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Vacuum energy and cosmological supersymmetry breaking in brane worlds

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Abstract

In the context of a toy model we discuss the phenomenon of colliding five-branes, with two of the extra space dimensions compactified on tori. In one of the branes (hidden world) the torus is magnetized. Assuming opposite-tension branes, we argue that the collision results eventually in a (time-dependent) cosmological vacuum energy, whose value today is tiny, lying comfortably within the standard bounds by setting the breaking of the four-dimensional supersymmetry at a TeV scale. The small value of the vacuum energy as compared with the supersymmetry-breaking scale is attributed to transient phenomena with relaxation times of order of the age of the Universe. An interesting feature of the approach is the absence of a cosmic horizon, thereby allowing for a proper definition of an S-matrix. As a result of the string non-criticality induced at the collision, our model does not provide an alternative to inflation, given that arguments can be given supporting the occurrence of an inflationary phase early after the collision. The physics before the collision is not relevant to our arguments on the cosmological constant hierarchy, which are valid for asymptotically long times after it.

One of the most important unsolved puzzles in theoretical particle physics is the issue of the smallness of the cosmological constant (or, better vacuum energy density) in comparison with other physical scales, for instance, the scale at which supersymmetry is broken in supersymmetric theories. The resolution of such puzzles may lie in the way by which supersymmetry is broken. One interesting idea is that supersymmetry is broken somehow cosmologically, in the sense of its breaking being linked to a non-zero cosmological constant. Such an idea has been studied recently in [1] in the modern context of brane (M) theory.

In Ref. [1], the vacuum energy has been assumed constant. This might not be necessarily the case, though. One might encounter transient situations, as in quintessence models [2], where the "vacuum" energy is relaxing to zero asymptotically by some power (usually quadratic) of the cosmological-frame time. Such scenaria are interesting, since they allow for an eventual exit from a de Sitter phase, implying non-eternally accelerating universes. This is a welcome fact from the point of view of string-theory [3], given that eternally accelerating (de Sitter) universes have cosmic horizons, which makes a definition of

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a S(cattering)-matrix connecting asymptotic states problematic [4,5].

In this Letter we shall adopt this latter point of view, and present a scenario, albeit crude, according to which colliding branes in superstring theory may result in a way of breaking supersymmetry on our four-dimensional world at a TeV scale, while maintaining a very small vacuum energy, decreasing with cosmological time. We should notice that scenaria with time-dependent vacuum energies have been considered by many authors in the past [6]. However, the physics of our model as well as its focus are different. We shall be interested in attempting to resolve the issue of the hierarchy between the cosmological vacuum energy and the supersymmetry-breaking scale. The relaxation rate of the vacuum energy is found to be proportional to $1/t^2$, where t is the Robertson–Walker time. This is argued to be sufficient for a resolution of the cosmic horizon problem as well.

To commence our discussion, let us consider for definiteness two five-branes of type IIB strings, embedded in a ten-dimensional bulk space–time. Two of the longitudinal brane dimensions are assumed compactified on a small torus, of radius R. In one of the branes, from now on called *hidden*, the torus is *magnetized* with a constant magnetic field of intensity H. This amounts to an effective four-dimensional vacuum energy in that brane of order: $V_{\text{hidd}} = R^2 H^2 > 0$. Notice that such compactifications provide alternative ways of breaking supersymmetry [7], which we shall make use of in the current Letter.

In scenaria with two branes embedded in higher-dimensional bulk space–times, e.g., in the scenario of [8], it is natural to assume (from the point of view of solutions to bulk field equations) that the two branes have *opposite* tensions. We, therefore, assume that before the collision the visible brane (our world) has negative tension $V_{\text{vis}} = -V_{\text{hidd}} < 0$. A negative tension brane is consistent with the possibility of accepting supersymmetric theories on it (anti-de Sitter-type).

The presence of opposite tension branes implies that the system is not stable. For our purposes we assume that the two branes are originally on collision course in the bulk, with a relative velocity u. The collision takes place at a given time moment. This constitutes an event, which in our scenario is identified with the *initial cosmological singularity* (big bang) on the observable world. We note that similar scenaria exist in the so-called ekpyrotic model for the universe [9]. It must be stressed, though, that the similarity pertains only to the brane-collision event. In our approach the physics is entirely different from the ekpyrotic scenario. First of all, the collision is viewed as an event resulting in non-criticality (departure from conformal invariance) of the underlying string theory, and hence in non-vanishing β functions at a σ-model level. On the contrary, in the scenario of [9] the underlying four-dimensional effective theory (obtained after integration of the bulk extra dimensions [9,10]) is assumed always critical, satisfying classical equations of motion, and hence vanishing σ-model β functions. Indeed this latter property leads only to contracting and *not expanding* four-dimensional universes according to the work of [10], which constitutes one of the main criticisms of the ekpyrotic universe.

On the other hand, in our non-critical description of the collision we do not assume classical solutions of the equations of motion, neither specific potentials associated with bulk branes, as in [9]. In our approach, we are interested only in the period *after the collision*. In fact, as we shall discuss in this Letter, in order to be able to use σ-model perturbation theory, one must restrict oneself at times much longer after the collision. Before the collision the moving branes may indeed be viewed as solutions of some classical equations, as in [9,10]. But the collision-induced deviations from conformal invariance, we advocate here, play a rôle analogous to a sort of (stringy) phase transition. In our Letter we shall only be interested in the phase after the collision, where the degrees of freedom of the system and its description may be different: the presence of non-criticality necessitates the introduction of a whole new target-space dimension, the Liouville mode [11].

Notably, our perturbative approach is not valid for times near the collision, where the string theory is strongly coupled, in contrast to the ekpyrotic universe case. Moreover, the deviation from conformal invariance, quantified through the appearance of a central charge deficit Q^2, which depends itself on time, is responsible for the entirely different way of obtaining the fate of the four-dimensional cosmology in our case. As we shall see, the generalized conformal invariance conditions (23), stemming from the Liouville dressing of the non-critical theory [11], encode the full dynamics of the four-dimensional theory in our approach. This dynamics, upon the identification of the

Liouville mode with time, in a sense that will be specified in our Letter below, leads to asymptotically expanding universes, in contrast to the contracting universe situation of the ekpyrotic scenario [10].

Most importantly, it must be stressed that our toy model should, by no means, be viewed as an alternative to inflation, as claimed to be the case of the ekpyrotic universe [9]. In fact this point appeared to be the main focus of criticism of that scenario [10]. In our case, inflationary phases of the universe do exist, as demonstrated recently in the context of non-supersymmetric type 0 strings upon deviations from criticality by either quantum fluctuations or brane collisions [3]. The presence of a time-dependent central-charge deficit $Q^2(t)$ is crucial to the effect. As argued in [3], the effective four-dimensional theory, obtained after appropriate compactification or integration over bulk dimensions, has at early times a phase, where inflation—at least in the sense of exponential expansion of the scale factor—always occur, succeeded by graceful exit from this de Sitter-type phase, which is not possible in critical strings.

In our Letter we shall not be interested in such early times or such important issues as density fluctuations etc. Our toy model is too crude to allow for a full study of a present-day cosmology with matter. Instead we would like to make an interesting observation, by means of this toy model, according to which the above-mentioned string non-criticality leads, for asymptotically long times after the collision (including present eras), to a natural explanation of a hierarchy between the vacuum energy and the scale of supersymmetry breaking, as well as the lack of a cosmic horizon. Nevertheless, we stress again, this toy non-critical string model is expected, on the basis of the work of [3], to exhibit an inflationary phase and eventual graceful exit from it.

We now make the plausible assumption that, during the collision, there is electric current transfer from the hidden to the visible brane, which results in the appearance of a magnetic field on the visible brane. We also assume that the entire effect is happening very slowly and amounts to a slow flow of energy and current density from the positive energy density brane to the one with negative tension. In turn, this results in a positive energy component of order H^2R^2 in the vacuum energy of the visible brane world. This energy component may be assumed to *cancel* the pre-existing negative tension asymptotically in time, leading to a vanishing cosmological constant at $t=\infty$. It is our aim to find, by a preliminary σ-model analysis, the asymptotic form (in large times) of this time-dependent four-dimensional vacuum energy, and relate this to supersymmetry breaking. Notice that such a scenario imitates a slow relaxation period of the Universe, which still goes on. This is in accordance with quintessence models [2] which have not yet reached their equilibrium state.

We should notice at this stage that the initial instability due to the negative tension brane disappears from the observable sector, given that the cosmological time flow begins from the moment of the collision. As we shall discuss in some detail in this Letter, at the moment of the collision the conformal invariance of the σ-model describing excitations on the observable world is spoiled, thereby implying the need for Liouville dressing [11,12]. This procedure restores conformal invariance at the cost of introducing an extra target-space coordinate (the Liouville mode ϕ), which in our model has time-like signature. Hence, initially, one faces a two-times situation. We argue, though, that our observable (cosmological) time X^0 parametrizes a certain curve, $\phi = \text{const} \times X_0 + \text{const}'$, on the two-times plane (X_0, ϕ), and hence one is left with one physical time.

The appearance of the magnetic field on the visible brane, on the dimensions $X^{4,5}$, is described (for times long after the collision) within a σ-model superstring formalism by the boundary deformation [13]:

$$\mathcal{V}_H = \int_{\partial\Sigma} A_5 \partial_\tau X^5 - i F_{05} \bar{\psi}^0 \rho^0 \psi^5 - i F_{45} \bar{\psi}^4 \rho^0 \psi^5, \tag{1}$$

where $A_5 = e^{\varepsilon X^0} H X^4$ and $F_{\mu\nu}$ is the (Abelian) field strength of A_μ, X^0 is the time and ∂_τ denotes tangential σ-model derivative on the world-sheet boundary. The σ-model deformation (1) describes open-string excitations attached to the brane world. In our approach, for convenience, we have set the charges at the end of the open string on the visible world equal to one. In (1) the presence of the quantity $\varepsilon \to 0^+$ reflects the *adiabatic* switching-on of the magnetic field after the collision. It should be remarked that in our approach the quantity ε is viewed as a renormalization-group scale parameter, which, as we shall argue below, flows in such a way that any contribution from the exponent εX^0 to H is cancelled after Liouville dressing.[1]

In addition to the magnetic field deformation, the σ-model contains also boundary deformations describing the 'recoil' of the visible world due to the collision:

$$\mathcal{V}_{\rm rec} = \int_{\partial\Sigma} Y_6(X_0)\partial_n X^6 + i\partial_0 Y_6 \bar{\psi}^0 \rho^1 \psi^6, \tag{2}$$

where $Y_6(X_0) = uX^0 e^{\varepsilon X^0}$, ∂_n denotes normal σ-model derivative on the world-sheet boundary and we have assumed for simplicity that the motion of the branes is along the sixth bulk dimension. In (2) u denotes the recoil velocity of the visible world, which is of the order of the incident velocity of the hidden brane.[2]

As can be seen straightforwardly, by an operator-product-expansion analysis with the free string world-sheet stress tensor, the presence of the exponential $e^{\varepsilon X^0}$ implies a small but negative world-sheet anomalous dimension $-\varepsilon^2/2 < 0$, and hence the relevance of both operators (1), (2) from a renormalization-group point of view. By virtue of the Zamolodchikov's c-theorem [14] there is a central-charge deficit Q^2, whose rate of change with the renormalization-group scale on the world sheet $\mathcal{T}$ is:

$$\frac{d}{d\mathcal{T}} Q^2 = -\beta^i \mathcal{G}_{ij} \beta^j. \tag{3}$$

A straightforward computation of the two point correlators between the operators $\mathcal{V}_H, \mathcal{V}_{\rm rec}$ yields the Zamolodchikov metric in coupling constant space [14]:

$$\begin{aligned} &\mathcal{G}_{HH} \\ &\quad = |z|^4 \langle e^{\varepsilon X^0(z)} X^4(z) \partial_\tau X^5(z) e^{\varepsilon X^0(0)} X^4(0) \partial_\tau X^5(0) \rangle \\ &\quad \sim e^{4\varepsilon^2 \ln|L/a|^2} \ln|L/a|^2 \end{aligned} \tag{4}$$

and similarly for $\mathcal{G}_{uu}$. The non-diagonal elements of $\mathcal{G}_{ij}$ vanish. It can be easily checked that the contributions from the world-sheet fermionic fields are subdominant as compared with the bosonic ones. We may identify

$$\varepsilon^{-2} \sim \ln|L/a|^2, \tag{5}$$

so that the above correlators scale as space–time length squared. It is a rather established fact that such an identification is natural, if not unavoidable, once one introduces simple operators with anomalous dimension related to a new space–time scale in their definition [15,16]. The above considerations imply that the Zamolodchikov metric is singular in the limit $\varepsilon \to 0^+$:

$$\mathcal{G}_{HH} \sim \mathcal{G}_{uu} \sim \frac{1}{\varepsilon^2}. \tag{6}$$

On the other hand, the σ-model β-functions for the couplings H and u, corresponding to the vertex operators (1) and (2), respectively, are: $\beta_{\overline{H}} = \frac{d\overline{H}}{d\mathcal{T}} = -\frac{\varepsilon^2}{2}\overline{H}$, $\beta_{\bar{u}} = \frac{d\bar{u}}{d\mathcal{T}} = -\frac{\varepsilon^2}{2}\bar{u}$, where the barred notation pertains to renormalized (scale-ε dependent) quantities, and $\mathcal{T} \propto \varepsilon^{-2} \sim \ln|L/a|^2$ from (5). These relations imply that the scale-ε dependent couplings have the form [17]: $\overline{H} \equiv \varepsilon H$, $\bar{u} \equiv \varepsilon u$, where H, u are scale-ε independent quantities.

From the above considerations one arrives at the following differential equation for the central charge deficit:

$$\begin{aligned} &\frac{d}{d\mathcal{T}} Q^2 \sim -\frac{H^2 + u^2}{\mathcal{T}^2} \\ &\quad \to \quad Q^2(\mathcal{T}) = Q_0^2 + \frac{H^2 + u^2}{\mathcal{T}}, \end{aligned} \tag{7}$$

where, for formal completeness we give here the more general case of n compactified tori, with $n = 1$ corresponding to a five-brane, $n = 2$ to a seven-brane and $n = 3$ to a nine-brane, which exhausts the possibilities in the case of type IIB superstring we are dealing for definiteness here. The quantity $Q_0^2 = Q^2(\infty)$ is a constant, and consists of the vacuum energy density contributions of the visible brane world $V_{\rm vis} < 0$ and the energy density of the magnetic field $H^2 R^{2n} > 0$. A physical meaning to (7) can be given by noting that its H-dependent term represents the electric-field energy density on the brane $\int_{\rm tori} F_{05}^2 \propto \varepsilon^2 H^2 R^{2n}$, induced by the time-varying magnetic field

[1] At this point we should remark that one could have used a different way of parametrizing the adiabatic switching-on of the magnetic field, for instance, a function $H(1 - e^{-\varepsilon X^0})$, $\varepsilon \to 0^+$. The conformal field theory analysis in that case is similar to the case considered above, and will not be presented here.

[2] Notice that a similar formalism describes also a plastic collision, where the two branes merge to a single one after the collision.

$He^{\varepsilon X^0}$. The u-dependent term on the other hand represents recoil kinetic energy contributions, which in our case are subleading. As we shall explain later on, the relaxation situation we encounter here implies that Q_0 is the equilibrium vacuum energy density, which we take to be zero $Q_0^2 = 0$ due to the cancellation between the initial vacuum energies of the colliding branes [8].

The non-conformal deformed σ-model can become conformal as usual by Liouville dressing [11]. Let φ be the Liouville mode with σ-model action

$$\mathcal{L}_\phi = \int_\Sigma Q^2(\mathcal{T})\partial\varphi\bar{\partial}\varphi + \int_\Sigma R^{(2)} Q^2(\mathcal{T})\varphi + \cdots, \qquad (8)$$

where the zero mode of φ is related to the renormalization-group scale $\ln|L/a|^2 \sim \mathcal{T}$, being viewed as a covariant renormalization scale on the world sheet [12]. The dots in (8) express possible world-sheet boundary extrinsic curvature terms, with which we shall not deal explicitly here.

It is customary [11] to normalize the kinetic term of the Liouville action by rescaling

$$\varphi \to \phi \equiv Q(\mathcal{T})\varphi, \qquad (9)$$

which plays the role of an extra target space–time dimension. Due to the supercritical nature of the central-charge deficit (7) at scales $\mathcal{T} < \infty$, $Q^2(\mathcal{T}) > 0$, the extra Liouville dimension is *time-like* [18], and therefore one faces a two-target-times situation.

We now come to discuss the dressing of the vertex operators $\mathcal{V}_H, \mathcal{V}_u$. This amounts to introducing the Liouville field in the definitions of A_5, Y_6, that operates as one more space–time coordinate resulting in conformally invariant boundary deformations. The new fields are given by

$$A_5(X_0, X_4, \phi) = HX^4 e^{\varepsilon X^0 + \alpha\phi},$$
$$Y_6(X_0, \phi) = uX^0 e^{\varepsilon X^0 + \alpha\phi}. \qquad (10)$$

The world-sheet supersymmetrized vertex operators are now given by

$$\mathcal{V}_H = \int_{\partial\Sigma} A_5 \partial_\tau X^5 - iF_{05}\bar{\psi}^0\rho^0\psi^5 - iF_{45}\bar{\psi}^4\rho^0\psi^5 - iF_{\phi 5}\bar{\psi}^\phi\rho^0\psi^5,$$
$$\mathcal{V}_{\rm rec} = \int_{\partial\Sigma} Y_6\partial_n X^6 + i\partial_0 Y_6\bar{\psi}^0\rho^1\psi^6 + i\partial_\phi Y_6\bar{\psi}^\phi\rho^1\psi^6, \qquad (11)$$

where ψ^ϕ is the supersymmetric partner of the Liouville field. The gravitational (Liouville) anomalous dimensions α are given by [11]:

$$\alpha = -\frac{Q(\mathcal{T})}{2} + \sqrt{\frac{Q^2(\mathcal{T})}{4} + \frac{\varepsilon^2}{2}}. \qquad (12)$$

As a σ-model, this $(d, 2)$ theory is conformal [11].

From the work of [7] it becomes clear that the coupling constant H is associated with supersymmetry-breaking mass splittings. This has to do with the different way fermions and bosons couple to an external magnetic field. The mass splittings squared of an open string are generically of order $\delta m^2 \sim H$. To be precise, in the case of a constant magnetic field, examined in [7], the supersymmetric mass splittings are

$$\Delta m^2_{\rm string} = 4H\Sigma_{45}, \qquad (13)$$

with Σ_{45} the spin operator on the plane of the torus. To ensure the phenomenologically reasonable order of magnitude of a TeV scale, one must assume very small [7] $H \sim 10^{-30} \ll 1$ in Planck units. In a similar manner, one assumes naturally that the velocities u are also much smaller than one, in order for our perturbative world-sheet analysis to be valid [15, 17]. For such small values of the couplings H, u one has from (7), (12) that the Liouville anomalous dimensions are of order $\alpha \sim \varepsilon/\sqrt{2}$, ignoring H, u dependent terms, which are subleading for $\varepsilon \ll 1$.

In our case, we have a slowly varying magnetic field $He^{\alpha\phi + \varepsilon X^0}$, from which we may deduce approximately the corresponding mass squared splittings:

$$\Delta m^2_{\rm string} \sim He^{\alpha\phi + \varepsilon X^0}\Sigma_{45}. \qquad (14)$$

The so-obtained mass splittings are constant upon the requirement that the flow of time X^0 and of Liouville mode ϕ are correlated in such a way that

$$\varepsilon X^0 + \varepsilon\phi/\sqrt{2} = \text{constant}, \qquad (15)$$

or at most slowly varying. Notice that deviations from the condition (15) would result in very large negative-mass squares, which are clearly unstable configurations. Hence, the identification (15) seems to provide a resolution of this problem.[3]

The condition (15) implies a connection of the zero mode of the Liouville field, $\mathcal{T}$, with the target time X^0. In this sense $Q_0^2 = Q^2(\infty)$ represents the central charge deficit of the theory asymptotically in time. Given that the initial vacuum energy on the observable brane is assumed to be cancelled during the collision with the hidden world, where the flow of our cosmic time (and hence the Liouville scale) starts, it is natural to assume that $Q_0^2 = 0$, which justifies our choice above. Note also that parametrizing this condition as $X^0 = -t$, $\phi_0 = \sqrt{2}\,t$, and taking into account that, for convergence of σ-model path integration, it is formally necessary to work with Euclidean signature X^0 [15], the induced metric on the hypersurface (15) in the extended space–time acquires a Minkowskian-signature Robertson–Walker form:

$$ds^2_{\text{hypersurf}} = -(d\phi_0)^2 + \left(dX^0\right)^2 + \cdots$$
$$= -(dt)^2 + \cdots, \tag{16}$$

where dots denote spatial parts.

At this stage an important comment is in order regarding the stability of the condition (15) in the context of Liouville strings. In our approach so far we have assumed a situation in which the magnetic field is adiabatically switched-on after the collision and then asymptotes to a constant value. On the other hand, one may consider an equally plausible situation in which the magnetic field is switched-on on our world, due to transient phenomena described above, and then relaxes to zero again. In such a case the magnetic field intensity on the brane world assumes the form

$$H\left(\Theta_\varepsilon\left(-X^0\right) + \Theta_\varepsilon\left(X^0\right)\right), \tag{17}$$

where $\Theta_\varepsilon(X) = -i\int \frac{d\omega}{\omega - i\varepsilon} e^{i\omega X}$, $\varepsilon \to 0^+$, denotes the regularized Heaviside function [15]. A contour integral representation yields $\Theta_\varepsilon(X) = \theta(X)e^{-\varepsilon X}$, with $X > 0$ and $\theta(X)$ the conventional Heaviside (unregularized) function.

[3] Note, however, that if one used the alternative representation of the adiabatic switching-on of the magnetic field $H(1 - e^{-\varepsilon X^0})$, $\varepsilon > 0$, the masses would be finite as $X^0 \to \infty$. Nevertheless the condition (15) would still be necessary from the physical point of view of having a single observable temporal coordinate in space–time. We discuss some consequences of the case where ϕ and X^0 are independent variables later on in the Letter.

In this case, one obtains *a pair* of independent σ-model deformations, corresponding to the two Θ functions in (17). The Liouville dressing procedure is now a bit more complicated, but as we shall argue below, this case yields indeed the dynamical stability requirements for the condition (15). To this end, we first remind the reader that in our previous analysis we have used the Liouville anomalous dimensions (12). The restoration of conformal invariance by Liouville dressing, however, actually requires in general two sets of anomalous dimensions $\alpha_\pm$ [11]

$$\alpha_\pm = -\frac{Q(\mathcal{T})}{2} \pm \sqrt{\frac{Q^2(\mathcal{T})}{4} + \frac{\varepsilon^2}{2}}. \tag{18}$$

In Liouville theory it is common to ignore the α_- as leading to states that “do not exist”, as leading to non-normalizable states in the semiclassical limit where the central charge of the theory goes to infinity. This is what we have done so far. However, in the context of string theory, with target-space–time interpretation, the “wrong sign” states corresponding to α_- may not be excluded, and under certain conditions such “wrong-sign” dressing leads to physical states. This is our case here, since as we shall see below, one actually does not face a situation with divergent central charge deficit which is cut off at a finite value at the ultraviolet world-sheet fixed point of the theory (Liouville scale $\phi_0 \to 0$). Using therefore opposite sign screening Liouville operators for the two vertex operators corresponding to the two Θ functions in (17), one encounters a supersymmetry-breaking mass spectrum for the string theory at hand of the form:

$$\Delta m^2_{\text{string}} \sim 2H\cosh\left(\frac{\varepsilon}{\sqrt{2}}\phi + \varepsilon X^0\right)\Sigma_{45}. \tag{19}$$

It is evident that in such a case minimization of the potential energy in target space based on (19) will lead to the condition (15), thereby providing us with a *dynamical stability argument* in favour of the identification of the Liouville world-sheet zero mode with the target time. Physically, one may interpret this result as implying that a time-varying magnetic field of the form (17) induces back reaction of strings onto the space–time in such a way that the mass splittings of the string excitation spectrum as a result of the field are actually stabilized.

From the Liouville action (8) we then observe that in our case the dilaton field is

$$\Phi = Q\phi = Q^2\varphi \sim \left(H^2 + u^2\right), \tag{20}$$

that is, one faces a situation with an asymptotically constant dilaton. This is a welcome fact, because otherwise, the space–time would not be asymptotically flat, and one could face trouble in appropriately defining masses.[4]

In the case of a constant dilaton the vacuum energy is determined by the central-charge charge deficit Q^2 (7), which in our case is:

$$\Lambda = \frac{R^{2n}}{\phi_0^2}\left(H^2 + u^2\right)^2, \tag{21}$$

where ϕ_0 is the world-sheet zero mode of the rescaled Liouville field (9).

It must be stressed that, due to the condition $\mathrm{Str}\,\mathcal{M}^2 = 0$, which is a characteristic feature of the magnetically-induced supersymmetry-breaking scenario of [7], there are no quadratically divergent terms in the one-loop effective potential of the low-energy theory, which assumes the form [19]:

$$\begin{aligned} V_1 = V_0 &+ \frac{1}{64\pi^2}\,\mathrm{Str}\,\mathcal{M}^0\Lambda_{uv}^4 \ln\frac{\Lambda_{uv}^2}{\mu^2} \\ &+ \frac{1}{32\pi^2}\,\mathrm{Str}\,\mathcal{M}^2\Lambda_{uv}^2 \\ &+ \frac{1}{64\pi^2}\,\mathrm{Str}\,\mathcal{M}^4 \ln\frac{\mathcal{M}^2}{\Lambda_{uv}^2} + \cdots, \\ \mathrm{Str}\,\mathcal{M}^n &= \sum_i (-1)^{2J_i}(2J_i - 1)m_i^n, \end{aligned} \tag{22}$$

where μ is a scale, and V_0 is a field-independent contribution. In our case V_0 is given by Λ in (21). Note also that in a supersymmetric theory (even if supersymmetry is broken) $\mathrm{Str}\,\mathcal{M}^0 = 0$, due to a balance between fermionic and bosonic degrees of freedom. If the supersymmetry is broken at a TeV scale, then, the remaining $\mathrm{Str}\,\mathcal{M}^4 \ln\frac{\mathcal{M}}{\Lambda_{uv}}$ term in (22), which induces quadratic corrections to the Higgs mass, produces a stable hierarchy.

We now remark that the restoration of the conformal invariance by the Liouville mode results in the following equations for the σ-model background fields/couplings g^i near a fixed-point of the world-sheet renormalization group (large-times cosmology) we restrict ourselves here [3,11,12,20]:

$$\left(g^i\right)'' + Q\left(g^i\right)' = -\beta^i(g), \tag{23}$$

where the prime denotes derivative with respect to the Liouville zero mode ϕ_0, and the sign on the right-hand side is appropriate for supercritical strings [18] we are dealing with here. In fact the β^i functions satisfy a gradient flow property

$$\beta^j\mathcal{G}_{ij} = \frac{\delta C[g]}{\delta g^i}, \tag{24}$$

where $\mathcal{G}_{ij} = z^2\bar{z}^2\langle V_i(z)V_j(0)\rangle$ is the Zamolodchikov metric in string theory space [14], with V_i the appropriate vertex operators corresponding to the couplings g^i, and $C[g]$ is the effective action which can be identified with the central charge deficit squared $Q^2[g,\phi_0]$ in our case.

It should be mentioned for completeness that the Liouville equations (23), (24), which restore conformal invariance, can always be viewed as conformal invariance conditions of a σ-model in $(d+1)$-dimensional space–time, with the extra coordinate provided by the Liouville mode ϕ. They themselves can be derived from a $(d+1)$-dimensional action, since the appropriate (Helmholtz) conditions are satisfied in this case [12]. Close to a fixed point, i.e., up to order g^2 in weak σ-model couplings/background fields, the action has the form [12]:

$$\mathcal{S} = \int d\phi_0\left(\frac{1}{2}g^{i\prime}\mathcal{G}_{ij}g^{j\prime} - C[g]\right). \tag{25}$$

Indeed, it can be readily checked that the Lagrange equations in theory space of this action reproduce the conformal invariance conditions (23), provided $\mathcal{G}'_{ij} = Q\mathcal{G}_{ij}$, a property, which as explained in detail in [12], characterizes Liouville dressing. Terms involving $g^{i\prime}\partial_m\mathcal{G}_{ij}g^{j\prime}$ are of order higher than g^2 and hence are ignored in our approach here. Notice that such an approach has also been used in [20] in discussing string cosmology and its relation to the renormalization-group on the world-sheet.

In our case g^i is the metric $G_{\mu\nu}$, the dilaton Φ and the electromagnetic field A_μ. The latter has already been discussed, and in our case, for asymptotic times we are interested in, the dilaton is constant (20). In

[4] For instance, in theories with linear dilatons in time asymptotically [18], q_0X^0, it is known that boson masses acquire tachyonic shifts $\delta m_B^2 = q_0^2$, while fermion masses remain unaffected.

what follows, therefore, we shall use (23) to determine the form of the metric $G_{\mu\nu}$, assuming a Robertson–Walker universe with scale factor $a(t,\phi_0)$. We shall be interested only in the effective four-dimensional low-energy theory, obtained by integrating our extra compact and bulk dimensions, as in [3].

The relevant four-dimensional equations are (ignoring contributions from the recoil velocities u assumed of order lower than (or at most similar to) H):

$$3\frac{\ddot a}{a} = 0,$$
$$-2\left(\frac{\dot a}{a}\right)^2 - \frac{\ddot a}{a} = 2\frac{a''}{a} + 2\left(\frac{a'}{a}\right)^2 + 2\frac{H^2}{\phi_0}\frac{a'}{a}, \qquad (26)$$

where the dot denotes derivative with respect to time $t = X^0$. From these equations we obtain the following solution for the scale factor (in string units):

$$a(t,\phi_0) = a_0\phi_0^b, \quad b = \frac{1}{2} - \frac{H^2}{2} \simeq \frac{1}{2}. \qquad (27)$$

We stress that this is the only acceptable solution from the point of view of Liouville dressing. The constant solution $b = 0$, which naively seems to be allowed, is excluded by the fact that such a solution corresponds to trivial gravitational dressing, $g' = 0$, which occurs *if and only if* $Q^2(\mathcal{T}) = 0$ [11] (critical-string, decoupling of the Liouville mode), in contradiction to our case, where $Q^2 > 0$ (7).

We now recall (15), according to which ϕ_0 is related linearly to the cosmic time $X^0 = -t$, $\phi_0 = \sqrt{2}\,t$. For asymptotically large times, therefore, this implies that the scale factor and the cosmological "vacuum" energy in our case behave as follows:

$$a(t) \sim a_0\sqrt{t}, \qquad \Lambda \sim \frac{H^4 R^{2n}}{t^2}. \qquad (28)$$

We should remark that, since the dilaton is constant, the dilaton equation does not yield any further information apart from consistency checks, which are easily performed. In particular, renormalizability of the σ-model requires an additional constrain, namely the Curci–Paffutti equation [21] which relates the dilaton β-function to the rest. This is valid for non-vanishing β-functions, and hence is applicable to our case as well [3]. It can be seen easily that from this equation one obtains no other information than a consistency check on the scaling behaviour of the central charge deficit Q^2 obtained above (7).

From (28) we observe that for times t of the order of the age of the observable universe, $t \sim 10^{60}$ in Planck units, and $H = 10^{-30}$ as required by TeV scale supersymmetry breaking, the cosmological vacuum energy is extremely suppressed at present according to this model. Significantly larger relaxation rates are obtained if the recoil effects are the dominant ones, a case which will be discussed briefly below. On the other hand, the $\sqrt{t}$ scaling of the scale factor implies an asymptotically decelerating universe, $\ddot a \sim -t^{-3/2}$, but on the other hand there is no cosmic horizon, and hence in this universe one can define properly asymptotic states, and thus an S-matrix.

Notice, therefore, that in our non-critical string scenario, one does indeed obtain an expanding universe, in contrast to standard ekpyrotic scenaria [9,10], based on critical strings and specific solutions to classical equations of motion. Such scenaria correspond to a vanishing Q^2, and hence $\beta^i = 0$ as discussed above in (23). In such a case, the effective actions used in [9,10] are given by the flow function $C[g]$ for the specific set of backgrounds used in those works. As we have seen, the presence of non-zero deficits Q^2 and Liouville dependence leads to very different physics.

One of the most important features of the existence of a non-equilibrium phase of string theory due to the collision is the possibility for an *inflationary phase*. Although the physics near the collision is strongly coupled, and the σ-model perturbation theory is not reliable, nevertheless one can give compelling physical arguments favouring the existence of an early phase of the brane world where the four-dimensional universe scale factor undergoes exponential growth (inflation). This can be understood as follows: in our model we encounter two type II string theory branes colliding, and then bouncing back. From a stringy point of view the collision and bounce will be described by a phase where open strings stretch between the two branes worlds (which can be thought of as lying a few string scales apart during the collision). During that early phase the excitation energy of the brane worlds can be easily computed by the same methods as those used to study scattering of type II D-branes in [22]. Essentially, the time integral of the relevant potential energy yields the scattering amplitude for the two branes, which was computed in [22]. According to standard arguments of type II string theory the exchange of open strings between two parallel D-branes is described by the emission of open-string pairs, and thus an *annulus* world-sheet diagram.[5] As a result of the annulus graphs, the exchange of pairs of open strings results in the appearance of "spin structure factors" in the scattering amplitude, which are expressed in terms of appropriate sums over Jacobi Θ functions. In particular, for small relative velocities $u \ll 1$ of the colliding branes, the appropriate spin structures start of at quartic order in u [22]:

$$\sum_{\alpha=2,3,4} e_\alpha \Theta_\alpha(u|\tau)\Theta_\alpha^3(0|\tau) \sim \mathcal{O}(u^4),$$
$$e_2 = -e_3 = e_4 = 1. \tag{29}$$

This is a result of the property of the Jacobi functions that are even functions of their argument, as well as that the Θ function satisfies by definition a "diffusion" equation: $[\partial_\tau + \frac{i}{4\pi}\partial_\nu^2]\Theta_\alpha(\nu|\tau) = 0$. The resulting excitation energy is therefore of order $\mathcal{O}(u^4)$ and may be thought of as an initial value of the central charge deficit of the non-critical string theory describing the physics of our brane-world after the collision. The deficit Q^2 is thus cut off at a finite value in the ultraviolet, and hence one never encounters a semiclassical limit for the underlying world-sheet field theory. This justifies, as already mentioned, the use of "wrong-sign" Liouville screening operators in this case. One may plausibly assume that the central charge deficit remains constant for some time, which is the era of *inflation*, as expressed by Eq. (23) for the scale factor. For (finite) constant $Q^2 = Q_*^2 = \mathcal{O}(u^4)$ it is easy to infer from (23) a scale factor exponentially growing with the Liouville zero mode $a(\phi_0) = e^{Q_*\phi_0/2}$. Upon the condition (15), then, one obtains an early inflationary phase after the collision, in contrast to the critical-string based arguments of [10]. The duration of the inflationary phase is $t_{\rm inf} \sim 1/Q_* \sim \mathcal{O}(u^{-2})$, which yields the conventional values of inflationary models of order $t_{\rm infl} \sim 10^9 t_{\rm Planck}$ for $u^2 \sim 10^{-9}$. This is compatible with the non-relativistic approximation for the D-branes, where our formalism is valid. Note that for such values of u the recoil effect is the dominant one in the relaxation of the vacuum energy (21), while the magnetic field is mainly responsible only for the supersymmetry breaking.

A final comment on the issue of inflation concerns the role of the dilaton as an inflaton field. During the phase of constant Q_* one may imagine the appearance of a scalar dilaton field in $(X^0, \vec{x})$ space–time which is linear in X^0: $\Phi = Q_* X^0$ such that after the condition (15) it cancels any dilaton effects, in the sense of a trivial world-sheet curvature coupling. This is a consistent solution of the conditions (23), implying a constant dilaton β-function Q_*^2, as required in non-critical strings with constant central-charge deficit [18]. Note that in this scenario, asymptotically in time, the dilaton $\Phi(X^0, \vec{x})$ tends to a constant, so *on the hypersurface* (15) of the $D+1$ extended space–time resulting after Liouville dressing the dilaton plays no actual rôle in the scenario. However, we stress that there is a dilaton field $\Phi(X^0, \vec{x})$ at the initial stage after the collision, which is non-trivial on the $(D+1)$-dimensional extended space–time, and hence one can safely speak about a dilaton acting like an inflaton field in this scenario.

Before closing we would like to make some brief remarks on the possible relevance of this toy model to realistic present-era cosmology, although we stress again, this is not the main purpose of our Letter. If one takes into account recent astrophysical claims [2] according to which the present era of the universe appears accelerating, then our results above seem to be ruled out by experiment. Of course the naive way out would be to observe that the above results are valid for times much later than the present era where one sees the acceleration.

Another interesting feature is the order of magnitude of the present-era vacuum energy. Physically, the relaxation to zero of the vacuum energy we find here seems quite plausible, given the transient nature of the collision of the two-branes. It is interesting to notice that its order of magnitude depends crucially on which is the dominant effect in the relaxation rate (21). If one insists on getting an inflationary era that lasts according to arguments of standard cosmology, then, as we have seen above, one requires recoil velocities of order $u < 10^{-4}$, which imply that recoil of the colliding branes is the dominant effect in providing (long after the scattering event) a vacuum-energy relaxation rate of order u^4/t^2 upon the identification (15) of Liouville mode with target time. This would then yield a vac-

[5] This is in contrast to the type I string case where the corresponding exchange of open strings is described by a world-sheet disk to lowest order.

uum energy which lies comfortably within the current observations [3,6].[6]

In this latter respect, however, an interesting observation can be made regarding our results. Notice that in case one does not care too much about the order of magnitude of the duration of the inflationary phase, but restricts oneself to the case where the recoil of the branes is subdominant as compared to the magnetic field effects, then the coefficient of the $1/\phi_0^2$ scaling in (21) is of order H^4, which by itself yields the order 10^{-120} in Planck units, in the case of supersymmetry breaking advocated here. This value is of the same order as the one claimed by the astrophysicists to have been "observed" from the preliminary supernovae data for the current era cosmological vacuum energy. Interestingly enough, therefore, the order of this coefficient by itself is what one needs [1] to resolve the supersymmetry-breaking/cosmological constant hierarchy. In our case naively, one could have obtained this latter result had one not made the connection of the Liouville scale with the time (15). Indeed, in such a case, where ϕ and X^0 are independent variables, which notably is mathematically consistent, one can freeze the renormalization group scale ϕ_0 to order R^n, to obtain a vacuum energy contribution (21) of the required magnitude. This vacuum energy is independent of time, and the solution of the metric equations (26) varies only with respect to the scale ϕ_0 (cf. (27)). Notice that this scenario is compatible with the alternative way of parametrizing the adiabatic switching-on of the magnetic field on the observable brane, $H(1-e^{-\varepsilon X^0})$.

However, as we have explained above, the transient nature of the colliding branes scenario, we are advocating here, seems to imply that the correct physical picture is the one in which (15) is valid and the excitation energy of the non-equilibrium system is given by a relaxing-to-zero time-dependent Λ (28), as in quintessence models [2]. The equilibrium state, which is the true ground state of the relaxing system, is then only reached asymptotically in time, and in our case has vanishing energy, due to the cancellation of the (positive) supersymmetry breaking energy contribution H^2 by the opposite in sign vacuum energy contribution of the negative tension brane, assumed to be our world. As we mentioned above, the initial instability due to the presence of negative tension branes is not necessarily a drawback in our cosmological non-equilibrium framework. The phenomenology of this transient scenario, therefore, seems to favour the recoil effect as the dominant one in the vacuum energy relaxation rate, without affecting our previous arguments on supersymmetry breaking which solely occurs due to the magnetic field.

This concludes our discussion on this toy model. It would be interesting to attempt and construct phenomenologically realistic supersymmetric brane–universe models along the lines outlined above, exhibiting an accelerating phase at late times. Such a situation is encountered in the non-supersymmetric case of [3]. The hope is that a realistic stringy universe model can be found, which has a late times accelerating phase and is capable of resolving the hierarchy between the supersymmetry-breaking scale and the present-era cosmological vacuum energy. This is left for future work.

[6] Notice that the coefficient of $1/t^2$ is of the same order as the initial vacuum energy $Q_*^2 \sim u^4$ during the inflationary era for type II strings. This suggests that a natural interpolating function for $Q^2(t)$ from the end of inflation $t_0 \sim 10^9$ (in Planck (string) units) until the present era would be $Q^2(t) = u^4/[(t-t_0)^2 + \mathcal{O}(1)]$. At present, however, we cannot support this by any quantitative analysis.

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