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Estimating the expediency of investing in the development of a regional electrical grid

Alexander S. Belenky^{a,*}, Dmitry S. Bolkunov^b

^a*Department of Mathematics, Faculty of Economic Sciences and the International Laboratory of Decision Choice and Analysis, The National Research University Higher School of Economics, Moscow, Russia, and Center for Engineering Systems Fundamentals, Massachusetts Institute of Technology, Cambridge, MA, USA.*

^b*The International Laboratory of Decision Choice and Analysis, The National Research University Higher School of Economics, Moscow, Russia.*

Abstract

Three situations associated with making decisions on investing in the development of a regional electrical grid are considered, and a game-theoretic approach to establishing the expediency of the investment in the grid and to finding optimal investment strategies for both the government and private investors in these situations is outlined. The implementation of this approach is associated with formulating and solving particular games on polyhedral sets of player strategies that are presented and discussed.

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1. Introduction

Developing new and improving already existing regional electrical grids always implies providing a certain level of investments in new technologies of and equipment for electricity production and transmission by both interested financial institutions and private investors. So recently, a rapid penetration of renewable sources of energy and systems for storing electricity in the structure of regional electrical grids has also drawn attention of potential investors to problems of estimating the profitability of investing in the development of such sources and systems. In all these problems, finding whether any investment in a) renting, acquisition, and production of particular systems for transforming solar and wind energy into electricity, and b) renting, acquisition, and production of electricity storing systems is profitable, and finding optimal investment strategies when such an investment is profitable, presents interest for both local and central governments, for business communities, and for the financiers in the energy sector of national economy in every country.

*Corresponding author.

E-mail address: abelenky@hse.ru.

Depending on what estimates of the needed investment volume in the development of a regional electrical grid are, three situations may emerge. In the first one, individual electricity producers or groups of them, transmitting companies, and large (mostly) industrial grid customers may turn out to be both capable of making their investments in particular parts or elements of the grid and interested in doing this at the required investment scale. They may provide the needed volume of investment either with the use of their own money only or by borrowing a part or all the money to be invested. In the second situation, a regional government with (or without) any support from the central government may be capable of making all or some of the investments in the grid that are needed. Finally, in the third situation, neither the private sector alone nor the regional and the central governments (even together) can provide the whole amount of the investment needed. If this is the case, a form of a partnership between the private and the public sector is expedient to raise an appropriate amount of money to invest in developing the grid.

In all the three situations, to quantitatively estimate both the maximum return on investment and the investment strategies leading to receiving this return, certain mathematical tools are needed. A game-theoretic approach to developing such tools is proposed, and the structure and features of the corresponding games are discussed in the present paper.

2. Finding optimal investment strategies of service providers in developing a regional electrical grid

In [1], it is proposed to consider the interaction between electricity producers and electricity consumers in the grid (both individual and group ones) on any particular time segment as a two-person, non-cooperative game on disjoint player strategies in the following form

$$\begin{cases} \langle \tilde{x}, Dy \rangle + \langle v, \tilde{x} \rangle \rightarrow \max_{\tilde{x} \in \tilde{M}} \\ \langle \tilde{x}, Dy \rangle \rightarrow \min_{y \in \Omega} . \end{cases}$$

Here

a) a part of the components of the vector variable \tilde{x} is formed by the volumes of the hourly electricity production of each electricity producer in the grid, another part of the components of the vector \tilde{x} corresponds to the capacities of the new sources of energy and new electricity storage systems to be developed in the time segment under consideration, whereas the other components are auxiliary variables,

b) components of the vector variable y are the prices for a unit volume of electricity that the grid customers are ready to pay to the electricity producers,

c) components of the vector v are parameters that are used in the above game model,

d) the matrix D is that of a linear operator that maps a set of the vectors y into an auxiliary set in a finite-dimensional space of the same dimension as that of the vectors \tilde{x} , and

e) \tilde{M} and Ω are polyhedra, which are defined by systems of linear inequalities $\tilde{M} = \{\tilde{x} \geq 0 : \tilde{A}\tilde{x} \geq \tilde{b}\}$ and $\Omega = \{y \geq 0 : By \geq d\}$ (where \tilde{A}, B are matrices whose elements are some parameters of the electrical grid, and \tilde{b}, d are vectors of such parameters).

In this game, all the electricity producers in the grid, which form a collective player in the game, try to maximize their revenue by choosing the volumes of electricity to be supplied that are acceptable to the consumers in the grid. The other collective player in this game is formed by all the electricity consumers, who try to minimize their expenditure associated with buying electricity in volumes acceptable to them.

As proven in [1], the pair of the vectors $(\tilde{x}^*, y^*) \in \tilde{M} \times \Omega$ form a Nash equilibrium point in the game if and only if there exist vectors $z^* \geq 0$ and $t^* \geq 0$ such that the pairs of the vectors (z^*, \tilde{x}^*) and (t^*, y^*) are solutions to the linear programming problems

$$\begin{aligned} \langle d, z \rangle + \langle v, \tilde{x} \rangle &\rightarrow \max_{(z, \tilde{x}) \in \{(z, \tilde{x}) \geq 0 : zB \leq \tilde{x}D, \tilde{A}\tilde{x} \geq \tilde{b}\}}, \\ \langle -b, t \rangle &\rightarrow \min_{(t, y) \in \{(t, y) \geq 0 : t\tilde{A} \leq -v - Dy, By \geq d\}}, \end{aligned}$$

forming a dual pair.

A detailed description of all the variables and parameters of the mathematical model underlying the above game, along with the proof of the above assertion, is presented in [1]. One of the features of this mathematical model consists of taking into consideration daily and seasonal changes in the electricity demand while remaining within the size of the model allowing one to solve the linear programming problems forming the dual pair in a time acceptable in practical calculations with the use of widely available standard software. That is, this demand is averaged by partitioning the time segment of several years (in which all the electricity producers and consumers in the grid interact [1]) into a set of so-called “typical” periods of several kinds. This partitioning is done in such a manner that the above-mentioned time segment turns out to be divided into “typical” periods each of which is repeated several times within each year from this time segment. Here, each “typical” period of each kind reflects regularities of the customers’ demand attributed to this period, and each “typical” period may last for a certain number of days or even weeks. This demand is met with the use of both the existing electricity generating and storing facilities and those to be developed as a result of the investments in volumes determined by the equilibrium vectors (\tilde{x}^*, y^*) . Thus, in [1], a game-theoretic approach to estimating the investment volume in the grid and to finding optimal investment strategies in the first situation, which is based on finding equilibrium investment strategies by solving the above two-person, non-cooperative game, is proposed. A practical application of this approach is demonstrated in [1] for an example that uses model data.

3. Developing economic mechanisms encouraging competition among service providers in tendering the right for “wiring-up” customers of a regional electrical grid

When the local government in a region of a country or the central government of the country or both intent to invest taxpayers’ money in any projects considered to be important for the region residents, including those in developing the regional electrical grid or any of its parts, this government (governments) is (are) responsible to the taxpayers on all the decisions it (they) makes (make) on the way the money is spent. “Wiring-up” particular areas in the region, which is one of the permanently existing problems in practically any region as its population grows and migrates, is one of the areas in which social interests related to the electrical grid functioning intersect with business ones. Solving this problem requires establishing rules that would give enough freedom to the local government authorities to promote a fair competition among all the interested businesses while encouraging these businesses to compete.

While large customers buy electricity from wholesalers on a long-term contractual basis, small end customers of the grid receive (buy) electricity mostly from distributors (retailers such as utility companies). In each fragment of the grid, these distributors obtain electricity from the transmission lines (at substations within the transmission network) and then deliver it to the end customers via low voltage lines. With storage facilities available throughout the grid, the number of distribution companies “wiring-up” regions and municipalities of the country is likely to increase. The conventional wisdom suggests that the distribution service (“wiring-up”) should be regulated, since it is considered to be a natural monopoly under the existing deregulations (in the U.S., such as, for instance, Energy Power Act of 1992, Federal Energy Regulatory Commission Order 888 of 1996, and the U.S. Supreme Court decision of 2000, regarding this Order). Nevertheless, if this were to happen, at least with the appearance of storage facilities throughout the grid, there would be enough room for developing incentives to compete for all the companies interested in delivering electricity to end customers.

Thus, at least theoretically, tenders for the right to provide electricity for end customers from the transmission network and storage facilities may eventually be held by both central and local authorities. As in any service business, holding such tenders has the potential to both reduce prices for end customers and improve the quality of the service. So it seems reasonable to explore whether local authorities would be interested in holding such tenders.

Generally, most tenders are conducted in the form of sealed-bid auctions, and several rules for determining the tender winner in such types of auctions may apply. The rules should encourage all the bidders to submit their true values of the tendered right to “wire-up” end customers to the transmission network and storage facilities as their bids. At the same time, these rules should discourage bidders who would like to win the tender at any cost from submitting artificially excessive prices as their bids.

As is known, generally, Vickrey auctions, sometimes referred to as the second-price sealed-bid auctions, solve the problem of encouraging the bidders to submit their real values of the subject of the bid in a single-step auction,

provided that profit is the only criterion to participate in the auction [2, 3]. However, Vickrey auctions may not necessarily discourage the submission of excessive prices by some bidders, for instance, by those for whom winning the tender even at a cost exceeding their values of the bid subject is still an acceptable and even a preferable strategy. This may happen when such bidders pursue other goals besides maximizing their own profit, such as, for instance, undermining opportunities of other bidders to succeed in the market [2, 3, 4]. If this were the case, in a particular tender, the absence of a mechanism discouraging the bidders from submitting excessive prices may discourage reputable service companies to participate in the tender, since they may have no chance to win. As is known, maximizing its own profit may not necessarily be the best strategy for a surrounding that plays against a coalition [5], so designing tender rules discouraging collusions among the bidders would contribute to equalizing chances of all the tender participants to win.

The above-mentioned problem may emerge when a tender for particular work or a set of works, for instance, for developing electricity transmission lines from remotely located alternative energy sources to a particular region is set by local or central government authorities. An example of a rule for determining the winner in a sealed ceiling bid in which a contract for providing services is offered to a set of eligible bidders can be found in [6, 7]. This rule is, in fact, an economic mechanism encouraging the bidders to submit bids (for the contract) that are close to a (calculated) price desired by the auctioneer (or by the organizer of the bid) while discouraging the bidders from submitting both damping prices and excessive ones. Under this rule, for each participating bidder, the probability of winning the contract at a particular price desirable to the auctioneer (or to the bid organizer) turns out to be higher than that under the traditional rule—when the contract is awarded to a bidder offering the lowest price for providing services stipulated in the contract. At the same time, for any bidder, the probability of winning the contract at a price exceeding this particular price (desirable to the auctioneer or to the bid organizer) turns out to be lower than that of winning the contract at this price.

The above-mentioned particular economic mechanism can be modified for organizing sealed-bid auctions for the right to “wire-up” particular regions, municipalities, etc., as well as those for other works relating to developing the national electrical grid in principle. Yet, developing new such mechanisms, as well as the mechanisms reducing the chances of forming corrupt ties, particularly, in the framework of possible public procurement activities associated with developing the grid remains challenging. One such mechanism, which reduces the chances of forming corrupt ties is proposed in [7] though this mechanism is not resistant to corrupt ties that may be formed between a cartel and a public administration. Also, since the right to provide several services associated with developing the grid may be tendered by a public administration in the form of packages, mechanisms for combinatorial auctions—under which packages of goods or services are tendered—are needed. Some examples of such general mechanisms are well known [8], whereas some particular mechanisms, for instance, the one for optimally partitioning all the needed works into a set of lots to be tendered can easily be developed.

4. Forming a public-private partnership in making a sizable investment in a regional electrical grid

As is known [9], public-private partnership (PPP) problems are those actively studied in economics, since the goals of these two sectors of any national economy not always coincide, and PPP is an economic mechanism that helps harmonize these goals. When a sizable investment in developing an electrical grid is needed, whereas neither the local government nor the central government (nor both of them) can secure the needed investment alone, and private investors interested in this market cannot secure the needed investment alone either, a PPP may make a difference.

However, to consider whether to deal with the central or with the local government (or with both) in the framework of a PPP of any kind, private investors need the government (or both governments) to guarantee them certain conditions of their participation in this PPP. Such conditions should make the private sector investments in the grid at least comparable (in terms of the level of profit) with other possible investments that interested private investors may make. So to make possible the formation of any PPP to provide sizable investment in electrical grids, one should estimate whether such a partnership can be formed in principle, proceeding from a) the conditions that the government (governments) can offer to potential private investors in the grid, and b) the options to allocate financial resources that the private investors have at the time of considering the government’s (governments’) offer to form this PPP.

For instance, the intent of the government (governments) to develop the grid by incorporating storage facilities and renewable sources of energy in it may be difficult to implement by the government (governments) on its (their) own due to financial reasons. However, the private sector may not be interested in investing in the corresponding projects or may consider any investments in these projects to be too risky financially. If this is the case, economic mechanisms encouraging the formation of a PPP between the public and the private sector are needed. Once the use of such mechanisms has generated a mutual interest in both parties to consider whether a potential PPP may be a) profitable enough to the private sector, and b) socially and financially beneficial to the government (governments), tools for estimating the economic effectiveness of this PPP should be available.

It turns out [9], that under certain natural, numerically verifiable assumptions, the analysis of the possibility to form an economically profitable PPP can mathematically be conducted by considering a three-person game on a polyhedral set of connected player strategies [10] with payoff functions of the players being linear functions of vector arguments. Moreover, an auxiliary game can be formed for this three-person game in such a manner that components of an equilibrium point in the auxiliary game can be used to construct an equilibrium point in the initial game. Thus, since necessary and sufficient conditions for the equilibrium points in the auxiliary game are verifiable, these conditions turn out to be verifiable sufficient conditions of equilibrium points in the initial game (though these conditions are not necessary ones for the initial game) [9].

The problem of analyzing the possibility to form a particular PPP for the purpose of investing in developing a regional electrical grid can be stated as follows: Both the local and the central governments are interested in developing a regional electrical grid (being part of the country's electrical grid), and they determine that n projects relating to particular parts of the grid or to particular functions of it should be developed. They try to convince private investors to finance the projects, and they plan to invite a management company to handle the implementation of these projects once the projects got financed. Upon the completion of the projects, the local government plans to tender the right to operate the objects, for instance, new storing systems, new generators, new solar and wind power stations, etc., to be created as a result of implementing the projects. Contracts for the right to operate these objects will be paid for by the tender winners, and these winners will be able to recuperate the money to be paid to the local government from the revenue to be generated as a result of running the developed objects during a certain period of time. Moreover, within this period of time, the local government also expects to return (with certain percentages to be stipulated at the time of negotiating the PPP) the invested capital to all its PPP partners. Once this period of time is over, the local government will continue to receive revenue from new owners of the created objects in the form of taxes, and the size of these taxes is to be in line with existing laws [10].

Proceeding from a) the investment volume needed for each of n projects on developing the grid, b) the investment volume that the government (governments) can secure for each project and in total, along with the estimate of the minimum investment volume needed by these n projects in total, c) the expected yearly revenue from the functioning of each project, d) the volume of financing expected to be requested by the management company, e) the investment volume that the private sector can offer for financing each project and in total, and f) a set of financial parameters determining conditions under which the government (governments) wishes (wish) to form a PPP, one can form a system of constraints determining a set of player strategies for each of the three players in the game of forming the PPP for investing in the grid—the government (governments), a group of interested private investors, and the management company. These strategies are connected, their sets form a polyhedron in a finite-dimensional space, and the payoff functions of the game players turn out to be linear ones of the corresponding vector arguments [9]. Under certain natural assumptions [9], this three-person game takes the form

$$\begin{cases} a - \langle \omega, w \rangle - \langle \chi, x \rangle \rightarrow \max_x \\ \langle (\sigma, 0), w \rangle \rightarrow \max_w \\ g - \langle (0, \tau), w \rangle \rightarrow \max_w \\ Aw + Bx \geq h \end{cases} \quad (4.1)$$

where a) components of the vector x are the investment volumes that the government (governments) can offer for each of n projects associated with the grid development, b) each component from a part of components of the vector w reflects the investment volume that the private sector can offer for the corresponding project, c) each component from the rest of components of the vector w is the volume of financing that the management company

requests for managing the corresponding project, d) τ , χ , σ , and ω are vectors of the above-mentioned financial parameters of the PPP, stipulated by the government (governments), e) a and g are positive, real numbers reflecting the revenue expected to be received from the functioning of all the n projects (within the above-mentioned period of time) by the government (governments) and by the group of private investors, respectively, and f) A , B , and h are matrices and a vector or corresponding dimensions, where the matrix elements and the vector components are formed by units, zeros, and real numbers calculated with the use of some of the above-mentioned financial parameters.

An auxiliary two-person game on a polyhedral set of connected player strategies

$$\begin{cases} a - \langle \omega, w \rangle - \langle \chi, x \rangle \rightarrow \max_x \\ g - \langle (0, \tau) w \rangle + \langle (\sigma, 0), w \rangle \rightarrow \max_w \\ Aw + Bx \geq h \end{cases} \quad (4.2)$$

is introduced, and as proven in [9], every pair of vectors that is an equilibrium point in Game 4.2 is an equilibrium point in Game 4.1. In any game on a polyhedral set of connected player strategies $S = \{(x, y) \geq 0 : Kx + Ly \geq h\}$ with the payoff function $\phi(x, y) = \langle p, x \rangle + \langle q, y \rangle$, where p , q , x , y , h are vectors, and K , L are matrices of corresponding dimensions, the pair of vectors is called an equilibrium point if the inequalities

$$\phi(x^*, y) \leq \phi(x^*, y^*) \leq \phi(x, y^*) \quad \forall (x^*, y) \in S, \forall (x, y^*) \in S$$

hold [11].

As proven in [11], equilibrium points in Game 4.2 exist if and only if an auxiliary system of quadratic inequalities

$$\begin{cases} Th \geq \delta, \\ \langle h, H_1 h \rangle + \langle \rho, h \rangle = 0, \\ \langle h, H_2 h \rangle + \langle \rho, h \rangle = 0, \end{cases}$$

where T is a matrix, H_1 and H_2 are symmetric quadratic matrices, whereas ρ , δ and h are vectors of corresponding dimensions and structures, is solvable. These necessary and sufficient conditions for the equilibrium points in Game 4.2 let one reduce finding equilibrium points in this game to solving so-called quadratic optimization problems [12], which can be done by methods of non-differentiable optimization, proposed in [12].

5. Conclusion

1. Though, in many countries, the electrical grid accounts for a sizable part of the energy consumption, providing investments even in regional electrical grids, especially in traditional electricity generators, presents a problem due to both the scale of the investment needed and risks associated with uncertainties in the electricity demand and with rapidly developing technologies for receiving electricity from renewable sources of energy. The necessity to properly manage the taxpayers' money within the existing laws by government agencies responsible for providing the population of a region with electricity and the above-mentioned risks for private investors require developing a) economic mechanisms that would encourage potential investors to at least consider investing in the grid, and b) particular tools for quantitatively evaluating the effectiveness of these mechanisms. Game-theoretic approaches to developing such tools seem promising, since they allow one to take into consideration conflict interests of the electricity suppliers and consumers. Moreover, when sets of player strategies can be described by linear constraints of a balance kind or by convex constraints allowing their linearization, for some classes of games on polyhedral sets with nonlinear payoff functions, equilibrium player strategies can be calculated with the use of well-developed optimization techniques many of which are implemented in the form of standard software packages.

2. Establishing both sufficient and necessary and sufficient conditions for the equilibria even in games on polyhedral sets presents considerable difficulties, as, for instance, this turns out to be the case for games with connected player strategies. So searching for classes of solvable games that describe any aspects of functioning of particular regional electrical grids seems reasonable. This search implies conducting an appropriate systems

analysis of investment problems arising in studying the functioning of regional electrical grids, and results of this analysis may help design new or choose known mathematical models and techniques to solve mathematical problems formulated on the basis of these models.

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