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Leptogenesis, Yukawa textures and weak-basis invariants

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Abstract

We show that a large class of sets of leptonic texture zeros considered in the literature imply the vanishing of certain CP-odd weak-basis invariants. These invariant conditions enable one to recognize a flavour model corresponding to a set of texture zeros, when written in an arbitrary weak-basis where the zeros are not manifest. We also analyse the rôle of texture zeros in allowing for a connection between leptogenesis and low-energy leptonic masses, mixing and CP violation. For some of the textures the variables relevant for leptogenesis can be fully determined in terms of low energy parameters and heavy neutrino masses.

1. Introduction

The evidence for nonvanishing neutrino masses provides a clear signal of physics beyond the Standard Model (SM), since in the SM neutrinos are strictly massless. The simplest extension of the SM which allows for nonvanishing but naturally small neutrino masses consists of the addition of right-handed neutrinos, leading to the seesaw mechanism [1].

In general, the seesaw mechanism framework contains a large number of free parameters, in fact many more than measurable quantities at low energies. In the literature, there have been various attempts at reducing the number of seesaw parameters either by introducing texture zeros and/or by reducing the number of right-handed neutrinos to two. One could be tempted to follow a bottom-up approach, using the observed pattern of lepton masses and mixing to infer about the appropriate set of texture zeros. Unfortunately this approach is not feasible, since texture zeros are not weak-basis (WB) invariant. This means that a given set of texture zeros which arise in a certain WB may not be present or may appear in different entries in another WB. Indeed, each texture zero ansatz corresponds to an infinite set of leptonic mass matrices, related to each other by WB transformations. Needless to say, two sets of leptonic mass matrices related by a WB transformation contain the same physics. This raises a number of questions, such as:

(i) How can one recognize a flavour model corresponding to a set of texture zeros, when written in a different WB, where the zeros are not explicitly present?
(ii) Do the sets of texture zeros considered in the literature imply the vanishing of certain WB invariants?
(iii) Can the physical content of a particular texture zero ansatz be expressed in terms of relations involving WB invariants?

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In this Letter, we address some of the above questions and in particular we show that some of the sets of texture zeros considered in the literature imply the vanishing of certain CP-odd invariants. Conversely, we show that starting from arbitrary leptonic mass matrices and imposing the vanishing of certain CP-odd invariants, together with the assumption of no conspiracy among the parameters of the Dirac and Majorana neutrino mass terms, one is automatically led to given sets of texture zeros. The relevance of CP-odd invariants in the analysis of texture zero ansätze was to be expected, since texture zeros lead in general to a decrease in the number of independent CP-violating phases.

This Letter is organized as follows. In Section 2, we describe the framework and set our notation. In Section 3, we reexamine the connection [2] between leptonic low energy physics and leptogenesis [3], in the case of one texture zero and two right-handed neutrinos. In Section 4 we study the relation between CP-odd WB invariants and texture zeros. Finally Section 5 contains our conclusions.

2. Framework

Let us consider the above mentioned extension of the SM which consists of the addition of one right-handed neutrino per fermion generation. After spontaneous gauge symmetry breaking, the following leptonic mass terms are generated:

$$\mathcal{L}_m = -\left[\overline{\nu_L^0} m_D \nu_R^0 + \frac{1}{2}\nu_R^{0T} C M_R \nu_R^0 + \overline{l_L^0} m_l l_R^0\right] + \text{h.c.} = -\left[\frac{1}{2} n_L^T C \mathcal{M}^* n_L + \overline{l_L^0} m_l l_R^0\right] + \text{h.c.}, \tag{1}$$

where M_R, m_D, and m_l denote the right-handed neutrino Majorana mass matrix, the neutrino Dirac mass matrix and the charged lepton mass matrix, respectively, with $n_L = (\nu_L^0, (\nu_R^0)^c)$ a column vector. The matrix $\mathcal{M}$ is given by

$$\mathcal{M} = \begin{pmatrix} 0 & m_D \\ m_D^T & M_R \end{pmatrix}. \tag{2}$$

It is always possible to choose a weak basis (WB) where the matrices M_R and m_l are both real and diagonal. The diagonalization of the $2n \times 2n$ matrix $\mathcal{M}$ is performed via the unitary transformation

$$V^T \mathcal{M}^* V = \mathcal{D}, \tag{3}$$

where $\mathcal{D} = \text{diag}(m_{\nu_1}, m_{\nu_2}, m_{\nu_3}, M_{\nu_1}, M_{\nu_2}, M_{\nu_3})$, with m_{ν_i} and M_{ν_i} denoting the physical masses of the light and heavy Majorana neutrinos, respectively. By writing V and $\mathcal{D}$ in the following block form:

$$V = \begin{pmatrix} K & R \\ S & T \end{pmatrix}, \tag{4}$$

$$\mathcal{D} = \begin{pmatrix} d & 0 \\ 0 & D \end{pmatrix} \tag{5}$$

the leptonic charged current interactions can be written in terms of mass eigenstates as

$$\mathcal{L}_W = -\frac{g}{\sqrt{2}}\left(\overline{l_{iL}}\gamma_\mu K_{ij}\nu_{jL} + \overline{l_{iL}}\gamma_\mu R_{ij} N_{jL}\right)W^\mu + \text{h.c.}, \tag{6}$$

where ν_j and N_j denote the light and heavy neutrinos. Since the right-handed neutrino Majorana mass term is $SU(2) \times U(1)$ invariant, the scale M of M_R can be much larger than the scale v of electroweak symmetry breaking. Assuming $M^2 \gg v^2$ the light neutrino masses are obtained to an excellent approximation from:

$$U_\nu^\dagger m_{\text{eff}} U_\nu^* = d, \tag{7}$$

where $m_{\text{eff}} = -m_D {M_R}^{-1} m_D^T$. The natural suppression of the eigenvalues of m_{eff} is the crucial point of the seesaw mechanism. The unitary matrix U_ν obtained from Eq. (7) is the so-called Pontecorvo, Maki, Nakagawa and Sakata matrix [4] and coincides with K in Eqs. (4) and (6) up to corrections of order v^2/M^2, which we shall ignore under the above assumption.

In the WB where m_l, M_R are diagonal, all mixing and CP violation are contained in m_D which is a complex 3×3 matrix. Three of its nine arbitrary phases can be eliminated by the simultaneous rephasing of ν_l, l_L, so one is left with six CP-violating phases. Therefore, the three eigenvalues of M_R, together with the 15 parameters of m_D give a total of eighteen parameters. This is to be compared with the nine parameters contained in the low energy data, namely the three mixing angles and three CP-violating phases contained in U_ν, together with the three light neutrino masses.

The fact that there are many more parameters in m_D, M_R than measurable quantities at low energies makes it impossible, in general, to derive the seesaw parameters from low energy data. A particularly fascinating question is whether it is possible to relate the size and sign [5] of the observed baryon asymmetry of the universe (BAU) to low energy CP violation, in a framework of baryogenesis through leptogenesis. It has been pointed out that this is only possible if further assumptions are introduced. This can be readily seen by noting that, from Eq. (7) and the definition of m_{eff}, the matrix m_D can be parametrized as [6]

$$m_D = iU_\nu \sqrt{d} G \sqrt{D_R} \tag{8}$$

with G an orthogonal complex matrix, $\sqrt{D_R}$ a diagonal real matrix verifying the relation $\sqrt{D_R}\sqrt{D_R} = D_R$ and $\sqrt{d}$ a real matrix with a maximum number of zeros such that $\sqrt{d}\sqrt{d}^T = d$. Note that $\sqrt{d}$ is not always a square matrix, as can be seen in Section 3. From Eq. (8), it follows that

$$m_D^\dagger m_D = -\sqrt{D_R} G^\dagger \sqrt{d}^T \sqrt{d} G \sqrt{D_R}. \tag{9}$$

Since the CP-violating phases relevant for leptogenesis are those contained in $m_D^\dagger m_D$ [7], it is clear that leptogenesis can occur even if there is no CP violation at low energies, i.e., no Majorana- or Dirac-type CP phases at low energies [8].

In the literature there have been various attempts at reducing the number of seesaw parameters by considering so-called minimal scenarios. Models with only two right-handed neutrinos immediately lead to one massless light neutrino, whereas models with only one right-handed neutrino would require two of the light neutrinos to be massless and are, therefore, ruled out in the context of type I seesaw, where no Higgs triplets are added.

3. Example with one texture zero and two right-handed neutrinos

In this section, we reexamine the connection between leptonic low energy physics and leptogenesis, in the case of one texture zero and two right-handed neutrinos [9]. From the definition of m_{eff}, it can be readily realized that in the case of only two right-handed neutrinos, one of the light neutrinos is massless and one has

$$U_\nu^\dagger m_{\mathrm{eff}} U_\nu^* = \begin{pmatrix} 0 & & \\ & m_2 & \\ & & m_3 \end{pmatrix}. \tag{10}$$

Let us assume now that m_D has one texture zero [10] and write it in the form

$$m_D = \begin{pmatrix} a_1 & 0 \\ b_1 & b_2 \\ c_1 & c_2 \end{pmatrix} \tag{11}$$

with arbitrary nonzero entries, in the WB where m_l and M_R are diagonal and real. Let us write m_D as in Eq. (8), taking into account that in this case, the complex orthogonal matrix G is two-by-two and can be parametrized by

$$G = \begin{pmatrix} \cos Z & \pm\sin Z \\ -\sin Z & \pm\cos Z \end{pmatrix} \tag{12}$$

with Z complex. We use the following parametrization [11] for U_ν:

$$U_\nu = \begin{pmatrix} c_{12}c_{13} & s_{12}c_{13} & s_{13} \\ -s_{12}c_{23} - c_{12}s_{23}s_{13}e^{i\delta} & c_{12}c_{23} - s_{12}s_{23}s_{13}e^{i\delta} & s_{23}c_{13}e^{i\delta} \\ s_{12}s_{23} - c_{12}c_{23}s_{13}e^{i\delta} & -c_{12}s_{23} - s_{12}c_{23}s_{13}e^{i\delta} & c_{23}c_{13}e^{i\delta} \end{pmatrix} \cdot P, \tag{13}$$

where $c_{ij} \equiv \cos\theta_{ij}$, $s_{ij} \equiv \sin\theta_{ij}$ and $P = \mathrm{diag}(1, e^{i\alpha/2}, 1)$; δ is a Dirac-type phase and α is a physical phase associated to the Majorana character of neutrinos. In the general case, with three nonzero neutrino masses, two Majorana phases would be present.

In the case of only two right-handed neutrinos one has

$$d = \begin{pmatrix} 0 & 0 & 0 \\ 0 & m_2 & 0 \\ 0 & 0 & m_3 \end{pmatrix}, \qquad \sqrt{d} = \begin{pmatrix} 0 & 0 \\ \sqrt{m_2} & 0 \\ 0 & \sqrt{m_3} \end{pmatrix}. \tag{14}$$

From Eq. (8) we then obtain

$$m_{Di1} = iU_{\nu i2}\sqrt{m_2}(\cos Z)\sqrt{M_1} + iU_{\nu i3}\sqrt{m_3}(-\sin Z)\sqrt{M_1}, \tag{15}$$

$$m_{Di2} = iU_{\nu i2}\sqrt{m_2}(\pm\sin Z)\sqrt{M_2} + iU_{\nu i3}\sqrt{m_3}(\pm\cos Z)\sqrt{M_2}. \tag{16}$$

The zero entry in m_D implies

$$s_{12}c_{13}e^{i\frac{\alpha}{2}}\sqrt{m_2}(\pm\sin Z) + s_{13}\sqrt{m_3}(\pm\cos Z) = 0 \tag{17}$$

so that

$$\cos^2 Z = \frac{s_{12}^2 c_{13}^2 \rho}{s_{12}^2 c_{13}^2 \rho + s_{13}^2 e^{-i\alpha}}, \qquad \frac{\sin Z}{\cos Z} = -\frac{s_{13}e^{-i\frac{\alpha}{2}}}{s_{12}c_{13}\sqrt{\rho}}, \tag{18}$$

where $\rho = m_2/m_3$. Thus the zero texture in the matrix m_D allows for a full determination the matrix G, up to a reflexion, in terms of low energy measurable quantities. From Eq. (9), it is clear that knowledge of G enables one to obtain the phases appearing in $m_D^\dagger m_D$ which are the ones relevant for leptogenesis. Therefore, the presence of the texture zero leads to a connection between leptogenesis and low energy measurable quantities.

It is instructive to consider the case of a degenerate M_R. The expression for $m_D^\dagger m_D$, obtained from Eq. (9), becomes particularly simple

$$m_D^\dagger m_D = \mathcal{N}\begin{pmatrix} s_{12}^2 c_{13}^2 \rho^2 + s_{13}^2 & \pm\sqrt{\rho}\, s_{12} c_{13} s_{13} (\rho e^{-i\alpha/2} - e^{i\alpha/2}) \\ \text{h.c.} & s_{13}^2 \rho + s_{12}^2 c_{13}^2 \rho \end{pmatrix} \tag{19}$$

with $\mathcal{N}$ given by

$$\mathcal{N} = M m_3 \frac{1}{(s_{12} c_{13} \sqrt{\rho})^2 \sqrt{|(1+\lambda^2)|^2}} \quad \text{with } \lambda = \frac{s_{13}}{s_{12} c_{13} \sqrt{\rho}} e^{-i\alpha/2}, \tag{20}$$

where M denotes the common heavy neutrino mass. This simple example illustrates again the rôle of texture zeros in enabling to establish a connection between leptonic low energy physics and leptogenesis. Of course, exact degeneracy would have to be lifted in order for leptogenesis to be possible. Almost degeneracy among heavy neutrinos leads to the very interesting scenario of resonant leptogenesis [12]. Ibarra and Ross [9] have analysed in detail the predictions from models with one and two texture zeros in m_D in the case of two right-handed neutrinos, including the constraints on leptogenesis and lepton flavour violating processes. As pointed out in Ref. [9], the case of only one texture zero has the special feature of fixing the matrix G without imposing any further restriction on light neutrino masses and mixing. This is clear from Eq. (8) which shows that each zero on each column of m_D corresponds to an orthogonality condition between that column of the matrix G and the corresponding row of the matrix $U_\nu \sqrt{d}$:

$$(m_D)_{ij} = 0: \quad (U_\nu)_{ik} \sqrt{d}_{kl} G_{lj} = 0. \tag{21}$$

With one zero in m_D this equation has always a solution for any U_ν and $\sqrt{d}$ of the form given in Eq. (14), independently of the hierarchy between M_1 and M_2. Obviously, the rôle of texture zeros is to introduce restrictions which lead to the decrease in the number of independent seesaw parameters. In particular texture zeros lead to a decrease in the number of independent CP-violating phases.

As we have previously emphasized, texture zeros are WB dependent, in the sense that a texture zero present in one basis may no longer exist in another WB. One may wonder whether it is possible to translate particular texture zeros into restrictions on the seesaw parameters expressed in terms of WB invariants. The fact that texture zeros lead in general to a decrease in the number of CP-violating phases, provides a hint that CP-odd WB invariants may be useful for introducing restrictions on the seesaw parameters. In the next section we address this question.

4. On the relation between CP-odd WB invariants and texture zeros

We start by recalling how CP-odd WB invariants can be constructed by studying the CP properties of the present minimal extension of the SM, which leads to the leptonic mass terms of Eq. (1). The starting point consists of writing the most general CP transformation which leaves invariant the gauge interactions. It can be readily seen that the CP transformation is given by

$$\begin{aligned} &\mathrm{CP} l_L^0 (\mathrm{CP})^\dagger = U' \gamma^0 C \overline{l_L^0}^T, \qquad \mathrm{CP} l_R^0 (\mathrm{CP})^\dagger = V' \gamma^0 C \overline{l_R^0}^T, \\ &\mathrm{CP} \nu_L^0 (\mathrm{CP})^\dagger = U' \gamma^0 C \overline{\nu_L^0}^T, \qquad \mathrm{CP} \nu_R^0 (\mathrm{CP})^\dagger = W' \gamma^0 C \overline{\nu_R^0}^T, \\ &\mathrm{CP} W_\mu^+ (\mathrm{CP})^\dagger = -(-1)^{\delta_{0\mu}} W_\mu^-, \end{aligned} \tag{22}$$

where U', V', W' are unitary matrices acting in flavour space. The inclusion of these matrices reflects the fact that in a WB, gauge interactions do not distinguish the various flavours. Invariance of the mass terms under the above CP transformation, requires that the following relations have to be satisfied:

$$W'^T M_R W' = -M_R^*, \qquad U'^\dagger m_D W' = m_D^*, \qquad U'^\dagger m_l V' = m_l^*. \tag{23}$$

From Eqs. (23) one can derive [13] various CP-odd WB invariants, which are constrained to vanish if CP invariance holds, following the procedure first outlined in Ref. [14]. This procedure has been widely applied in Ref. [15] to the study of CP violation in many different scenarios. An example is the following condition:

$$\mathrm{Tr}\big[m_{\mathrm{eff}} m_{\mathrm{eff}}^\dagger, h_l\big]^3 = 0, \tag{24}$$

where $h_l = m_l m_l^\dagger$. This condition is satisfied in the limit of no CP violation of Dirac type, at low energies.

CP invariance requires the vanishing of certain WB invariants. In the minimal seesaw model which we are considering, with an equal number of left-handed and right-handed neutrinos, in general the number of CP-violating phases equals $n^2 - n$, where

n denotes the number of lepton flavours. In the presence of flavour symmetries leading to texture zeros and/or relations among parameters, one may have a smaller number of CP-violating phases and some of the CP-odd WB invariants may automatically vanish.

Next we analyse the possible connection between texture zeros and the vanishing of certain CP-odd invariants, in the cases of two and three right-handed neutrinos.

4.1. Two right-handed neutrinos and two texture zeros

In the case of two right-handed neutrinos all ansätze with two texture zeros in m_D have been studied in [9]. It can be readily verified that in all two texture zero ansätze the following WB invariant condition is satisfied:

$$I_1 \equiv \mathrm{tr}\big[m_D M_R^\dagger M_R m_D^\dagger, h_l\big]^3 = 0. \tag{25}$$

One may ask whether the converse is also true, i.e., whether the imposition of the condition of Eq. (25) on arbitrary complex leptonic mass matrices automatically leads to one of the two zero ansätze classified in [9]. We show that Eq. (25), together with a reasonable assumption of no conspiracy among the parameters of m_D and those of M_R, does require the matrix m_D to have two texture zeros in the WB where both M_R and m_l are diagonal real. Note that the hypothesis of "no conspiracy" is quite natural, since m_D and M_R originate in different terms of the Lagrangian.

In order to fix the notation let us write

$$m_D M_R^\dagger M_R m_D^\dagger = \begin{bmatrix} r_1 & \alpha_1 & \alpha_2 \\ \alpha_1^* & r_2 & \alpha_3 \\ \alpha_2^* & \alpha_3^* & r_3 \end{bmatrix}, \tag{26}$$

where r_i are real and α_i are complex elements, which depend on the heavy right-handed neutrino masses and the matrix elements of m_D. Writing

$$m_D \equiv \begin{bmatrix} a_1 & a_2 \\ b_1 & b_2 \\ c_1 & c_2 \end{bmatrix} \tag{27}$$

we obtain for the α_i in Eq. (26)

$$\alpha_1 = M_1^2 a_1 b_1^* + M_2^2 a_2 b_2^*, \qquad \alpha_2 = M_1^2 a_1 c_1^* + M_2^2 a_2 c_2^*, \qquad \alpha_3 = M_1^2 b_1 c_1^* + M_2^2 b_2 c_2^*. \tag{28}$$

The WB invariant I_1, calculated in the WB where m_l is also diagonal, is given by

$$I_1 = 6i\big(m_\tau^2 - m_\mu^2\big)\big(m_\tau^2 - m_e^2\big)\big(m_\mu^2 - m_e^2\big)\,\mathrm{Im}\big[\alpha_1 \alpha_2^* \alpha_3\big]. \tag{29}$$

Clearly, $I_1 = 0$, if and only if one of the α_i's is equal to zero or else the α_i's have cyclic phases in such a way that $\arg[\alpha_1\alpha_2^*\alpha_3] = 0, \pi$. If one adopts the above "no conspiracy" hypothesis, it is clear that the solutions where one of the α_i's vanishes, would require that each one of the two zeros contributing to that α_i should vanish. It can then be readily verified hat solutions of Eq. (25) in which one of the α_i's vanishes, correspond to textures with one zero in each column.

For example the requirement $\alpha_1 = 0$ is verified in the case of the following four m_D textures:

$$\begin{bmatrix} 0 & 0 \\ b_1 & b_2 \\ c_1 & c_2 \end{bmatrix}, \qquad \begin{bmatrix} 0 & a_2 \\ b_1 & 0 \\ c_1 & c_2 \end{bmatrix}, \qquad \begin{bmatrix} a_1 & 0 \\ 0 & b_2 \\ c_1 & c_2 \end{bmatrix}, \qquad \begin{bmatrix} a_1 & a_2 \\ 0 & 0 \\ c_1 & c_2 \end{bmatrix}. \tag{30}$$

All other possible textures with one zero in each column correspond to either $\alpha_2 = 0$ or $\alpha_3 = 0$. We consider now the solutions of Eq. (25) corresponding to cyclic α_i's. It can be readily verified that cyclic solutions correspond to textures with two zeros in the same column. Indeed, textures with two zeros in the first column eliminate the terms with M_2^2 whilst terms with two zeros in the second one eliminate the terms with M_1^2. An example is

$$\begin{bmatrix} 0 & a_2 \\ 0 & b_2 \\ c_1 & c_2 \end{bmatrix}, \qquad \alpha_1 = M_2^2 a_2 b_2^*, \qquad \alpha_2 = M_2^2 a_2 c_2^*, \qquad \alpha_3 = M_2^2 b_2 c_2^* \tag{31}$$

which obviously leads to $\arg[\alpha_1\alpha_2^*\alpha_3] = 0$.

All the fifteen different textures with two zeros in m_D are thus obtained from the invariant condition $I_1 = 0$, together with the "no-conspiracy" hypothesis. The low energy predictions arising from all these textures were analysed in Ref. [9], where it was shown that only five of them are allowed by present experiment. It was also pointed out in [9] that in general texture zeros in m_D may appear in a weak basis where neither M_R nor m_l are diagonal. Implications for low energy physics in the case of nondiagonal M_R were also discussed [9], under certain restrictive assumptions on m_l. At this stage, it is worth noting that there are other CP-odd

WB invariants which vanish for all the two zero textures considered above, even if they arise in a basis where M_R is not diagonal. An example is the following WB invariant condition:

$$I' \equiv \mathrm{tr}\big[m_D m_D^\dagger, h_l\big]^3 = 0 \tag{32}$$

which is verified for any texture with two zeros in m_D in a WB where m_l is diagonal, while M_R is arbitrary.

It should be pointed out that two zeros in m_D, in the WB where M_R and m_l are diagonal, still allow for CP-violation. In fact, with two zeros in m_D one can only eliminate at most two independent CP-violating phases out of the three present in the general case, with two right-handed neutrinos. As a result, not all CP-odd WB invariants vanish in the case of two texture zeros. An example of a CP-odd invariant [13] which does not vanish is given by

$$I'' \equiv \mathrm{Im}\,\mathrm{tr}\big[\big(m_D^\dagger m_D\big) M_R^* M_R M_R^* \big(m_D^\dagger m_D\big)^* M_R\big]. \tag{33}$$

In the WB where M_R is diagonal, it can be written as

$$I'' = M_1 M_2 \big(M_2^2 - M_1^2\big)\,\mathrm{Im}\, h_{12}^2, \tag{34}$$

where

$$h_{12} = \big(m_D^\dagger m_D\big)_{12} = a_1^* a_2 + b_1^* b_2 + c_1^* c_2 \tag{35}$$

so that $\mathrm{Im}\, h_{12}^2$ can differ from zero in the case of two texture zeros. This invariant is sensitive to the combination $m_D^\dagger m_D$ which is relevant for leptogenesis. Furthermore, two texture zeros in m_D also allow for CP violation of Dirac type in the leptonic charged weak currents. This can be seen from the condition written in Eq. (24).

4.2. Three right-handed neutrinos and three texture zeros

Let us now consider the case of three right-handed neutrinos and analyse the conditions under which the invariant I_1 vanishes. In this case m_D is a three-by-three matrix which can be written as

$$m_D = \begin{bmatrix} a_1 & a_2 & a_3 \\ b_1 & b_2 & b_3 \\ c_1 & c_2 & c_3 \end{bmatrix}. \tag{36}$$

The parameters α_i of Eq. (26) are now given by

$$\alpha_1 = M_1^2 a_1 b_1^* + M_2^2 a_2 b_2^* + M_3^2 a_3 b_3^*, \qquad \alpha_2 = M_1^2 a_1 c_1^* + M_2^2 a_2 c_2^* + M_3^2 a_3 c_3^*,$$
$$\alpha_3 = M_1^2 b_1 c_1^* + M_2^2 b_2 c_2^* + M_3^2 b_3 c_3^* \tag{37}$$

for M_R diagonal. Eq. (29) remains valid in the WB where m_l is also diagonal. There are, as before, two types of possible solutions. Solutions in which one of the α_i's is zero (irrespective of M_R) are all those corresponding to three zeros in m_D-one in each column leaving one row without zeros, as for example in

$$\begin{bmatrix} 0 & 0 & a_3 \\ b_1 & b_2 & 0 \\ c_1 & c_2 & c_3 \end{bmatrix} \quad (\alpha_1 = 0), \qquad \begin{bmatrix} 0 & a_2 & a_3 \\ b_1 & b_2 & b_3 \\ c_1 & 0 & 0 \end{bmatrix} \quad (\alpha_2 = 0), \qquad \begin{bmatrix} a_1 & a_2 & a_3 \\ b_1 & 0 & b_3 \\ 0 & c_2 & 0 \end{bmatrix} \quad (\alpha_3 = 0). \tag{38}$$

Solutions with three zeros in the same row would lead to one vanishing α_i, but they are physically unacceptable since they correspond to the decoupling of one generation at low energies.

Any one of the m_D matrices with three zeros has six real independent parameters and three independent CP-violating phases. Furthermore, we have three Majorana masses M_1, M_2, M_3. This is to be compared to three light neutrino masses, three mixing angles and three physical CP-violating phases at low energies.

In addition to the solutions in Eq. (38), we obtain a set of cyclic solutions, similar to the case of two right-handed neutrinos in Eq. (31), such that $\arg[\alpha_1 \alpha_2^* \alpha_3] = 0$ but with $\alpha_i \neq 0$. However, these solutions correspond to four texture zeros and therefore will be discussed in the following subsection, which is dedicated to the study of the connection between low and high energy CP violation in the context of models with four texture zeros.

4.3. Three right-handed neutrinos and four texture zeros

Cyclic solutions of Eq. (25), in the case of three right-handed neutrinos, require, for arbitrary phases in m_D, four zeros in this matrix. In this case one column has no zeros, the other two columns have two zeros each, as for example in

$$\begin{bmatrix} 0 & 0 & a_3 \\ b_1 & 0 & b_3 \\ 0 & c_2 & c_3 \end{bmatrix}, \qquad \begin{bmatrix} 0 & a_2 & 0 \\ 0 & b_2 & b_3 \\ c_1 & c_2 & 0 \end{bmatrix}. \tag{39}$$

Cyclic solutions where all zeros are grouped in one square, i.e., one column and one row have no zeros, are physically unacceptable, as they lead to $U_{i1} = 0$ for some i.

For cyclic solutions of Eq. (25), thus having four texture zeros, the number of parameters in m_D is much reduced. One has five real parameters and two complex phases in m_D. In particular, for the nonsquared cyclic solutions in Eq. (39), one may easily find the connection between leptogenesis and low energy physics. Let us consider as an example, the first matrix in Eq. (39). The a_i, b_i and c_i in m_D can be expressed as functions of the neutrino masses, mixing angles and CP-violating phases through Eq. (8). In this example, the matrix G can be fully determined by Eq. (21) due to the existence of four zeros. With three right-handed neutrinos the matrix $\sqrt{d}$ is diagonal with nonzero entries $\sqrt{m_i}$ and we have, e.g.,

$$(m_D)_{12} = 0: \quad (U_\nu)_{1k}\sqrt{m_k}G_{k2} = 0 \tag{40}$$

leading to

$$(\vec{G}_1)_i = \bigl(\varepsilon_{ijk}(U_\nu)_{1j}\sqrt{m_j}(U_\nu)_{3k}\sqrt{m_k}\bigr)\frac{1}{N_1}, \tag{41}$$

$$(\vec{G}_2)_i = \bigl(\varepsilon_{ijk}(U_\nu)_{1j}\sqrt{m_j}(U_\nu)_{2k}\sqrt{m_k}\bigr)\frac{1}{N_2}, \tag{42}$$

$$(\vec{G}_3)_i = \varepsilon_{ijk}(\vec{G}_1)_j(\vec{G}_2)_k = \frac{1}{N_3}(U_\nu)_{1i}\sqrt{m_i} \quad \text{(no sum in } i\text{)}, \tag{43}$$

where the $\vec{G}_i$ are the columns of the matrix G and the N_i are complex normalization factors, with phases such that $\vec{G}_i^2 = 1$.

Let us now consider the nonzero entries of m_D, for example b_1, which corresponds to

$$(m_D)_{21} = b_1 = i\bigl((U_\nu)_{2k}\sqrt{m_k}\bigr)G_{k1}\sqrt{M_1}. \tag{44}$$

Once the G_{kj} are replaced by the explicit formulas obtained above, the coefficients of m_D can be fully expressed in terms of physical quantities only, up to nonphysical phases which can be rotated away.

In this example we have

$$-m_{\text{eff}} = m_D\frac{1}{D}m_D^T = \begin{bmatrix} a_3^2M_3^{-1} & a_3b_3M_3^{-1} & a_3c_3M_3^{-1} \\ a_3b_3M_3^{-1} & b_1^2M_1^{-1} + b_3^2M_3^{-1} & b_3c_3M_3^{-1} \\ a_3c_3M_3^{-1} & b_3c_3M_3^{-1} & c_2^2M_2^{-1} + c_3^2M_3^{-1} \end{bmatrix}. \tag{45}$$

Only five entries in m_{eff} are independent. We can relate $(m_{\text{eff}})_{23}$ to other entries by

$$(m_{\text{eff}})_{11}(m_{\text{eff}})_{23} - (m_{\text{eff}})_{12}(m_{\text{eff}})_{13} = 0. \tag{46}$$

This relation implies low energy constraints and furthermore guarantees the orthogonality of columns one and two of the matrix G, as defined above. It is clear from the definition of m_{eff} that G does not play any rôle in low energy physics. However in the matrix $m_D^\dagger m_D$, which is the matrix relevant for leptogenesis, the elements of G play an important rôle since they do not cancel out. In this example, there is a strong relation between leptogenesis and low energy physics due to the fact that G can be fully expressed in terms of measurable low energy parameters. With four texture zeros, there are constraints in the low energy physics which result from the reduction of the independent parameters in m_{eff} as expressed in this example by Eq. (46). This relation excludes scenarios with direct or inverse hierarchical light neutrinos, i.e., the case of one neutrino mass much smaller than the other two. Likewise, in the three zero textures of Section 4.2, there are also low energy constraints which in this case translate into the existence of one zero in one of the off-diagonal elements of m_{eff} (and its symmetric entry). For instance, for the first matrix in Eq. (38), corresponding to the case $\alpha_1 = 0$, one has $(m_{\text{eff}})_{12} = 0$, or equivalently

$$m_1U_{11}U_{21} + m_2U_{12}U_{22} + m_3U_{13}U_{23} = 0. \tag{47}$$

In Ref. [16] the stability of zeros in neutrino mass matrices under quantum corrections, in type I seesaw models, has been studied. It was found that some of the two-zero textures for the neutrino mass matrix that have been classified as incompatible with experimental data, are not excluded. A detailed study of the phenomenology of three and four texture zeros in m_D is beyond the scope of this Letter.

Four texture zeros may also be obtained from the solutions with three texture zeros considered in the previous subsection in which one of the α_i's is zero. However, for these cases, the extra zero has to be imposed by demanding that a new invariant I_2 vanishes.[1] Taking

$$I_2 \equiv \operatorname{tr}\bigl[M_R^\dagger M_R, m_D^\dagger m_D\bigr]^3 \tag{48}$$

[1] With respect to the cyclic solutions, we do not need to consider this invariant, as they automatically obey $I_2 = 0$.

and computing I_2 for, e.g., the $\alpha_1 = 0$ case in Eq. (38), one finds

$$I_2 = 6i\left(M_3^2 - M_2^2\right)\left(M_3^2 - M_1^2\right)\left(M_2^2 - M_1^2\right)|c_3|^2 \operatorname{Im}\left[b_1^* b_2 c_1 c_2^*\right]. \tag{49}$$

It is clear that $I_2 = 0$, if one of the parameters[2] b_1, b_2, c_1, c_2 vanishes. The case $c_3 = 0$ is of no physical interest as it leads to vanishing solar neutrino mixing, which is clear by computing m_{eff}. Taking, e.g., $c_1 = 0$, one then obtains for m_D

$$m_D = \begin{bmatrix} 0 & 0 & a_3 \\ b_1 & b_2 & 0 \\ 0 & c_2 & c_3 \end{bmatrix} \tag{50}$$

which has 4 texture zeros.

It is interesting to note that imposing I_2 equal to zero, irrespective of condition $I_1 = 0$, for nondegenerate M_i, requires:

$$\operatorname{Im}\left[\left(m_D^\dagger m_D\right)_{12}\left(m_D^\dagger m_D\right)_{31}\left(m_D^\dagger m_D\right)_{23}\right] = 0, \tag{51}$$

where

$$\left(m_D^\dagger m_D\right)_{12} = a_1^* a_2 + b_1^* b_2 + c_1^* c_2, \qquad \left(m_D^\dagger m_D\right)_{13} = a_1^* a_3 + b_1^* b_3 + c_1^* c_3, \qquad \left(m_D^\dagger m_D\right)_{23} = a_2^* a_3 + b_2^* b_3 + c_2^* c_3, \tag{52}$$

Matrices m_D, with three zeros, one on each row leaving one column without zeros, such as

$$\begin{bmatrix} 0 & a_2 & a_3 \\ 0 & b_2 & b_3 \\ c_1 & 0 & c_3 \end{bmatrix}, \qquad \begin{bmatrix} a_1 & a_2 & 0 \\ 0 & b_2 & b_3 \\ 0 & c_2 & c_3 \end{bmatrix}, \qquad \begin{bmatrix} a_1 & 0 & a_3 \\ b_1 & 0 & b_3 \\ c_1 & c_2 & 0 \end{bmatrix} \tag{53}$$

verify this condition. These matrices are the transposed of the solutions found in Section 4.2.

5. Conclusions

We have shown that CP-odd WB invariants can be useful in the analysis of lepton-flavour models with texture zeros. In particular, we have pointed out that there is a large class of sets of texture zeros considered in the literature which lead to the vanishing of certain CP-odd invariants. Conversely, it was shown that starting from arbitrary complex leptonic mass matrices, the imposition of the vanishing of certain CP-odd invariants together with a reasonable assumption of no conspiracy among the parameters of m_D and M_R, automatically leads to given sets of texture zeros. These WB invariants enable one to recognize models characterized by texture zeros in m_D in the WB where m_l, M_R are diagonal, when these same models are written in a different WB where the texture zeros are not manifest.

We have also discussed the rôle of texture zeros in allowing for a connection between leptogenesis and low energy data, such as leptonic masses, mixing and CP violation. We have done the analysis in the context of two, three and four texture zeros. The crucial point is the fact that in the presence of texture zeros, the matrix G defined in Eq. (8), can be expressed in terms of low energy parameters. Recall that G enters in $m_D^\dagger m_D$ which in turn plays a crucial rôle in leptogenesis. Furthermore texture zeros lead in general to specific predictions at low energies.

An important step towards the understanding of the flavour puzzle would be finding a theoretical framework which would naturally lead to the vanishing of the CP-odd invariants considered in this Letter or else to specific texture zeros.

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References

[1] P. Minkowski, Phys. Lett. B 67 (1977) 421;
T. Yanagida, in: Proc. of the Workshop on Unified Theory and Baryon Number in the Universe, KEK, March 1979;
S.L. Glashow, in: M. Lévy, et al. (Eds.), Quarks and Leptons, Cargèse, Plenum, New York, 1980, p. 707;

[2] Cases where, e.g., $\arg[b_1^* b_2 c_1 c_2^*] = 0$, or with other phase relations amongst the a's, b's and c's will not be studied here.

M. Gell-Mann, P. Ramond, R. Slansky, in: Supergravity, Stony Brook, September 1979;
R.N. Mohapatra, G. Senjanovic, Phys. Rev. Lett. 44 (1980) 912.
[2] See for example: A.S. Joshipura, E.A. Paschos, W. Rodejohann, JHEP 0108 (2001) 029, hep-ph/0105175;
G.C. Branco, T. Morozumi, B.M. Nobre, M.N. Rebelo, Nucl. Phys. B 617 (2001) 475, hep-ph/0107164;
F. Buccella, D. Falcone, F. Tramontano, Phys. Lett. B 524 (2002) 241, hep-ph/0108172;
W. Rodejohann, K.R.S. Balaji, Phys. Rev. D 65 (2002) 093009, hep-ph/0201052;
G.C. Branco, R. González Felipe, F.R. Joaquim, M.N. Rebelo, Nucl. Phys. B 640 (2002) 202, hep-ph/0202030;
J.R. Ellis, M. Raidal, Nucl. Phys. B 643 (2002) 229, hep-ph/0206174;
Z.Z. Xing, Phys. Lett. B 545 (2002) 352, hep-ph/0206245;
S. Davidson, A. Ibarra, Nucl. Phys. B 648 (2003) 345, hep-ph/0206304;
M.N. Rebelo, Phys. Rev. D 67 (2003) 013008, hep-ph/0207236;
P.H. Frampton, S.L. Glashow, T. Yanagida, Phys. Lett. B 548 (2002) 119, hep-ph/0208157;
T. Endoh, S. Kaneko, S.K. Kang, T. Morozumi, M. Tanimoto, Phys. Rev. Lett. 89 (2002) 231601, hep-ph/0209020;
M. Raidal, A. Strumia, Phys. Lett. B 553 (2003) 72, hep-ph/0210021;
T. Hambye, hep-ph/0210048;
G.C. Branco, R. González Felipe, F.R. Joaquim, I. Masina, M.N. Rebelo, C.A. Savoy, Phys. Rev. D 67 (2003) 073025, hep-ph/0211001;
S.F. King, Phys. Rev. D 67 (2003) 113010, hep-ph/0211228;
S. Pascoli, S.T. Petcov, W. Rodejohann, Phys. Rev. D 68 (2003) 093007, hep-ph/0302054;
S. Davidson, JHEP 0303 (2003) 037, hep-ph/0302075;
W. Buchmuller, P. Di Bari, M. Plumacher, Nucl. Phys. B 665 (2003) 445, hep-ph/0302092;
E. Nezri, J. Orloff, JHEP 0304 (2003) 020, hep-ph/0004227;
S. Kaneko, M. Katsumata, M. Tanimoto, JHEP 0307 (2003) 025, hep-ph/0305014;
E.K. Akhmedov, M. Frigerio, A.Y. Smirnov, JHEP 0309 (2003) 021, hep-ph/0305322;
A. Broncano, M.B. Gavela, E. Jenkins, Nucl. Phys. B 672 (2003) 163, hep-ph/0307058;
L. Velasco-Sevilla, JHEP 0310 (2003) 035, hep-ph/0307071;
B. Dutta, R.N. Mohapatra, Phys. Rev. D 68 (2003) 113008, hep-ph/0307163;
G.F. Giudice, A. Notari, M. Raidal, A. Riotto, A. Strumia, Nucl. Phys. B 685 (2004) 89, hep-ph/0310123;
V. Barger, D.A. Dicus, H.J. He, T.J. Li, Phys. Lett. B 583 (2004) 173, hep-ph/0310278;
R. González Felipe, F.R. Joaquim, B.M. Nobre, Phys. Rev. D 70 (2004) 085009, hep-ph/0311029;
W. Rodejohann, Eur. Phys. J. C 32 (2004) 235, hep-ph/0311142;
W. Grimus, L. Lavoura, J. Phys. G 30 (2004) 1073, hep-ph/0311362;
A. Ibarra, G.G. Ross, Phys. Lett. B 591 (2004) 285, hep-ph/0312138;
W. Buchmuller, P. Di Bari, M. Plumacher, Ann. Phys. 315 (2005) 305, hep-ph/0401240;
W. Buchmuller, P. Di Bari, M. Plumacher, New J. Phys. 6 (2004) 105, hep-ph/0406014;
R.N. Mohapatra, S. Nasri, Phys. Rev. D 71 (2005) 033001, hep-ph/0410369;
M.C. Chen, K.T. Mahanthappa, Phys. Rev. D 71 (2005) 035001, hep-ph/0411158;
N. Sahu, S. Uma Sankar, hep-ph/0501069;
K.S. Babu, A. Bachri, H. Aissaoui, hep-ph/0509091;
R.N. Mohapatra, et al., hep-ph/0510213.
[3] M. Fukugita, T. Yanagida, Phys. Lett. B 174 (1986) 45.
[4] B. Pontecorvo, Sov. Phys. JETP 7 (1958) 172, Zh. Eksp. Teor. Fiz. 34 (1957) 247;
Z. Maki, M. Nakagawa, S. Sakata, Prog. Theor. Phys. 28 (1962) 870;
B. Pontecorvo, Sov. Phys. JETP 26 (1968) 984, Zh. Eksp. Teor. Fiz. 53 (1967) 1717.
[5] The first example where both the size and sign are related was given by P.H. Frampton, S.L. Glashow, T. Yanagida in [2].
[6] J.A. Casas, A. Ibarra, Nucl. Phys. B 618 (2001) 171, hep-ph/0103065.
[7] J. Liu, G. Segre, Phys. Rev. D 48 (1993) 4609, hep-ph/9304241;
M. Flanz, E.A. Paschos, U. Sarkar, Phys. Lett. B 345 (1995) 248;
M. Flanz, E.A. Paschos, U. Sarkar, Phys. Lett. B 382 (1995) 447, Erratum;
L. Covi, E. Roulet, F. Vissani, Phys. Lett. B 384 (1996) 169;
A. Pilaftsis, Phys. Rev. D 56 (1997) 5431, hep-ph/9707235;
W. Buchmüller, M. Plümacher, Phys. Lett. B 431 (1998) 354, hep-ph/9710460.
[8] See M.N. Rebelo in [2];
M.N. Rebelo, Invited talk at (BEYOND 03), Castle Ringberg, Tegernsee, Germany, 9–14 June 2003, hep-ph/0311226.
[9] See A. Ibarra, G.G. Ross, in [2].
[10] This is the particular texture chosen by R. González Felipe, F.R. Joaquim, B.M. Nobre, in the work cited in [2].
[11] Particle Data Group Collaboration, S. Eidelman, et al., Phys. Lett. B 592 (2004) 1.
[12] A. Pilaftsis, Nucl. Phys. B 504 (1997) 61, hep-ph/9702393;
J.R. Ellis, M. Raidal, T. Yanagida, Phys. Lett. B 546 (2002) 228, hep-ph/0206300;
A. Pilaftsis, T.E.J. Underwood, Nucl. Phys. B 692 (2004) 303, hep-ph/0309342;
R. González Felipe, F.R. Joaquim, B.M. Nobre in [2];
T. Hambye, Y. Lin, A. Notari, M. Papucci, A. Strumia, Nucl. Phys. B 695 (2004) 169, hep-ph/0312203;
K. Turzynski, Phys. Lett. B 589 (2004) 135, hep-ph/0401219;
T. Hambye, J. March-Russell, S.M. West, JHEP 0407 (2004) 070, hep-ph/0403183;
G.C. Branco, R. González Felipe, F.R. Joaquim, B.M. Nobre, hep-ph/0507092.
[13] See A. Pilaftsis in [7];
G.C. Branco, T. Morozumi, B.M. Nobre, M.N. Rebelo in [2].
[14] J. Bernabeu, G.C. Branco, M. Gronau, Phys. Lett. B 169 (1986) 243.
[15] G.C. Branco, M.N. Rebelo, Phys. Lett. B 173 (1986) 313;

M. Gronau, A. Kfir, R. Loewy, Phys. Rev. Lett. 56 (1986) 1538;
G.C. Branco, L. Lavoura, M.N. Rebelo, Phys. Lett. B 180 (1986) 264;
G.C. Branco, V.A. Kostelecky, Phys. Rev. D 39 (1989) 2075;
G.C. Branco, M.N. Rebelo, J.W.F. Valle, Phys. Lett. B 225 (1989) 385;
L. Lavoura, J.P. Silva, Phys. Rev. D 50 (1994) 4619, hep-ph/9404276;
F.J. Botella, J.P. Silva, Phys. Rev. D 51 (1995) 3870, hep-ph/9411288;
F. del Aguila, J.A. Aguilar-Saavedra, Phys. Lett. B 386 (1996) 241, hep-ph/9605418;
F. del Aguila, J.A. Aguilar-Saavedra, M. Zralek, Comput. Phys. Commun. 100 (1997) 231, hep-ph/9607311;
J.A. Aguilar-Saavedra, J. Phys. G 24 (1998) L31, hep-ph/9703461;
O. Lebedev, Phys. Rev. D 67 (2003) 015013, hep-ph/0209023;
S. Davidson, R. Kitano, JHEP 0403 (2004) 020, hep-ph/0312007;
F.J. Botella, M. Nebot, O. Vives, hep-ph/0407349;
G.C. Branco, M.N. Rebelo, J.I. Silva-Marcos, Phys. Lett. B 614 (2005) 187, hep-ph/0502118;
S. Davidson, H.E. Haber, Phys. Rev. D 72 (2005) 035004, hep-ph/0504050;
J.F. Gunion, H.E. Haber, hep-ph/0506227;
P.F. Harrison, W.G. Scott, Phys. Lett. B 628 (2005) 93, hep-ph/0508012.

[16] C. Hagedorn, J. Kersten, M. Lindner, Phys. Lett. B 597 (2004) 63, hep-ph/0406103;
S. Antusch, J. Kersten, M. Lindner, M. Ratz, M.A. Schmidt, JHEP 0503 (2005) 024, hep-ph/0501272.