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Optimal dividend problem with a terminal value for spectrally positive Lévy processes

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HIGHLIGHTS

- The risk process is modeled by a general spectrally positive Levy process before dividends are deducted.
- We give an explicit expression of the value function of a barrier strategy.
- We show that the optimal dividend strategy is formed by a barrier strategy.

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ABSTRACT

In this paper we consider a modified version of the classical optimal dividend problem taking into account both expected dividends and the time value of ruin. We assume that the risk process is modeled by a general spectrally positive Lévy process before dividends are deducted. Using the fluctuation theory of spectrally positive Lévy processes we give an explicit expression of the value function of a barrier strategy. Subsequently we show that a barrier strategy is the optimal strategy among all admissible ones. Our work is motivated by the recent work of Bayraktar, Kyprianou and Yamazaki (2013a).

1. Introduction

In this paper we consider the classical optimal dividend problem of de Finetti for an insurance company whose risk process evolves as a spectrally positive Lévy process in the absence of dividend payments. Over the last decade there has been a great deal of interest in the insurance risk process as modeled by a Lévy process, going back to the work of Klüppelberg et al. (2004). In this literature, the traditional compound Poisson model is replaced by a general Lévy process, allowing for much greater flexibility in modeling as well as access to the growing literature on Lévy processes which has also developed greatly in the last few decades. In particular, spectrally positive Lévy processes form a very tractable subclass which has been much exploited recently both in the theoretical and in the actuarial literature, for example, Avanzi et al. (2007), Avanzi and Gerber (2008), Bayraktar and Egami (2008), Li and Wu (2008), Ng (2009), Yao et al. (2010), Dai et al. (2010, 2011), Avanzi et al. (2011), Bayraktar et al. (2013a,b) and Yin and Wen (2013a,b) to name but a few.

The recent paper of Bayraktar et al. (2013a) studied the classical de Finetti's optimal dividend problem where the surplus of a company is modeled by a spectrally positive Lévy process. We continue this optimal dividend problem but add a component to the dividend-value function that penalizes early ruin of controlled risk processes. We now state the control problem considered in this paper. Let $X = \{X(t)\}_{t\geq 0}$ be a Lévy process without negative jumps defined on a filtered probability space $(\Omega, \mathcal{F}, \mathbb{F}, P)$, where $\mathbb{F} = (\mathcal{F}_t)_{t\geq 0}$ is generated by the process X and satisfies the usual conditions. As the process X has no negative jumps, its Laplace

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exponent exists and is given by

$$\Psi(\theta) = \frac{1}{t}\ln Ee^{-\theta X(t)} = c\theta + \frac{1}{2}\sigma^2\theta^2 + \int_0^\infty (e^{-\theta x} - 1 + \theta x \mathbf{1}_{\{|x|<1\}})\Pi(dx), \tag{1.1}$$

where $\mathbf{1}_A$ is the indicator function of a set A, $c > 0$, $\sigma \geq 0$ and Π is a measure on $(0, \infty)$ satisfying ((c, σ, Π) is called the Lévy triplet of X)

$$\int_0^\infty (1 \wedge x^2)\Pi(dx) < \infty.$$

Denote by P_x for the law of X when $X(0) = x$. Let E_x be the expectation associated with P_x. For short, we write P and E when $X(0) = 0$. To avoid trivialities, it is assumed that X does not have monotone sample paths. In the sequel, we assume that $-\Psi'(0+) = \mathbb{E}(X(1)) > 0$ which implies that the process X drifts to $+\infty$. It is well known that if $\int_1^\infty y\Pi(dy) < \infty$, then $\mathbb{E}(X(1)) < \infty$, and $\mathbb{E}(X(1)) = -c + \int_1^\infty y\Pi(dy)$. Note that X has paths of bounded variation if and only if

$$\sigma = 0 \quad \text{and} \quad \int_0^\infty (1 \wedge x)\Pi(dx) < \infty.$$

In this case, we write (1.1) as

$$\Psi(\theta) = c_0\theta + \frac{1}{2}\sigma^2\theta^2 + \int_0^\infty (e^{-\theta x} - 1)\Pi(dx), \tag{1.2}$$

with $c_0 = c + \int_0^1 x\Pi(dx)$ the so-called drift of X. In particular, in the case where $\sigma = 0$, $\Pi(dx) = \lambda P(dx)$, the process becomes the dual model in Avanzi et al. (2007), and in the case where $\Pi(dx) = \lambda P(dx)$, the process becomes the dual model with diffusion in Avanzi and Gerber (2008). For more details on spectrally positive Lévy processes, the reader is referred to Bertoin (1996), Sato (1999) and Kyprianou (2006).

The process X is an appropriate model of a company driven by inventions or discoveries, or the cash fund of an investment company before dividends are deducted. Let $\pi = \{L_t^\pi : t \geq 0\}$ be a dividend strategy consisting of a nondecreasing, right-continuous and $\mathbb{F}$-adapted process starting at 0, where L_t^π stands for the lump-sums of dividends paid out by the company up until time t. The risk process with initial capital $x \geq 0$ and controlled by a dividend strategy π is defined by $U^\pi = \{U_t^\pi : t \geq 0\}$, where

$$U_t^\pi = X(t) - L_t^\pi, \quad t \geq 0.$$

The ruin time is then given by

$$\tau_\pi = \inf\{t > 0 | U_t^\pi = 0\}.$$

A dividend strategy is called admissible if $L_t^\pi - L_{t-}^\pi \leq U_{t-}^\pi$, for all $t < \tau_\pi$, in other words the lump sum dividend payment is smaller than the size of the available capital. We define the dividend-value function V_π by

$$V_\pi(x) = E\left[\int_0^{\tau_\pi} e^{-qt} dL_t^\pi + Se^{-q\tau_\pi} | U_0^\pi = x\right],$$

where $q > 0$ is an interest force for the calculation of the present value and $S \in \mathbb{R}$ is the terminal value. Let Ξ be the set of all admissible dividend policies. De Finetti's dividend problem consists of solving the following stochastic control problem:

$$V(x) = \sup_{\pi \in \Xi} V_\pi(x), \tag{1.3}$$

and to find an optimal policy $\pi^* \in \Xi$ that satisfies $V(x) = V_{\pi^*}(x)$ for all $x \geq 0$.

Next, we shall have a review on the related literature. This optimal dividend problem has recently gained a lot of attention in actuarial mathematics for spectrally negative Lévy processes. Avram et al. (2007), Loeffen (2008) and Kyprianou et al. (2010) studied the case of $S = 0$ for spectrally negative Lévy processes. The case $S < 0$ was studied by Thonhauser and Albrecher (2007) for the compound Poisson model and Brownian motion risk process. The case $S \in \mathbb{R}$ was studied by Loeffen (2009) and Loeffen and Renaud (2010) for spectrally negative Lévy processes. It was shown that the optimal dividend strategy is formed by a barrier strategy for this type model under some conditions imposed on the Lévy measure. Moreover, Azcue and Muler (2005) have provided a counterexample for the case $S = 0$ showing that a barrier strategy cannot be optimal. However, this is in contrast to the dividend problem in the case of $S = 0$ for spectrally positive Lévy processes considered by Bayraktar et al. (2013a), which shows that there a barrier strategy always forms the optimal strategy, no matter the form of the jump measure. Motivated by the work of Bayraktar et al. (2013a), the purpose of this paper is to examine the analogous question for the same model in the case of $S \neq 0$.

The rest of the paper is organized as follows. In Section 2 we state some facts about scale functions. In Section 3 we give the main results. Explicit expressions for the expected discounted value of dividend payments are obtained, and it is shown that the optimal dividend strategy is formed by a barrier strategy.

2. Scale functions

For an arbitrary spectrally positive Lévy process, the Laplace exponent Ψ is strictly convex and $\lim_{\theta\to\infty}\Psi(\theta) = \infty$. Moreover, Ψ is strictly increasing on $[\Phi(0), \infty)$, where $\Phi(0)$ is the largest zero of Ψ. Thus there exists a function $\Phi : [0, \infty) \to [\Phi(0), \infty)$ defined by $\Phi(q) = \sup\{\theta \geq 0 : \Psi(\theta) = q\}$ (its right-inverse) and such that $\Psi(\Phi(q)) = q,\ q \geq 0$.

We now recall the definition of the q-scale function $W^{(q)}$ and some properties of this function. For each $q \geq 0$ there exists a continuous and increasing function $W^{(q)} : \mathbb{R} \to [0, \infty)$, called the q-scale function defined in such a way that $W^{(q)}(x) = 0$ for all $x < 0$ and on $[0, \infty)$ its Laplace transform is given by

$$\int_0^\infty e^{-\theta x} W^{(q)}(x)dx = \frac{1}{\Psi(\theta) - q}, \quad \theta > \Phi(q). \tag{2.1}$$

Closely related to $W^{(q)}$ is the scale function $Z^{(q)}$ given by

$$Z^{(q)}(x) = 1 + q\int_0^x W^{(q)}(y)dy, \quad x \in \mathbb{R}.$$

We will also use the following functions:

$$\overline{W}^{(q)}(x) = \int_0^x W^{(q)}(z)dz,$$

$$\overline{Z}^{(q)}(x) = \int_0^x Z^{(q)}(z)dz, \quad x \in \mathbb{R}.$$

Note that

$$Z^{(q)}(x) = 1, \qquad \overline{Z}^{(q)}(x) = x, \quad x \leq 0.$$

If X has paths of bounded variation then, for all $q \geq 0$, $W^{(q)}|_{(0,\infty)} \in C^1(0, \infty)$ if and only if Π has no atoms. In the case that X has paths of unbounded variation, then for all $q \geq 0$, $W^{(q)}|_{(0,\infty)} \in C^1(0, \infty)$. Moreover if $\sigma > 0$ then $C^2(0, \infty)$. Further, if the Lévy measure has a density, then the scale functions are always differentiable (see e.g. Chan et al., 2011).

The initial values of $W^{(q)}$ and its derivative can be derived from (2.1):

$$W^{(\delta)}(0+) = \begin{cases} \dfrac{1}{c_0}, & \text{if } X \text{ has paths of bounded variation,} \\ 0, & \text{otherwise,} \end{cases}$$

and

$$W^{(\delta)'}(0+) = \begin{cases} \dfrac{2}{\sigma^2}, & \text{if } \sigma \neq 0, \\ \dfrac{q+\Pi(0,\infty)}{c_0^2}, & \text{if } X \text{ is compound Poisson}, \\ \infty, & \text{if } \sigma = 0 \text{ and } \Pi(0,\infty) = \infty. \end{cases}$$

The functions $W^{(q)}(x)$ and $Z^{(q)}(x)$ play a key role in the solution of two-sided exit problem. The following results can be extracted directly out of the existing literature. See for example Korolyuk et al. (1976), Bertoin (1997), Avram et al. (2004) and Kuznetsov et al. (2013) in a somewhat different form. Define the first passage times for $a < b$, with the convention $\inf \emptyset = \infty$,

$$T_b^+ = \inf\{t \geq 0 : X(t) > b\},$$

$$T_a^- = \inf\{t \geq 0 : X(t) < a\}, \qquad \tau_{ab} = T_a^- \wedge T_b^+.$$

Then we have for $x, y \in (a, b)$, $q \geq 0$, $z \geq b$,

$$E_x(e^{-qT_a^-}\mathbf{1}_{\{T_a^- < T_b^+\}}) = \frac{W^{(q)}(b-x)}{W^{(q)}(b-a)}, \tag{2.2}$$

$$E_x(e^{-qT_b^+}\mathbf{1}_{\{T_b^+ < T_a^-\}}) = Z^{(q)}(b-x) - W^{(q)}(b-x) \times \frac{Z^{(q)}(b-a)}{W^{(q)}(b-a)}, \tag{2.3}$$

$$E_x\left(e^{-q\tau_{ab}}\mathbf{1}_{\{X(\tau_{ab})=b\}}\right) = \frac{\sigma^2}{2}\left(W^{(q)'}(b-x) - W^{(q)}(b-x) \times \frac{W^{(q)'}(b-a)}{W^{(q)}(b-a)}\right), \tag{2.4}$$

$$E_x\left(e^{-q\tau_{ab}}\mathbf{1}_{\{X(\tau_{ab}-)\in dy, X(\tau_{ab})\in dz\}}\right) = u^{(q)}(x,y)\Pi(dz-y)dy, \tag{2.5}$$

where

$$u^{(q)}(x,y) = W^{(q)}(b-x)\frac{W^{(q)}(y-a)}{W^{(q)}(b-a)} - W^{(q)}(y-x).$$

The identities (2.2) and (2.3) together with the strong Markov property imply that

$$e^{-q(t\wedge\tau_{ab})}W^{(q)}(b - X(t\wedge\tau_{ab})), \qquad e^{-q(t\wedge\tau_{ab})}Z^{(q)}(b - X(t\wedge\tau_{ab}))$$

and

$$e^{-q(t\wedge\tau_{ab})}\left(Z^{(q)}(b - X(t\wedge\tau_{ab})) - W^{(q)}(b - X(t\wedge\tau_{ab})) \times \frac{Z^{(q)}(b-a)}{W^{(q)}(b-a)}\right)$$

are martingales.

3. Main results

Denoted by $\pi_b = \{L_t^b, t \leq \tau^b\}$ the constant barrier strategy at level b and let $U_b = \{U_b(t) : t \geq 0\}$ be the corresponding risk process, where $U_b(t) = X(t) - D_b(t)$, with $L_{0-}^b = 0$ and $L_t^b = (b \vee \sup_{0\leq s\leq t} X(s)) - b$. Note that $U_b(t)$ is a spectrally positive Lévy process reflected at b, $\pi_b \in \Xi$ and $L_0^b = x - b$ if $X(0) = x > b$. Denote by $V_b(x)$ the dividend-value function when using the dividend strategy π_b, that is,

$$V_b(x) = E_x\left[\int_0^{T_b} e^{-qt}dL_t^b + Se^{-qT_b}\right], \quad 0 \leq x \leq b,$$

where $T_b = \inf\{t > 0 : U_b(t) = 0\}$ with $T_b = \infty$ if $U_b(t) > 0$ for all $t \geq 0$. Here $q > 0$ is the discount factor and $S \in \mathbb{R}$ is the terminal value.

We will now present the main results of this paper.

Theorem 3.1. *The dividend-value function of the barrier strategy at level $b \geq 0$ is given by*

$$V_b(x) = \begin{cases} \Lambda(b)Z^{(q)}(b-x) - \overline{Z}^{(q)}(b-x) - \dfrac{\Psi'(0+)}{q}, & \text{if } 0 \leq x \leq b, \\ x - b + \Lambda(b) - \dfrac{\Psi'(0+)}{q}, & \text{if } x > b, \end{cases} \tag{3.1}$$

where

$$\Lambda(b) = \left(\overline{Z}^{(q)}(b) + \frac{\Psi'(0+)}{q} + S\right)\frac{1}{Z^{(q)}(b)}.$$

Theorem 3.2. *The barrier strategy at b^* is an optimal strategy for the control problem (1.3) regardless of the Lévy measure, i.e. $V(x) = V_{b^*}(x)$, where*

$$b^* = \begin{cases} (\overline{Z}^{(q)})^{-1}\left(-\dfrac{\Psi'(0+)}{q} - S\right), & \text{if } -\dfrac{\Psi'(0+)}{q} - S > 0, \\ 0, & \text{if } -\dfrac{\Psi'(0+)}{q} - S \leq 0, \end{cases} \tag{3.2}$$

$$V_{b^*}(x) = \begin{cases} -\overline{Z}^{(q)}(b^*-x) - \dfrac{\Psi'(0+)}{q}, & \text{if } -\dfrac{\Psi'(0+)}{q} - S > 0, \\ x + S, & \text{if } -\dfrac{\Psi'(0+)}{q} - S \leq 0, \end{cases} \tag{3.3}$$

for any $x \geq 0$.

To prove Theorems 3.1 and 3.2, we need several lemmas.

Denote by $\mathcal{A}$ the extended generator of the process X, which acts on C^2 function g defined by

$$\mathcal{A}g(x) = \frac{1}{2}\sigma^2 g''(x) - cg'(x) + \int_0^\infty [g(x+y) - g(x) - g'(x)y\mathbf{1}_{\{|y|<1\}}]\Pi(dy). \tag{3.4}$$

Lemma 3.1. *Let $S = 0$. Assume that $V_b(x)$ is bounded and twice continuously differentiable on $(0, b)$ with a bounded first derivative. Then $V_b(x)$ satisfies the following integro-differential equation:*

$$\mathcal{A}V_b(x) = qV_b(x), \quad 0 < x < b,$$

together with the boundary conditions

$$V_b(0) = 0, \qquad V_b'(b) = 1,$$

$$V_b(x) = x - b + V_b(b) \quad \text{for } x > b.$$

Proof. Similar to the case of jump–diffusion (cf. Yin et al., 2013 or, Yin and Wen, 2013a,b), applying the Itô's formula for semimartingales one has

$$E_x\left[e^{-q(t_n\wedge T_b)}V_b(U_b(t_n\wedge T_b))\right] = V_b(x) + E_x\int_0^{t_n\wedge T_b} e^{-qs}[(\mathcal{A}-q)V_b(U_b(s))]ds - E_x\left[\int_0^{t_n\wedge T_b} e^{-qt}dL_t^b\right],$$

where $\{t_n, n \geq 1\}$ is an appropriate localization sequence of stopping times. Letting $n \to \infty$ and note that $V_b(0) = 0$ we have

$$V_b(x) = E_x\left[\int_0^{T_b} e^{-qt}dL_t^b\right].$$

This ends the proof.

Remark 3.1. With $\Pi(dx) = \lambda P(dx)$, where $\lambda > 0$ and $P(y)$ is a probability distribution function on $(0, \infty)$, the result of Lemma 3.1 reduces to the results of Avanzi and Gerber (2008, (2.3)–(2.6)).

Lemma 3.2. *For $b, q \geq 0$ and $0 \leq x \leq b$, we have*

$$E_x\left[e^{-qT_b}\right] = \frac{Z^{(q)}(b-x)}{Z^{(q)}(b)}. \tag{3.5}$$

Proof. Let $Y_b(t) = b - U_b(t)$, then Y_b is a reflected Lévy process with initial value $b - x$. Define $\tilde{T}_b = \inf\{t > 0 : Y_b(t) \geq b\}$, then

$$E\left[e^{-qT_b}|U_b(0) = x\right] = E\left[e^{-q\tilde{T}_b}|Y_b(0) = b - x\right] = \frac{Z^{(q)}(b-x)}{Z^{(q)}(b)},$$

where in the last step we have used the result of Proposition 2(i) in Pistorius (2004); see also Theorem 2.8(i) in Kuznetsov et al. (2013). This ends the proof.

The following lemma due to Bayraktar et al. (2013a). Here we give a different proof.

Lemma 3.3. *For $b, q \geq 0$ and $0 \leq x \leq b$, define*

$$V(x, b) = E_x\left[\int_0^{T_b} e^{-qt} dL_t^b\right],$$

then we have

$$V(x, b) = \frac{\overline{Z}^{(q)}(b)}{Z^{(q)}(b)} Z^{(q)}(b-x) - \overline{Z}^{(q)}(b-x) + \frac{\Psi'(0+)}{q}\left(\frac{Z^{(q)}(b-x)}{Z^{(q)}(b)} - 1\right). \tag{3.6}$$

Proof. By the law of total probability and the strong Markov property as in Yin et al. (2013), we have

$$V(x, b) = h_1(x)V(b, b) + h_2(x), \tag{3.7}$$

where

$$h_1(x) = E_x\left(e^{-qT_b^+}\mathbf{1}_{\{T_b^+ < T_0^-\}}\right),$$

and

$$h_2(x) = E_x\left(e^{-qT_b^+}(X(T_b^+) - b)\mathbf{1}_{\{T_b^+ < T_0^-\}}\right).$$

By (2.3),

$$h_1(x) = Z^{(q)}(b-x) - W^{(q)}(b-x)\frac{Z^{(q)}(b)}{W^{(q)}(b)}. \tag{3.8}$$

By (2.5),

$$\begin{aligned} E_x\left(e^{-qT_b^+}X(T_b^+)\mathbf{1}_{\{T_b^+ < T_0^-\}}\right) &= \int_{y=0}^{b}\int_{z=b}^{\infty} zu^{(q)}(x, y) \\ &\quad \times \Pi(dz - y)dy \\ &\equiv I_1(x) - I_2(x), \end{aligned} \tag{3.9}$$

where

$$\begin{aligned} I_1(x) &= \int_{y=0}^{b}\int_{z=b}^{\infty} \frac{W^{(q)}(b-x)}{W^{(q)}(b)} W^{(q)}(y) z\Pi(dz - y)dy \\ &= -\frac{bZ^{(q)}(b)}{W^{(q)}(b)} W^{(q)}(b-x) + bcW^{(q)}(b-x) \\ &\quad + \frac{W^{(q)}(b-x)}{W^{(q)}(b)}\left(\overline{Z}^{(q)}(b) - \frac{\Psi'(0+)}{q} Z^{(q)}(b) \right. \\ &\quad \left. + \frac{\Psi'(0+)}{q}\right), \end{aligned} \tag{3.10}$$

$$\begin{aligned} I_2(x) &= \int_{y=0}^{b}\int_{z=b}^{\infty} W^{(q)}(y-x) z\Pi(dz - y)dy \\ &= -bZ^{(q)}(b-x) + bcW^{(q)}(b-x) \\ &\quad + \overline{Z}^{(q)}(b-x) - \frac{\Psi'(0+)}{q} Z^{(q)}(b-x) + \frac{\Psi'(0+)}{q}, \end{aligned} \tag{3.11}$$

Substituting (3.10) and (3.11) into (3.9) we get

$$\begin{aligned} &E_x\left(e^{-qT_b^+}X(T_b^+)\mathbf{1}_{\{T_b^+ < T_0^-\}}\right) \\ &= \frac{W^{(q)}(b-x)}{W^{(q)}(b)}\left(\overline{Z}^{(q)}(b) - \Psi'(0+)\overline{W}^{(q)}(b) - bZ^{(q)}(b)\right) \\ &\quad - \overline{Z}^{(q)}(b-x) + \Psi'(0+)\overline{W}^{(q)}(b-x) + bZ^{(q)}(b-x), \end{aligned}$$

from which and (3.8) we arrive at

$$\begin{aligned} h_2(x) &= \frac{W^{(q)}(b-x)}{W^{(q)}(b)}\left(\overline{Z}^{(q)}(b) - \Psi'(0+)\overline{W}^{(q)}(b)\right) \\ &\quad - \overline{Z}^{(q)}(b-x) + \Psi'(0+)\overline{W}^{(q)}(b-x). \end{aligned} \tag{3.12}$$

Substituting (3.8) and (3.12) into (3.7) and using the boundary condition $V'(b, b) = 1$ in Lemma 3.1, we get

$$V(b, b) = \frac{\overline{Z}^{(q)}(b)}{Z^{(q)}(b)} + \frac{\Psi'(0+)}{qZ^{(q)}(b)} - \frac{\Psi'(0+)}{q},$$

and the result follows.

Proof of Theorem 3.1. The result follows from Lemmas 3.2 and 3.3.

Proof of Theorem 3.2. By differentiating (3.1), we obtain that

$$V_b'(x) = \begin{cases} -q\Lambda(b)W^{(q)}(b-x) \\ \quad + Z^{(q)}(b-x), & \text{if } 0 < x \leq b, \\ 1, & \text{if } x > b. \end{cases} \tag{3.13}$$

It follows that $V_b'(b) = 1$ if and only if $\Lambda(b) = 0$, or, equivalently $\overline{Z}^{(q)}(b) = -\frac{\Psi'(0+)}{q} - S$. Denote our candidate barrier level by

$$b^* = \begin{cases} (\overline{Z}^{(q)})^{-1}\left(-\frac{\Psi'(0+)}{q} - S\right), & \text{if } -\frac{\Psi'(0+)}{q} - S > 0, \\ 0, & \text{if } -\frac{\Psi'(0+)}{q} - S \leq 0. \end{cases} \tag{3.14}$$

The dividend-value function when using the barrier strategy π_{b^*} is given by

$$V_{b^*}(x) = \begin{cases} -\overline{Z}^{(q)}(b^* - x) \\ \quad - \frac{\Psi'(0+)}{q}, & \text{if } -\frac{\Psi'(0+)}{q} - S > 0, \\ x + S, & \text{if } -\frac{\Psi'(0+)}{q} - S \leq 0, \end{cases} \tag{3.15}$$

for any $x \geq 0$.

According to Lemma 5 in Loeffen (2008), to prove the theorem, it suffices to show that $V_{b^*}(x)$ satisfies

$$(\mathscr{A} - q)V_{b^*}(x) \leq 0 \quad \text{for } x > b^*. \tag{3.16}$$

As in the proof of Theorem 2.1 in Bayraktar et al. (2013a), when $b^* = 0$, then $V_{b^*}(x) = x + S$ and $(\mathscr{A} - q)V_{b^*}(x) = -\Psi'(0+) - q(x + S) \leq -qx \leq 0$ for $x \geq 0$.

Now suppose that $b^* > 0$. Since $(e^{-q(t\wedge\tau_{0b^*})}[\overline{Z}^{(q)}(b^* - X(t \wedge \tau_{0b^*})) + \frac{\Psi'(0+)}{q}])_{t\geq 0}$ is a P_x-martingale, by the Itô's formula one can deduce that

$$(\mathscr{A} - q)V_{b^*}(x) = 0 \quad \text{for } 0 < x < b^*. \tag{3.17}$$

In particular (3.17) holds for $x = b^*$ since V_{b^*} is sufficiently smooth. Hence $V_{b^*}(x)$ satisfies the condition (3.16) since $(\mathcal{A} - q)V_{b^*}(x)$ is decreasing on $[b^*, \infty)$. This completes the proof of Theorem 3.2.

Remark 3.2. Letting $S \to 0$ in Theorem 3.2, the result reduces to the result of Theorem 2.1 in Bayraktar et al. (2013a).

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