Weakly-non-linear interactions of modulated T-S waves in the boundary layer of an airfoil

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Abstract

Weakly non-linear interactions of Tollmien-Schlichting (T-S) waves are investigated experimentally for an incompressible 2D boundary layer. The number of waves introduced in the boundary layer is varied, covering regimes composed of few waves or bands of T-S waves (frequency and wavenumber). The experiments are carried out on the pressure side of an airfoil designed to achieve a long region of laminar flow. The results show that modulations of fundamental wave tend to generate disturbances in a range of low frequencies, via nonlinear wave interaction, which can act as self-generated seeds for subharmonic resonance. This sequence of weakly non-linear events were recently described for regimes composed of few waves. The results shown here, for wave bands, are in qualitative agreement with previous findings. The mechanism seems to be a possible route for transition in the case of Natural Laminar Flow airfoils under ‘natural’ environment disturbance conditions.

Keywords: Boundary-Layer; Transition; Weakly-nonlinear;

1. Introduction

Drag reduction by delaying the transition from laminar to turbulent flow is still one of the few technologies that can significantly improve performance of today’s highly optimized airplane configurations. For the design of airfoils with long extend of laminar flow, it is necessary to have an accurate prediction of the transition location in order to achieve the desired drag reduction without imposing a significant penalty to the overall airfoil performance. Methods to predict transition based on Linear Stability Theory (LST) were developed in the early 50’s and still are the most widely used tool in the design of natural laminar flow airfoils. However, for many practical applications these methods do not provide accurate enough predictions, especially when the extension of the region where the linear theory is valid...
decreases in comparison to the region where nonlinear effects play a role. For two-dimensional boundary layers, the instabilities of Tollmien-Schlichting (TS) waves are often the dominant route for transition. At a called “natural” disturbance conditions, nonlinear interactions of T-S waves can occur in a broad band of frequencies and spanwise wavenumbers.

The role of transition to turbulence in a boundary layer under “natural” environmental disturbance conditions is still a challenging problem. Although many aspects of the phenomenon are well described by theory, the whole process is still not completely understood. Since Gaster & Grant, the use of localized disturbances in space and time (wave packet) became common as a model for the investigation of ‘natural’ transition. Important aspects of phenomenon were clarified by experiments involving wavepackets. Some of those works reported a higher nonlinear activity of perturbations developing inside the wavepackets, especially for strong modulation of the disturbance amplitude in time. From the spectral viewpoint such stronger modulation corresponds to a broader frequency spectrum of the wavepacket. Different explanations were proposed previously. However, no any definitive, indubitable arguments were found. Therefore, the present investigation deals with the study of interactions involving bands of deterministic T-S waves at weakly nonlinear stages of boundary layer transition.

Simplified regimes composed by few fundamental waves with different frequencies were recently investigated. Such scenario could be interpreted as a single fundamental wave with amplitude modulation in time, similar to wavepackets. Their results have shown that the modulation of fundamental waves did not alter the growth of subharmonics due to resonant interactions. Also, the production of subharmonics in the nonlinear stages of the wave evolution could be clearly described and substantiated. The self production of subharmonics was also observed for wave packets. The current investigation is a continuation of previous works towards the understanding of mechanisms present at the weakly nonlinear stages of ‘natural’ transition in an airfoil boundary layer.

2. Set-up and Results

The experiments were conducted in the Laminar Wind Tunnel of the IAG. The measurements are carried out on an airfoil at controlled disturbance conditions. T-S waves are excited in the boundary layer by a slit source, which was mounted flush with the surface. Below the slit, 116 equally spaced pneumatic tubes are located and connected to 116 loud-speakers, which are driven by power amplifiers and a 128 channel arbitrary waveform generator. This device enables the generation of well-defined disturbances within a broad band of frequencies and spanwise wavenumbers. Phase-locked hot-wire measurements with respect to the disturbance generation are performed downstream of the slit. A complete description of the set up can be found in de Paula et. al(2013).

In previous experiments, amplitude modulations of the disturbances were obtained by exciting two or three 2-D fundamental waves with different frequencies. The frequency of modulation was given by the difference in frequency of the fundamental waves. Here, the same conditions are kept and the carrier frequency is set to 634Hz with modulation frequencies (ΔF) of 92Hz and 156Hz. Figures 1(a) and (b) show the evolution of two excited 2-D fundamental waves, introduced at the source, and fluctuations having the modulation frequency (ΔF). It is important to note that difference modes are not introduced directly by the source; they arise from non-linear interaction between the excited fundamental waves. Linear stability calculations (LST) are also displayed in the figure. A good agreement with the theory is seen, except for the ΔF modes.

Measurements along the spanwise direction are carried out and the spectra of spanwise wavenumbers of the waves at ΔF frequency are depicted in figures 2(a) and 2(b). These measurement are performed at one displacement thickness from the wall, which corresponds to the maximum in the eigenfunction profiles given by LST for 2-D and 3D waves at ΔF frequency. The curves are shifted by a factor of 10, for sake of clarity. The spectra show that, initially, ΔF modes consist mainly of 2-D waves (β = 0rad/mm). Further downstream, a band of 3-D modes is amplified. Additional insight is gained by an amplitude scaling of the curves with the initial amplitude of the excited fundamentals. Note that in all cases A_j1(Δs = 0) = A_j2(Δs = 0) and the reference cases (A_R) in figures 2(a) and 2(b) correspond to those shown in figures 1(a) and 1(b) respectively. It can be seen that at Δs = 40 and 80mm the normalized spectra are well scaled by the square of fundamental waves amplitude. This was previously reported and it is an indicative that pure product of fundamentals are forcing the excitation of difference modes. In that work the nature of three-dimensionality seen already in this non-resonant stage was associated with the efficiency of the non-linear interaction...
Fig. 1. Streamwise evolution of amplitudes of fundamental (black symbols) and ΔF (open symbols) modes. Cases with modulations of 92Hz (a) and of 156Hz (b). In the experiments only the two fundamental waves are excited directly by the T-S source.

Fig. 2. Normalized spanwise-wavenumber spectra of difference frequency modes. Spectral amplitudes are normalized by the initial fundamental-wave amplitude squared. Cases with modulations of 92Hz (a) and of 156Hz (b).

between fundamentals waves. Further downstream the quadratic scaling does not offer a good correlation of measured spanwise spectra and resonance is likely to occur.

The above sequence of nonlinear events was observed and substantiated in de Paula et. al² for a few modes. However, it is not clear if these nonlinear events do also occurs in a scenario where several waves are present simultaneously in the boundary layer. Therefore, an experiment was set to address this question. To this end, bands of waves having the same central frequency of 634Hz and two modulation frequencies are investigated. Differently from the studies involving wavepackets, here the phase of the waves at each frequency was randomly selected in order to avoid high amplitudes of disturbances within a small time instant. The maximum amplitude within the time series of the driving signal is also maintained constant and equals to the case composed of few waves. The frequency-spanwise-wavenumber spectra shown in figure 3 is obtained for a modulation frequency of 92Hz. The spectral distributions of figure 3 show qualitatively a similar behavior when compared to the case where only two fundamental waves were excited. It can be seen that at ΔX = 20mm only disturbances introduced by the source are visible. Further downstream, at ΔX = 80mm, some seeds of disturbances in the range of ΔF frequencies are already noticeable and at the last station, 3-D waves of such modes reach high amplitudes. At the last station, traces of detuned subharmonic resonance can be observed in the figure 3. According to Kachanov et. al.¹, detuned subharmonic resonances can occur even with very large frequency detunings of the subharmonics, but in these cases the resonance is weaker and present a symmetric mode with respect to the exact subharmonic frequency. Here, the detuned resonance is suggested by the
filling of the spectral distribution at frequencies slightly lower than the band of excited 2D fundamental waves, i.e. below the dashed line.

Fig. 3. Spectral distribution of streamwise velocity fluctuations for the regime with 2-D band of T-S waves. The 2-D fundamental waves are excited with a bandwidth of 92Hz. The gray scale corresponds to \( \log(A) \) in % of \( U_e \)

For the case with a modulation frequency of 156Hz, the spectral distribution, shown in figure 4, do also present a similar behavior when compared to the results obtained for few interacting modes, which are displayed in the figure 1(b). Again the same picture is observed, where only the disturbances excited are visible close to the source and further downstream traces of detuned subharmonic resonance become noticeable. It is important to note here, that evidences of detuned resonance are rather stronger for the case of the figure 4 when compared to the figure 3. This is related to the bandwidth of difference modes. According to the argument of Kachanov et. al\(^1\) the resonance is enhanced when the frequency of the detuned modes approaches subharmonic frequencies. In the present experiments, the frequency detuning is reduced in the case with modulation of 156Hz.

Fig. 4. Spectral distribution of streamwise velocity fluctuations for the regime with 2-D band of T-S waves. The 2-D fundamental waves are excited with a bandwidth of 156Hz. The gray scale corresponds to \( \log(A) \) in % of \( U_e \)

Thus, it can be inferred that if the band of waves fundamental waves is broad enough to enable the generation of difference modes within the range of subharmonics, then the subharmonic resonance can be enhanced. These observations fully support the mechanism described in the work of Craik\(^10\). Apparently, this is the most probably route for the amplification of the disturbances toward the late stages of transition. The scenario observed here, is in agreement with the findings for wavepackets and wavetrains and is also considered to occur in the case of 'natural' transition.

3. Conclusions

The results show that the evolution of the \( \Delta F \) modes is primarily dependent only on the nonlinear interactions between fundamental waves. It is seen that quasi-2D initial disturbances produce quickly some low-frequency (\( \Delta F \)) waves in the frequency range, which corresponds to the width of the frequency band of the fundamental modes. In a
first non-linear stage the seeds of 3-D low frequency waves are generated by an interaction of the fundamental waves. Further downstream the produced low frequency disturbances became 3-D. In the regime with frequency bands of fundamental waves a similar behavior occurs. An indicative of detuned subharmonic resonance with the fundamental waves is seen. Apparently, this is the most probably route for the amplification of the disturbances toward the late stages of transition. The scenario observed here, is in agreement with the findings for wavepackets and wavetrains and is also considered to occur in the case of “natural” transition.

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References