



# Complex-mass renormalization in chiral effective field theory

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## ABSTRACT

We consider a low-energy effective field theory of vector mesons and Goldstone bosons using the complex-mass renormalization. As an application we calculate the mass and the width of the  $\rho$  meson.

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## 1. Introduction

The setting up of a consistent power-counting scheme for chiral effective field theories with heavy degrees of freedom is a non-trivial endeavor. For example, in baryon chiral perturbation theory the usual power counting is not satisfied if the dimensional regularization is used in combination with the minimal subtraction scheme [1]. The current solutions to this problem either involve the heavy-baryon approach [2] or the use of a suitably chosen renormalization condition [3–6]. Because the mass difference between the nucleon and the  $\Delta(1232)$  is small in comparison to the nucleon mass, the  $\Delta$  resonance can also be consistently included in the framework of effective field theory [7–11].

On the other hand, the treatment of the  $\rho$  meson is more complicated. While the  $\Delta$  resonance decays into a (heavy) nucleon and a (light) pion, the main decay of the  $\rho$  meson involves two pions with vanishing masses in the chiral limit. Therefore, for energies of the order of the  $\rho$ -meson mass, the loop diagrams develop large imaginary parts. Unlike in the baryonic sector, these power-counting-violating contributions, being imaginary, cannot be absorbed in the redefinition of the parameters of the Lagrangian as long as the usual renormalization procedure is used. Despite this

feature, the heavy-particle approach has been considered in Refs. [12–16], treating the vector mesons as heavy static matter fields.

In Refs. [17] and [18] we considered the inclusion of virtual vector mesons in the framework of (baryon) chiral perturbation theory for low-energy processes in which the vector mesons cannot be generated explicitly. The present work extends the applicability of the chiral effective field theory to the momentum region near the complex pole corresponding to the vector mesons. We tackle the power-counting problem by using the complex-mass renormalization scheme [19–24], which is an extension of the on-mass-shell renormalization scheme to unstable particles. As an application we consider the mass and the width of the  $\rho$  meson which are of particular interest in the context of lattice extrapolations [25,26]. For a different approach to these problems using the infrared regularization, see Refs. [27,28].

## 2. Lagrangian

We start from the most general effective Lagrangian for  $\rho$  and  $\omega$  mesons and pions in the parametrization of the model III of Ref. [29], where the  $\rho$ -vector fields transform in-homogeneously under chiral transformations,

$$\mathcal{L} = \mathcal{L}_\pi + \mathcal{L}_{\rho\pi} + \mathcal{L}_\omega + \mathcal{L}_{\omega\rho\pi} + \dots$$

Displaying explicitly only those terms relevant for the calculations of this work, the individual expressions read

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$$\begin{aligned}
\mathcal{L}_\pi &= \frac{F^2}{4} \text{Tr}[\partial_\mu U (\partial^\mu U)^\dagger] + \frac{F^2 M^2}{4} \text{Tr}(U^\dagger + U), \\
\mathcal{L}_{\rho\pi} &= -\frac{1}{2} \text{Tr}(\rho_{\mu\nu} \rho^{\mu\nu}) + \left[ M_\rho^2 + \frac{c_x M^2 \text{Tr}(U^\dagger + U)}{4} \right] \\
&\quad \times \text{Tr} \left[ \left( \rho^\mu - \frac{i\Gamma_\mu}{g} \right) \left( \rho_\mu - \frac{i\Gamma_\mu}{g} \right) \right], \\
\mathcal{L}_\omega &= -\frac{1}{4} (\partial_\mu \omega_\nu - \partial_\nu \omega_\mu) (\partial^\mu \omega^\nu - \partial^\nu \omega^\mu) + \frac{M_\omega^2 \omega_\mu \omega^\mu}{2}, \\
\mathcal{L}_{\omega\rho\pi} &= \frac{1}{2} g_{\omega\rho\pi} \epsilon_{\mu\nu\alpha\beta} \omega^\nu \text{Tr}(\rho^{\alpha\beta} u^\mu), \tag{1}
\end{aligned}$$

where

$$\begin{aligned}
U &= u^2 = \exp\left(\frac{i\vec{\tau} \cdot \vec{\pi}}{F}\right), \\
\rho^\mu &= \frac{\vec{\tau} \cdot \vec{\rho}^\mu}{2}, \\
\rho^{\mu\nu} &= \partial^\mu \rho^\nu - \partial^\nu \rho^\mu - ig[\rho^\mu, \rho^\nu], \\
\Gamma_\mu &= \frac{1}{2} [u^\dagger \partial_\mu u + u \partial_\mu u^\dagger], \\
u_\mu &= i[u^\dagger \partial_\mu u - u \partial_\mu u^\dagger]. \tag{2}
\end{aligned}$$

In fact, at the beginning all the fields and parameters of Eqs. (1) and (2) should be regarded as bare quantities which are usually indicated by a subscript 0. However, in order to increase the readability of the expressions we have omitted this index. In Eqs. (1),  $F$  denotes the pion-decay constant in the chiral limit,  $M^2$  is the lowest-order expression for the squared pion mass,  $M_\rho$  and  $M_\omega$  refer to the bare  $\rho$  and  $\omega$  masses,  $g$ ,  $c_x$ , and  $g_{\omega\rho\pi}$  are coupling constants. Demanding that the dimensionless and dimensionfull couplings are independent, the consistency condition for the  $\rho\pi\pi$  coupling [30] leads to the KSFR relation [31,32]

$$M_\rho^2 = 2g^2 F^2. \tag{3}$$

### 3. Renormalization and power counting

To perform the renormalization we use the standard procedure of expressing the bare quantities (parameters and fields, now indicated by a subscript 0) in terms of renormalized ones, leading to the generation of counterterms. Below, we show explicitly only those which are relevant for calculations of this work,

$$\begin{aligned}
\rho_0^\mu &= \sqrt{Z_\rho} \rho^\mu, \\
Z_\rho &= 1 + \delta Z_\rho, \\
M_{\rho,0} &= M_R + \delta M_R, \\
c_{x,0} &= c_x + \delta c_x. \tag{4}
\end{aligned}$$

We apply the complex-mass renormalization scheme [19–24] and choose  $M_R^2 = (M_\chi - i\Gamma_\chi/2)^2$  as the pole of the  $\rho$ -meson propagator in the chiral limit. The pole mass and the width of the  $\rho$  meson in the chiral limit are denoted by  $M_\chi$  and  $\Gamma_\chi$ , respectively. Both are input parameters in our approach. We include  $M_R$  in the propagator and the counterterms are treated perturbatively. In the complex-mass renormalization scheme, the counterterms are also complex quantities.

Since the  $\rho$  mass will not be treated as a small quantity, the presence of large external four-momenta, e.g., in terms of the zeroth component, leads to a considerable complication regarding the power counting of loop diagrams. To assign a chiral order to a given diagram it is first necessary to investigate all possibilities how the external momenta could flow through the internal lines

of that diagram. Next, when assigning powers to propagators and vertices, one needs to determine the chiral order for a given flow of external momenta. Finally, the smallest order resulting from the various assignments is defined as the chiral order of the given diagram.

The power counting rules are as follows. Let  $q$  collectively stand for a small quantity such as the pion mass. A pion propagator counts as  $\mathcal{O}(q^{-2})$  if it does not carry large external momenta and as  $\mathcal{O}(q^0)$  if it does. On the other hand, a vector-meson propagator counts as  $\mathcal{O}(q^0)$  if it does not carry large external momenta and as  $\mathcal{O}(q^{-1})$  if it does. The pion mass counts as  $\mathcal{O}(q^1)$ , the vector-meson mass as  $\mathcal{O}(q^0)$ , and the width as  $\mathcal{O}(q^1)$ . Vertices generated by the effective Lagrangian of Goldstone bosons  $\mathcal{L}_\pi^{(n)}$  count as  $\mathcal{O}(q^n)$ . Derivatives acting on heavy vector mesons, which cannot be eliminated by field redefinitions, count as  $\mathcal{O}(q^0)$ . While the diagrams with vector meson loops satisfy the power counting after renormalization, the contributions of vector meson loops can systematically be absorbed in the renormalization of the parameters of the effective Lagrangian. Therefore, it is more convenient to drop such loop diagrams and include their contributions in the redefinition of the parameters.

### 4. Evaluation of the two-point function

The mass and width of the  $\rho$  meson are extracted from the complex pole of the two-point function. The undressed propagator of the vector meson reads

$$iS_{0\mu\nu}^{ab}(p) = -i\delta^{ab} \frac{g_{\mu\nu} - \frac{p_\mu p_\nu}{M_R^2}}{p^2 - M_R^2 + i0^+}, \tag{5}$$

with complex  $M_R^2$ . We parameterize the sum of all one-particle-irreducible diagrams contributing to the two-point function as

$$i\Pi_{\mu\nu}^{ab}(p) = i\delta^{ab} [g_{\mu\nu} \Pi_1(p^2) + p_\mu p_\nu \Pi_2(p^2)]. \tag{6}$$

The dressed propagator, expressed in terms of the self energy, has the form

$$iS_{\mu\nu}^{ab}(p) = -i\delta^{ab} \frac{g_{\mu\nu} - p_\mu p_\nu \frac{1 + \Pi_2(p^2)}{M_R^2 + \Pi_1(p^2) + p^2 \Pi_2(p^2)}}{p^2 - M_R^2 - \Pi_1(p^2) + i0^+}. \tag{7}$$

The pole of the propagator is found as the (complex) solution to the following equation:

$$z - M_R^2 - \Pi_1(z) = 0. \tag{8}$$

In the vicinity of the pole  $z$ , the dressed propagator can be written as

$$iS_{\mu\nu}^{ab}(p) = -i\delta^{ab} \left[ \frac{Z_\rho^r (g_{\mu\nu} - \frac{p_\mu p_\nu}{z})}{p^2 - z + i0^+} + R \right], \tag{9}$$

where

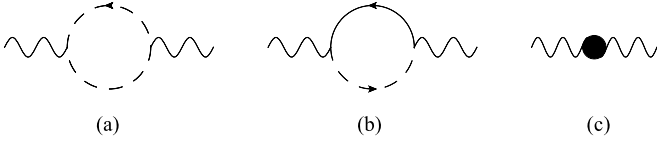
$$Z_\rho^r = \frac{1}{1 - \Pi_1'(z)},$$

and  $R$  stands for the non-pole part. The counterterms  $\delta M_R$  and  $\delta Z_\rho$  are fixed by requiring that, in the chiral limit,  $M_R^2$  is the pole of the dressed propagator and that the residue  $Z_\rho^r$  is equal to one.

The solution to Eq. (8) can be found perturbatively as an expansion

$$z = z^{(0)} + z^{(1)} + z^{(2)} + \dots, \tag{10}$$

where the superscripts ( $i$ ) denote the  $i$ th-loop order. Each of these terms can be expanded in small quantities in the chiral expansion.



**Fig. 1.** One-loop contributions to the  $\rho$ -meson self-energy at  $\mathcal{O}(q^3)$ . The dashed, solid, and wiggly lines correspond to the pion, the  $\omega$  meson, and the  $\rho$  meson, respectively.

Up to and including third chiral order, the tree-order result for  $\Pi_1$  is

$$\Pi_1^{(0)} = c_\chi M^2. \quad (11)$$

At tree order, the pole obtained from Eq. (8) reads

$$z^{(0)} = M_R^2 + c_\chi M^2. \quad (12)$$

The one-loop contributions to the vector self-energy up to  $\mathcal{O}(q^3)$  are shown in Fig. 1. The contributions of diagrams (a) and (b) to  $\Pi_1$  are given by

$$D_a = -\frac{g^2 \mu^{4-n} [2I_M - (p^2 - 4M^2)I_{MM}]}{n-1},$$

$$D_b = \frac{(n-2)g_{\omega\rho\pi}^2 \mu^{4-n}}{4(n-1)} [M^4 I_{MM\omega} - (2I_{MM\omega} M_\omega^2 + I_M - I_{M\omega} + 2I_{MM\omega} p^2) M^2 + I_{MM\omega} p^2 + M_\omega^2 (I_{MM\omega} M_\omega^2 + I_M - I_{M\omega}) - (2I_{MM\omega} M_\omega^2 + I_M + I_{M\omega}) p^2], \quad (13)$$

where the loop integrals are defined as

$$I_{m_1 m_2} = \frac{i}{(2\pi)^n} \int \frac{d^n k}{[k^2 - m_1^2 + i0^+][(p+k)^2 - m_2^2 + i0^+]},$$

$$I_m = \frac{i}{(2\pi)^n} \int \frac{d^n k}{k^2 - m^2 + i0^+}, \quad (14)$$

with  $n$  the space-time dimension and  $p$  the four-momentum of the vector meson.

Due to the large momenta flowing through the  $\rho\pi\pi$  vertex in diagram (a), this vertex should, in principle, count as  $\mathcal{O}(q^0)$ . However, its large component is proportional to  $p^\mu$  and, thus, does not contribute to  $\Pi_1$ . Therefore, the  $\rho\pi\pi$  vertex actually contributes as  $\mathcal{O}(q^1)$ . Hence, diagram (a) contributes to  $\Pi_1$  starting at  $\mathcal{O}(q^4)$ , which is beyond the accuracy of our calculation. Diagram (c) contains the contributions of the counterterms.

Diagram (a) contains a power-counting-violating imaginary part (which is proportional to the  $\rho$ -meson mass for an “on-shell” resonance and hence does not vanish in the chiral limit). It is impossible to cancel this imaginary part by contributions of counterterms unless we use the complex-mass renormalization scheme, where the counterterm contributions become complex quantities. It is this new feature which makes a crucial difference and allows one to solve the power-counting problem for the “on-shell”  $\rho$  meson. In diagram (b) we take  $M_\omega = M_R$  which is a good approximation for the purposes of this work.

We fix the counterterms contributing to the pole of the  $\rho$ -meson propagator such that the pole at chiral limit stays at  $M_R^2$ . This gives:

$$\delta M_R = -\frac{1}{3} g^2 M_R \lambda + \frac{g^2 M_R (-3 \ln \frac{M_R^2}{\mu^2} + 3i\pi + 5)}{288\pi^2} + \frac{1}{3} g_{\omega\rho\pi}^2 M_R^3 \lambda + \frac{g_{\omega\rho\pi}^2 M_R^3 (3 \ln \frac{M_R^2}{\mu^2} + 1)}{288\pi^2},$$

$$\delta c_\chi = 4g^2 \lambda - \frac{g^2 (1 - \ln \frac{M_R^2}{\mu^2} + i\pi)}{8\pi^2} + g_{\omega\rho\pi}^2 M_R^2 \lambda - \frac{g_{\omega\rho\pi}^2 M_R^2 (1 - \ln \frac{M_R^2}{\mu^2})}{32\pi^2}, \quad (15)$$

where

$$\lambda = \frac{1}{16\pi^2} \left\{ \frac{1}{n-4} - \frac{1}{2} [\ln(4\pi) + \Gamma'(1) + 1] \right\}. \quad (16)$$

The contributions of diagrams (a), (b) and (c) to the pole, expanded up to  $\mathcal{O}(q^4)$ , read

$$z^{(1)} = \frac{g^2 M^4}{16\pi^2 M_R^2} \left( 3 - 2 \ln \frac{M^2}{M_R^2} - 2i\pi \right) - \frac{g_{\omega\rho\pi}^2 M^3 M_\chi}{24\pi} - \frac{g_{\omega\rho\pi}^2 M^4 (\ln \frac{M^2}{M_\chi^2} - 1)}{32\pi^2} + \frac{i g_{\omega\rho\pi}^2 M^3 \Gamma_\chi}{48\pi}. \quad (17)$$

As is seen from Eq. (17), the contribution of diagram (c) is indeed of  $\mathcal{O}(q^4)$  within the complex-mass renormalization scheme.

Using the renormalized version of Eq. (3), i.e.,  $M_R^2 = 2g^2 F^2$ , to eliminate  $g^2$  from Eq. (17), we obtain for the pole mass and the width of the  $\rho$  meson to  $\mathcal{O}(q^4)$

$$M_\rho^2 = M_\chi^2 + c_\chi M^2 - \frac{g_{\omega\rho\pi}^2 M^3 M_\chi}{24\pi} + \frac{M^4}{32\pi^2 F^2} \left( 3 - 2 \ln \frac{M^2}{M_\chi^2} \right) - \frac{g_{\omega\rho\pi}^2 M^4 (\ln \frac{M^2}{M_\chi^2} - 1)}{32\pi^2}, \quad (18)$$

$$\Gamma = \Gamma_\chi + \frac{\Gamma_\chi^3}{8M_\chi^2} - \frac{c_\chi \Gamma_\chi M^2}{2M_\chi^2} - \frac{g_{\omega\rho\pi}^2 M^3 \Gamma_\chi}{48\pi M_\chi} + \frac{M^4}{16\pi F^2 M_\chi}. \quad (19)$$

The non-analytic terms of Eq. (18) agree with the corresponding results of Refs. [12,14,26]. Note that both mass  $M_\chi$  and width  $\Gamma_\chi$  in the chiral limit are input parameters in our approach.

To estimate the numerical values of contributions of different orders we substitute

$$F = 0.092 \text{ GeV}, \quad M = 0.139 \text{ GeV},$$

$$g_{\omega\rho\pi} = 16 \text{ GeV}^{-1}, \quad M_\chi \approx M_\rho = 0.78 \text{ GeV}$$

and obtain in units of  $\text{GeV}^2$  and  $\text{GeV}$ , respectively,

$$M_\rho^2 = M_\chi^2 + 0.019c_\chi - 0.0071 + 0.0014 + 0.0013, \\ \Gamma \approx \Gamma_\chi + 0.21\Gamma_\chi^3 - 0.016c_\chi\Gamma_\chi - 0.0058\Gamma_\chi + 0.0011. \quad (20)$$

For pion masses larger than  $M_\rho/2$  the  $\rho$  meson becomes a stable particle. For such values of the pion mass the series of Eq. (19) should diverge.

## 5. Conclusions

To summarize, we have considered an effective field theory of vector mesons interacting with Goldstone bosons using the complex-mass renormalization scheme. A systematic power counting for the momentum region near the  $\rho$  meson pole, emerging within this scheme, allows one to calculate the physical quantities

in powers of small parameters. While we cannot give the general proof, it can, in close analogy with Ref. [33], be demonstrated that properly renormalized multi-loop diagrams satisfy the power counting within the complex mass renormalization scheme. As an application we have calculated the pole mass and the width of the  $\rho$  meson which are of particular interest in the context of lattice extrapolations [25,26]. In the isospin-symmetric limit, we calculated these quantities to  $\mathcal{O}(q^3)$  in terms of the light quark mass and the width of the vector meson in the chiral limit. To estimate the contributions of higher orders we also retained  $\mathcal{O}(q^4)$  terms of  $\mathcal{O}(q^3)$  diagrams.

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### References

- [1] J. Gasser, M.E. Sainio, A. Svarc, Nucl. Phys. B 307 (1988) 779.
- [2] E.E. Jenkins, A.V. Manohar, Phys. Lett. B 255 (1991) 558.
- [3] H.B. Tang, arXiv:hep-ph/9607436.
- [4] T. Becher, H. Leutwyler, Eur. Phys. J. C 9 (1999) 643.
- [5] J. Gegelia, G. Japaridze, Phys. Rev. D 60 (1999) 114038.
- [6] T. Fuchs, J. Gegelia, G. Japaridze, S. Scherer, Phys. Rev. D 68 (2003) 056005.
- [7] E.E. Jenkins, A.V. Manohar, Phys. Lett. B 259 (1991) 353.
- [8] T.R. Hemmert, B.R. Holstein, J. Kambor, J. Phys. G 24 (1998) 1831.
- [9] V. Pascalutsa, D.R. Phillips, Phys. Rev. C 67 (2003) 055202.
- [10] V. Bernard, T.R. Hemmert, U.G. Meißner, Phys. Lett. B 565 (2003) 137.
- [11] C. Hacker, N. Wies, J. Gegelia, S. Scherer, Phys. Rev. C 72 (2005) 055203.
- [12] E.E. Jenkins, A.V. Manohar, M.B. Wise, Phys. Rev. Lett. 75 (1995) 2272.
- [13] J. Bijnens, P. Gosdzinsky, Phys. Lett. B 388 (1996) 203.
- [14] J. Bijnens, P. Gosdzinsky, P. Talavera, Nucl. Phys. B 501 (1997) 495.
- [15] J. Bijnens, P. Gosdzinsky, P. Talavera, JHEP 9801 (1998) 014.
- [16] J. Bijnens, P. Gosdzinsky, P. Talavera, Phys. Lett. B 429 (1998) 111.
- [17] T. Fuchs, M.R. Schindler, J. Gegelia, S. Scherer, Phys. Lett. B 575 (2003) 11.
- [18] M.R. Schindler, J. Gegelia, S. Scherer, Phys. Lett. B 586 (2004) 258.
- [19] R.G. Stuart, in: J. Tran Thanh Van (Ed.),  $Z^0$  Physics, Editions Frontieres, Gif-sur-Yvette, 1990, p. 41.
- [20] A. Denner, S. Dittmaier, M. Roth, D. Wackeroth, Nucl. Phys. B 560 (1999) 33.
- [21] A. Denner, S. Dittmaier, Nucl. Phys. B (Proc. Suppl.) 160 (2006) 22.
- [22] A. Denner, S. Dittmaier, M. Roth, L.H. Wieders, Nucl. Phys. B 724 (2005) 247.
- [23] S. Actis, G. Passarino, Nucl. Phys. B 777 (2007) 100.
- [24] S. Actis, G. Passarino, C. Sturm, S. Uccirati, Phys. Lett. B 669 (2008) 62.
- [25] D.B. Leinweber, T.D. Cohen, Phys. Rev. D 49 (1994) 3512.
- [26] D.B. Leinweber, A.W. Thomas, K. Tsushima, S.V. Wright, Phys. Rev. D 64 (2001) 094502.
- [27] P.C. Bruns, U.-G. Meißner, Eur. Phys. J. C 40 (2005) 97.
- [28] P.C. Bruns, U.-G. Meißner, Eur. Phys. J. C 58 (2008) 407.
- [29] G. Ecker, J. Gasser, H. Leutwyler, A. Pich, E. de Rafael, Phys. Lett. B 223 (1989) 425.
- [30] D. Djukanovic, M.R. Schindler, J. Gegelia, G. Japaridze, S. Scherer, Phys. Rev. Lett. 93 (2004) 122002.
- [31] K. Kawarabayashi, M. Suzuki, Phys. Rev. Lett. 16 (1966) 255.
- [32] Riazuddin, Fayyazuddin, Phys. Rev. 147 (1966) 1071.
- [33] M.R. Schindler, J. Gegelia, S. Scherer, Nucl. Phys. B 682 (2004) 367.