thus, depend only on s

$$\xi^V_a(s) \equiv \frac{|\mathcal{A}^V_+(s)|^2 + |\mathcal{A}^V_-(s)|^2}{2} \,,$$
$$\xi^V_{a_i}(s) \equiv \frac{-(|\mathcal{A}^V_+(s)|^2 + |\mathcal{A}^V_-(s)|^2) + 2|\mathcal{A}^V_0(s)|^2}{4} \,, \tag{27}$$
$$\xi^V_b(s) \equiv \frac{|\mathcal{A}^V_+(s)|^2 - |\mathcal{A}^V_-(s)|^2}{2} \,.$$

[2] For $V = J/\psi$, we integrate over the J/ψ decay angle here so that the interference term disappears.

In fact, for $V=\gamma$, the longitudinal amplitude vanishes ($\mathcal{A}_0^\gamma=0$), which simplifies the above expressions, giving as a result $a^\gamma=-2a_1^\gamma$.

The coefficients $c_{1,2}$ are related to the form factors in Eq. (17) as:

$$c_1(s,s_{13},s_{23})=\mathcal{C}_1(s,s_{13},s_{23})|\vec{p}_1|,$$
$$c_2(s,s_{13},s_{23})=\mathcal{C}_2(s,s_{13},s_{23})|\vec{p}_2|,$$

where we wrote explicitly the Dalitz variables dependence. The angle δ (with $0<\delta<\pi$) is defined as

$$\cos\delta=\frac{\vec{p}_1\cdot\vec{p}_2}{|\vec{p}_1||\vec{p}_2|}.$$

Let us also remind that all the relevant kinematical variables can be expressed in terms of the Dalitz variables:

$$|\vec{p}_{1,2}|^2=E_{1,2}^2-m_{1,2}^2,\quad \vec{p}_1\cdot\vec{p}_2=E_1E_2-\frac{s_{12}-m_1^2-m_2^2}{2},$$
$$E_{1,2}=\frac{s-s_{23,13}+m_{1,2}^2}{2\sqrt{s}}.$$

5. Photon polarization: relating the $B\to K_1\gamma$ and $B\to K_1 J/\psi$ amplitudes

The photon polarization in the $B\to K_1\gamma$ process which we want to determine is defined as following:

$$\mathcal{P}_\gamma\equiv\frac{|\mathcal{A}_+^\gamma(s)|^2-|\mathcal{A}_-^\gamma(s)|^2}{|\mathcal{A}_+^\gamma(s)|^2+|\mathcal{A}_-^\gamma(s)|^2}. \tag{28}$$

Strictly speaking, $\mathcal{P}_\gamma$ is different from the "polarization parameter"

$$\lambda_\gamma\equiv\frac{|C_+|^2-|C_-|^2}{|C_+|^2+|C_-|^2} \tag{29}$$

where $C_\pm$ represents only the short-distance $b\to s\gamma$ decay, i.e. $C_+/C_-\simeq m_{s(b)}/m_{b(s)}$ for $\overline{B}(B)$ decays, while the amplitude $\mathcal{A}_\pm^\gamma(s)$ is written as the product of $C_\pm$ and the hadronic form factor $T_1(0)$ which contains the long-distance effect. Now, when we consider only one K_1 final state, we expect a single form factor for both $\pm$ polarization, i.e. $\mathcal{A}_\pm^\gamma(s)\propto T_1(0)$. Thus, the long-distance part cancels out and $\mathcal{P}_\gamma$ becomes equivalent to λ_γ. On the other hand, the so-called charm loop contributions deviate the form factors for the $\pm$ polarization, which induces a small difference between $\mathcal{P}_\gamma$ and λ_γ. We will come back to this issue later-on. Note that $\mathcal{P}_\gamma$ is s-independent even after including a possible charm loop contribution as the s-dependence part is the same for $\mathcal{A}_\pm^\gamma(s)$ for radiative decays. We will also discuss on a possible s-dependence of $\mathcal{P}_\gamma$ later-on.

Now using Eq. (27), one can find

$$\mathcal{P}_\gamma=\frac{\xi_b^\gamma}{\xi_a^\gamma}. \tag{30}$$

We show now that this can be determined from the measurement of angular coefficients of $B\to K_1\gamma$ and $B\to K_1 J/\psi$, i.e. a^V, a_i^V, a^γ, b^γ in Eq. (21) *in a model independent way*. The result, which is our main finding, is:

$$\mathcal{P}_\gamma=\frac{\xi_b^\gamma}{\xi_a^\gamma}=\mp\frac{b^\gamma(s,s_{13},s_{23})}{a^\gamma(s,s_{13},s_{23})}\times\frac{1}{\sqrt{1-\left(\frac{a_2^V(s,s_{13},s_{23})}{a_1^V(s,s_{13},s_{23})}\right)^2-\left(\frac{a_3^V(s,s_{13},s_{23})}{a_1^V(s,s_{13},s_{23})}\right)^2}}. \tag{31}$$

Let us briefly derive this equation. First, we obtain ξ_a^γ via:

$$\xi_a^\gamma=\frac{a^\gamma(s,s_{13},s_{23})}{N_s^\gamma\left[|c_1|^2+|c_2|^2-2\mathrm{Re}(c_1c_2^*)\cos\delta\right]}. \tag{32}$$

The term in the square brackets in the denominator is common for $V=J/\psi,\gamma$ and can be obtained for given point of (s,s_{13},s_{23}) as

$$|c_1|^2+|c_2|^2-2\mathrm{Re}(c_1c_2^*)\cos\delta=\frac{a^V(s,s_{13},s_{23})}{N_s^V\xi_a^V(s)}=\frac{a_1^V(s,s_{13},s_{23})}{N_s^V\xi_{a_i}^V(s)}. \tag{33}$$

Next, we determine ξ_b^γ from the experimental measurement of $b^\gamma(s,s_{13},s_{23})$:

$$\xi_b^\gamma=-\frac{b^\gamma(s,s_{13},s_{23})}{N_s^\gamma\left[2\,\mathrm{Im}(c_1c_2^*)\sin\delta\right]}. \tag{34}$$

Now we obtain the denominator factor $2\mathrm{Im}(c_1c_2^*)\sin\delta$. By writing

$$\mathrm{Im}(c_1c_2^*)=\pm\sqrt{|c_1|^2|c_2|^2-[\mathrm{Re}(c_1c_2^*)]^2},$$

we find that we need to obtain independently these two factors, $|c_1|^2|c_2|^2$ and $\mathrm{Re}(c_1c_2^*)$, from the above equations. Then, by using Eqs. (23)–(25), we find

$$2\,\mathrm{Im}(c_1c_2^*)\sin\delta=\pm\frac{1}{N_s^V\xi_{a_i}^V(s)}\times\sqrt{(a_1^V(s,s_{13},s_{23}))^2-(a_2^V(s,s_{13},s_{23}))^2-(a_3^V(s,s_{13},s_{23}))^2} \tag{35}$$

Finally, the sign ambiguity remains, which can not be resolved at this point.

Now by inserting Eqs. (32)–(35) into Eq. (30), we can obtain the polarization which we want to determine as Eq. (31)

The main result in Eq. (31) implies:

- The photon polarization in $B\to K_1\gamma$ can be obtained from the measurement of the angular coefficients $a^\gamma(s,s_{13},s_{23})$, $b^\gamma(s,s_{13},s_{23})$ which can be measured only with the standard $\cos\theta$ distribution, together with the coefficients $a_{1,2,3}^V(s,s_{13},s_{23})$ which requires the azimuthal angle ϕ distribution. The advantage is that the latter coefficients can be measured equally by using either $B\to J/\psi K_1$ or $B\to K_1\gamma$ decays. Therefore, we can take advantage of the much higher statistics of the J/ψ process.
- The final results depend only on the ratio of the angular coefficients so that there is no need for the normalization.
- The photon polarization $\mathcal{P}_\gamma$ does not depend on s nor any Dalitz variables (sub-dominant effects which could induce s-dependence are discussed below), which implies that the expression in Eq. (31) is constant at any point of the (s,s_{13},s_{23}) plane. When we use the J/ψ to determine the denominator of this term, we simply need to map point by point on the Dalitz plane.
- Concerning the sign ambiguity, in practice, we may measure the absolute value of the polarization parameter $|\mathcal{P}_\gamma|$. In this way, we are left with the sign ambiguity of overall sign of $\mathcal{P}_\gamma$ but we can neglect the sign variation of b^γ/a^γ term since $\mathcal{P}_\gamma$ must be constant in the (s,s_{13},s_{23}) plane.

We should make a brief comment on the s-dependence of $\mathcal{P}_\gamma$. Although it is sub-dominant, a contamination from the $K_1(1400)$ resonance could cause the s-dependence. Also, the large width of the $K_1(1270)$ itself inducing an s-dependence can not be impossible [23]. However, for both cases, the s-dependence would appear only at far the K_1 pole. Therefore, in studying the amplitudes in the vicinity of the peak, we expect the final s-dependence to be very moderate.

As stressed earlier, the polarization $\mathcal{P}_\gamma$ differs in principle from λ_γ due to the charm loop contribution, which is not short distance, and is not included therefore in the $C_\pm$ coefficients. The evaluation of this effect is very difficult. It has been discussed quantitatively only in the simpler cases $B \to K^*\gamma$ and $B \to K^* l^+ l^-$ where rather different evaluations have been proposed: one being a parametric one in the $1/m_b$ expansion [24], another being through QCD sum rules [25,26]. In our paper [1], we have tried to discuss the connection between the two evaluations. On the other hand, an evaluation of charm contributions to $B \to K_1\gamma$ has not been done so far. Since the short-distance contributions, including new physics effects, should be the same for $B \to K^*\gamma$ and $B \to K_1\gamma$, an observation of different photon polarizations between these two channels should be attributed to the long-distance effect, in particular, to the charm contributions. Therefore, such an observation could provide an important key to understand the charm loop contributions.

Before closing the section, let us discuss the reliability of the method. Our argument below is only qualitative since for a quantitative discussion, detailed Monte Carlo simulations would be needed. $B \to J/\psi K\pi\pi$ has been studied by the Belle Collaboration [20]. In order to separate $B \to J/\psi K_1$ event from the $J/\psi K\pi\pi$ spectrum, a careful resonance study has to be done, namely vetoing other charmonium channels such as $B \to \psi(2S)K_1$ as well as the exotic resonances which decay into $J/\psi\pi\pi$, i.e. $B \to X(3872)K$ or $B \to Y(4260)K$. Nearly 2.5×10^3 events are identified as $B \to J/\psi K_1$ in [20]. Approximately 20(100) times more events are expected at Belle II with 10(50) ab^{-1} of data, which will allow easily to extract detailed Dalitz and angular distributions of K_1 decays. Therefore the errors expected in the second part of Eq. (31) (those written in terms of a_i^V) would be nearly negligible.

The main uncertainty will come from the first part of Eq. (31), i.e. the ratio of the angular coefficient of $B \to K_1\gamma$, $b^\gamma(s, s_{13}, s_{23})/a^\gamma(s, s_{13}, s_{23})$. In the recent analysis of Babar [14], about 2.5×10^3 $B^+ \to K^+\pi^+\pi^-\gamma$ events are reconstructed, among which 60% are known to come from $B^+ \to K_1^+(1270)\gamma$. Thus, with Belle II with 10(50) ab^{-1} of data, we expect $5(25) \times 10^3$ $B \to K_1(1270)\gamma$ events. With LHCb run one data (3 fb^{-1}), 1.4×10^4 $B^+ \to K^+\pi^+\pi^-\gamma$ events are reconstructed, which extrapolate to $\sim 2.2 \times 10^4$ events for $B^+ \to K_1^+(1270)\gamma$ at the end of LHCb run II (8 fb^{-1}). With this size of data, we can easily make over a hundred of bins on the Dalitz plane, which can be further optimized by using the known decay property of $K_1(1270)$. This naive estimate tells that we can have order of 10 MeV resolution on $\pi\pi$ and $K\pi$ invariant mass, which can lead to a high enough sensitivity to $\mathcal{P}_\gamma$.

6. Conclusions

The angular distribution in the polar angle θ of the $B \to K_{\text{res}}\gamma \to K\pi\pi\gamma$ process has recently been measured by the LHCb Collaboration [16]. Among various kaonic resonances K_{res}, a large $B \to K_1(1270)\gamma$ contribution has been identified, confirming the previous result [14,21,22]. The extraction of the $b \to s\gamma$ photon polarization from this data requires a detailed knowledge of the K_1 decays, in particular, the imaginary part of the product of the two form factors, $\text{Im}(c_1 c_2^*)$. The imaginary part is, in general, very sensitive to the resonance structure of the decay while there are many uncertainties in the resonance decay structure of $K_1(1270)$, especially due to i) the limited phase space for the main decay channel $K_1(1270) \to \rho K$ resulting in strong distortion effects, ii) a possible $K_1(1270) \to \kappa\pi$ contributions, neither well determined experimentally nor theoretically tractable.

In order to circumvent this problem, we propose a determination of the strong interaction factor $\text{Im}(c_1 c_2^*)$ independent of an isobar model for the K_1 decay. This method requires the Dalitz plot of the angular coefficients including both polar and azimuthal angles. In this article, we have shown that the same Dalitz plot analysis can be also obtained through the $B \to J/\psi K_1 \to J/\psi K\pi\pi$ channel. The B decay part of these two channels are very different while we found that we have enough observables to separate the B decay part. The realization of our proposal would require a detailed Monte Carlo studies, in particular by evaluating the binning effect.

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