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An approximate solution to a moving boundary problem with space–time fractional derivative in fluvio-deltaic sedimentation process

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Abstract A mathematical model of the movement of the shoreline in a sedimentary ocean basin is discussed. The model includes space–time fractional derivative in Caputo sense and variable latent heat term. An approximate solution of the problem is obtained by Adomian decomposition method and the results thus obtained are compared graphically with an exact solution of integer order ($\beta = 1$, $\alpha = 1$). Three particular cases, the standard diffusion, the time-fractional and the space-fractional diffusions are also discussed. The model and solution are generalization of previous works.

1. Introduction

An interesting moving boundary problem in the field of earth surface science involves the movement of the shoreline in a sedimentary ocean basin (a shoreline problem). The classical diffusion transport models [1–3] provide a reliable means of modeling the sediment transport in fluvial depositional systems. The assumptions of the classical diffusion equation are thin-tailed periods of inactivity and thin-tailed transport distances for sediment particles. From the literature [4–7], the deviation from normal (Fickian) diffusion in sediment tracer dispersion is observed that violates the assumption of statistical convergence to a Gaussian. Therefore, the fractional diffusion equations are widely used for the investigation of the mechanism of anomalous diffusion in transport processes through complex and/or disordered systems including fractal media [8,9]. It is well known that fractional derivative is a good tool for taking into account memory mechanism, particularly in some diffusive processes [10]. Both space and time fractional operators correspond to the diffusion limit of continuous time random walk models with long-tailed waiting time and/or jump length distributions [10,11]. Li et al. [12] used Caputo derivative $\beta \in (0,1]$ and Riesz-Feller derivative $\alpha \in (0,2]$ operators for the first order time derivative and second order space derivative, respectively and presented an analytic solution to fractional form of a moving boundary problem in drug release devices in term of Fox H function. Voller [13] presented

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Nomenclature

h	height of the earth's crust (basement) above datum, m
x	space variable, m
t	time
$z(t)$	ocean level above datum, m
$s(t)$	shoreline position, m
$u(t)$	position of intersection between offshore sediment wedge and basement, m
q	prescribed sediment line flux, $m^3\,m^{-1}\,t^{-1}$
a	slope of off-shore sediment wedge
b	slope of basement

Greek symbols

η	height of sediment above datum, m
γ	a constant
ν	diffusion coefficient, $m^2\,t^{-1}$
α	fractional order of space derivative
β	fractional order of time derivative

fractional (non-integer) form of a limit Stefan problem using Caputo derivatives for both space and time, and discussed exact solution of the problem. Recently, some researchers [14–17] also discussed various mathematical models governed with different fractional derivatives for both the space and time.

The most commonly used definitions in mathematical models are the Riemann–Liouville and Caputo. Riemann–Liouville fractional derivative requires initial conditions to be expressed in terms of fractional integrals and their derivatives which have no obvious physical interpretation. So, Riemann–Liouville fractional derivative is not always the most convenient definition for real applications [18]. However, Caputo fractional derivative requires the initial conditions (including the mixed boundary conditions) in the same form as that of ordinary differential equations with integer derivatives [18]. These integer-order derivatives represent well-understood features of a physical situation and therefore their values can be measured accurately. Another advantage is that the Caputo derivative of a constant is zero, whereas the Riemann–Liouville fractional derivative of a constant is not zero. Therefore, it is interesting and applicable to use Caputo fractional derivative in diffusion model of sediment transport on earth surface. It can be seen in [19,20] that a pure power-law, heavy-tailed probability density function for the periods of inactivity without any truncation leads to a time-fractional diffusion equation which describes the evolution of surface elevation in time. Voller and Paola [21] presented the deviation of fluvial profiles from ones predicted by classical diffusion and proposed the exploration of fractional diffusive model to describe the observed steady-state fluvial profiles in a depositional system. Ganti et al. [22] discussed time fractional diffusion model for the surface dynamics of depositional systems by considering the fact that the periods of inactivity are heavy-tailed. They also discussed physical mechanisms constrain the occurrence of extremes in depositional systems and how these constraints reflect in the probability distributions of the random variables. They also presented that preliminary thoughts on continuum models for surface evolution of depositional systems are consistent with the documented probability distributions for erosional, depositional and inactivity events. Martin et al. [23] also discussed the physical basis for anomalous diffusion in bed load transport. Rajeev and Kushwaha [24] also discussed a mathematical model with time-fractional derivative for a moving boundary problem which occurs in sedimentation process. These models motivate to discuss space–time fractional diffusion model in sedimentation process to study the physical effect in complex domain.

The diffusion equation with a moving boundary (moving boundary problem) is a special nonlinear problem which is difficult to get the exact solution [25,26]. Hence, many approximate and numerical methods have been used to solve the moving boundary problems [27–33]. The approximate analytical approach taken in this literature is Adomian decomposition method (ADM). Adomian decomposition method was developed by Adomian [34–36] and has been applied to solve a wide class of non-linear differential and partial differential equations [37,38]. Grzymkowski and Slota [39] presented the solution of One-phase inverse Stefan problem by Adomian decomposition method. Das and Rajeev [29] also used and Adomian decomposition method to solve time-fractional diffusion equation with a moving boundary condition which is related to the diffusional release of a solute from a polymer matrix in which the initial loading is higher/lower than the solubility.

In this study, we consider the non-classical or non-Fickian, anomalous sediment transport in braided networks. Our attention in this paper is to discuss a moving boundary problem governed by fractional space–time derivative in Caputo sense which arises during the movement of the shoreline in a sedimentary ocean basin. The main physical purpose for adopting and investigating diffusion equations with fractional space–time derivative is to describe phenomena of anomalous (non-Fickian) sediment transport through complex and/or disordered systems including fractal media which occurs in sedimentation process. Adomian decomposition method is successfully applied to solve the proposed problem. The obtained results are compared with the existing exact solutions.

2. The fluvio-deltaic sedimentation model

Fluvio-deltaic sedimentation problem involves the shoreline propagation in a sedimentary ocean basin due to a sediment line flux, tectonic subsidence of the earth's crust, and sea level change. The mathematical model of fluvio-deltaic sedimentation process is discussed in [1–3]. In this paper, we consider a fixed line flux, a constant ocean level ($z = 0$), no tectonic subsidence of the earth's crust, and a constant sloping basement $b < a$. This scenario is a reasonable approximation for some modern continental margins. A schematic cross section of such a basin indicating the variables is revealed in Fig. 1 [2]. Under this limit case, the dynamics of the sedimentation process become a moving boundary problem with variable latent heat (see in [2]) which is as follows

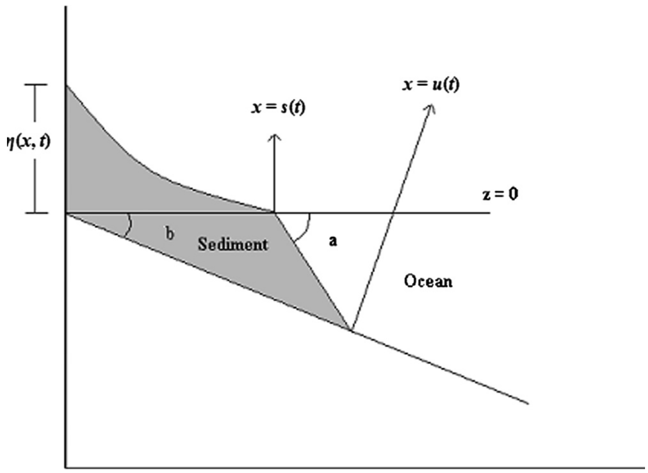

Figure 1 A schematic cross section of a basin with no tectonic subsidence and sea level change.

$$\frac{\partial \eta}{\partial t} = v\frac{\partial^2 \eta}{\partial x^2}, \quad 0 < x < s(t) \tag{1}$$

with initial and boundary conditions

$$v\frac{\partial \eta}{\partial x}\bigg|_{x=0} = -q(t) \tag{2}$$

and

$$\eta(s,t) = 0 \tag{3}$$

where $\eta(x,t)$ is height of sediment above datum, v is a diffusion coefficient, $q(t)$ is the time-dependent sediment line flux and $s(t)$ is the moving contact point (moving interface).

The additional conditions on the moving interface are

$$-v\frac{\partial \eta}{\partial x}\bigg|_{x=s(t)} = \gamma s\frac{ds}{dt} \tag{4}$$

and

$$s(0) = 0 \tag{5}$$

where $a(u-s) = \frac{abs}{a-b} = \gamma s$.

3. The fractional model

In order to describe phenomena of anomalous (non-Fickian) sediment transport through complex and/or disordered systems including fractal media, we consider above moving boundary problem with fractional space–time derivatives. Using Caputo fractional derivatives for both space and time as given in [13], a space–time fractional form of the Eqs. (1)–(5) can be described as follows:

$$D_t^{\beta}\eta(x,t) = v\frac{\partial}{\partial x}\left(D_x^{\alpha}\eta(x,t)\right) \quad (0 < x < s(t),\ 0 < \alpha,\ \beta \leqslant 1) \tag{6}$$

with the following posed conditions

$$vD_x^{\alpha}\eta(0,t) = -q \tag{7}$$

$$\eta(s,t) = 0. \tag{8}$$

The additional conditions on the moving interface are

$$-vD_x^{\alpha}\eta(s(t),t) = \gamma s D_t^{\beta}s(t) \tag{9}$$

and

$$s(0) = 0 \tag{10}$$

where $a(u-s) = \frac{abs}{a-b} = \gamma s$ and q is prescribed sediment line flux that is considered as a constant. The operator, Caputo fractional derivative [13] is defined as

$$D_t^{\beta}f(t) = D_t^{\beta-n}[f^{(n)}(t)] = \frac{1}{\Gamma(n-\beta)}\int_0^t \frac{f^{(n)}(\tau)}{(t-\tau)^{\beta+1-n}}d\tau \quad (n-1 < \beta \leqslant n,\ n \in N)$$

and

$$D_t^{-\beta}f(t) = \int_0^t \frac{(t-\tau)^{\beta-1}}{\Gamma(\beta)}f(\tau)d\tau \quad (\beta > 0)$$

where Γ (.) is the Gamma function. In this paper, following properties of fractional derivative [13,30] are used

(i) the Caputo derivative of a constant C is

$$D^{\beta}C = 0$$

(ii) for any real value $m \geqslant 1 \geqslant \beta > 0$ or $0 < m \leqslant \beta \leqslant 1$

$$D_t^{\beta}t^m = \frac{\Gamma(1+m)}{\Gamma(1+m-\beta)}t^{m-\beta} \tag{11}$$

In particular, if $\beta > 0$.
$D_t^{\beta}t^{\beta+1} = \Gamma(\beta+2)t$ and $D_t^{\beta}t^{\beta} = \Gamma(\beta+1)$.

4. Solution of the problem by Adomian decomposition method

We first write the Eq. (6) in operator form

$$L_{xx}\eta(x,t) = D_x^{1-\alpha}\left(\frac{1}{v}\frac{\partial^{\beta}\eta(x,t)}{\partial^{\beta}t}\right) \tag{12}$$

where $L_{xx} = \frac{\partial^2}{\partial x^2}$.

Assuming that the inverse operator L_{xx}^{-1} exists and

$$L_{xx}^{-1}(.) = \int_0^x \int_0^x (.)dx\,dx$$

Applying the inverse operator L_{xx}^{-1} on the both side of Eq. (12)

$$\eta(x,t) - \eta(0,t) = L_{xx}^{-1}\left(D_x^{1-\alpha}\left(\frac{1}{v}\frac{\partial^{\beta}\eta(x,t)}{\partial^{\beta}t}\right)\right) \tag{13}$$

Choosing the following initial approximations of $\eta(x,t)$ and $s(t)$ as given in [29]

$$\eta_0 = c\left(s_0^{\alpha} - x^{\alpha}\right)$$

where $c = \frac{q}{v\Gamma(1+\alpha)}$

and $s_0 = a_0 t^{\beta/2}$

where $a_0 = \left(\frac{cv}{\gamma}\frac{\Gamma(1-\beta/2)\Gamma(1+\alpha)}{\Gamma(1+\beta/2)}\right)^{1/2}$.

According to the Adomian decomposition method [34], decomposing the unknown function $\eta(x,t)$ as follows:

$$\eta(x,t) = \eta_0 + \eta_1 + \eta_2 + \cdots \tag{14}$$

From the Eqs. (13) and (14), the components $\eta_0, \eta_1, \eta_2 \ldots$ are recursively determined by

$$\eta_0 = \eta(0,t) = c\left(s_0^{\alpha} - x^{\alpha}\right)$$

$$\eta_1 = L_{xx}^{-1}\left(D_x^{1-\alpha}\left(\frac{1}{v}\frac{\partial^\beta \eta_0(x,t)}{\partial^\beta t}\right)\right) = \frac{ca_0^\alpha}{v}\frac{\Gamma\left(1+\frac{\alpha\beta}{2}\right)}{\Gamma\left(1+\frac{\alpha\beta}{2}-\beta\right)\Gamma(\alpha+2)}t^{\frac{\alpha\beta}{2}-\beta}x^{\alpha+1}$$

$$\eta_2 = L_{xx}^{-1}\left(D_x^{1-\alpha}\left(\frac{1}{v}\frac{\partial^\beta \eta_1(x,t)}{\partial^\beta t}\right)\right) = \frac{ca_0^\alpha}{v^2}\frac{\Gamma\left(1+\frac{\alpha\beta}{2}\right)}{\Gamma\left(1+\frac{\alpha\beta}{2}-2\beta\right)\Gamma(2\alpha+3)}t^{\frac{\alpha\beta}{2}-2\beta}x^{2\alpha+2}$$

$$\eta_3 = L_{xx}^{-1}\left(D_x^{1-\alpha}\left(\frac{1}{v}\frac{\partial^\beta \eta_2(x,t)}{\partial^\beta t}\right)\right) = \frac{ca_0^\alpha}{v^3}\frac{\Gamma\left(1+\frac{\alpha\beta}{2}\right)}{\Gamma\left(1+\frac{\alpha\beta}{2}-3\beta\right)\Gamma(3\alpha+4)}t^{\frac{\alpha\beta}{2}-3\beta}x^{3\alpha+3}$$

$$\eta_4 = L_{xx}^{-1}\left(D_x^{1-\alpha}\left(\frac{1}{v}\frac{\partial^\beta \eta_3(x,t)}{\partial^\beta t}\right)\right) = \frac{ca_0^\alpha}{v^4}\frac{\Gamma\left(1+\frac{\alpha\beta}{2}\right)}{\Gamma\left(1+\frac{\alpha\beta}{2}-4\beta\right)\Gamma(4\alpha+5)}t^{\frac{\alpha\beta}{2}-4\beta}x^{4\alpha+4}$$

$$\vdots$$

and so on. Thus,

$$\eta(x,t) = \eta_0+\eta_1+\eta_2+\cdots = \sum_{n=0}^{\infty}\frac{c(a_0)^\alpha}{v^n}\frac{\Gamma\left(1+\frac{\alpha\beta}{2}\right)}{\Gamma\left(1+\frac{\alpha\beta}{2}-n\beta\right)\Gamma(n\alpha+n+1)}t^{\frac{\alpha\beta}{2}-n\beta}x^{n(\alpha+1)} - cx^\alpha \quad (15)$$

which give height of the sediment above the datum.

Now, using (15) and writing the interface condition (9) in operator form

$$s(t) = s_0 - D_t^{-\beta}(F(s)) \quad (16)$$

where initial approximation $s_0 = a_0 t^{\frac{\beta}{2}}$ and

$$F(s) = -\frac{v}{\gamma s(t)}\frac{\partial^\alpha}{\partial x^\alpha}(\eta(s(t),t)) = -\frac{cv}{\gamma}\left(\frac{-\Gamma(1+\alpha)}{s(t)} + \frac{a_0^\alpha}{v}\frac{\Gamma\left(1+\frac{\alpha\beta}{2}\right)}{\Gamma\left(1+\frac{\alpha\beta}{2}-\beta\right)}t^{\frac{\alpha\beta}{2}-\beta} + \frac{a_0^\alpha}{v^2}\frac{(s(t))^{\alpha+1}\Gamma\left(1+\frac{\alpha\beta}{2}\right)}{\Gamma\left(1+\frac{\alpha\beta}{2}-2\beta\right)\Gamma(\alpha+3)}t^{\frac{\alpha\beta}{2}-2\beta} + \frac{a_0^\alpha}{v^3}\frac{(s(t))^{2\alpha+2}\Gamma\left(1+\frac{\alpha\beta}{2}\right)}{\Gamma\left(1+\frac{\alpha\beta}{2}-3\beta\right)\Gamma(2\alpha+4)}t^{\frac{\alpha\beta}{2}-3\beta} + \ldots\ldots\ldots\ldots\right)$$

Accordingly [36], decomposing $s(t)$ as

$$s(t) = \sum_{n=0}^{\infty} s_n \quad (17)$$

Using (16) and (17), we have

$$\sum_{n=0}^{\infty} s_n = s_0 - D_t^{-\beta}\left(\sum_{n=0}^{\infty} A_n\right) \quad (18)$$

where A_n are so-called Adomian polynomials for non-linear terms and defined as

$$A_0 = F(s_0)$$

$$A_1 = s_1 F'(s_0)$$

$$A_2 = s_2 F'(s_0) + \frac{1}{2}s_1^2 F''(s_0)$$

and so on. The components of $s_n(t)$, $n \geqslant 1$, can be completely determined as

$$s_1 = D_t^{-\beta}(A_0)$$

where

$$A_0 = -\frac{cv}{\gamma}\left(\frac{-\Gamma(1+\alpha)}{s_0} + \frac{a_0^\alpha}{v}\frac{\Gamma\left(1+\frac{\alpha\beta}{2}\right)}{\Gamma\left(1+\frac{\alpha\beta}{2}-\beta\right)}t^{\frac{\alpha\beta}{2}-\beta} + \frac{a_0^\alpha}{v^2}\frac{(s_0)^{\alpha+1}\Gamma\left(1+\frac{\alpha\beta}{2}\right)}{\Gamma\left(1+\frac{\alpha\beta}{2}-2\beta\right)\Gamma(\alpha+3)}t^{\frac{\alpha\beta}{2}-2\beta} + \frac{a_0^\alpha}{v^3}\frac{(s_0)^{2\alpha+2}\Gamma\left(1+\frac{\alpha\beta}{2}\right)}{\Gamma\left(1+\frac{\alpha\beta}{2}-3\beta\right)\Gamma(2\alpha+4)}t^{\frac{\alpha\beta}{2}-3\beta} + \cdots\right)$$

$$s_2 = D_t^{-\beta}(A_1)$$

where

$$A_1 = s_1\frac{d}{ds_0}\left[-\frac{cv}{\gamma}\left(\frac{-\Gamma(1+\alpha)}{s_0} + \frac{a_0^\alpha}{v}\frac{\Gamma\left(1+\frac{\alpha\beta}{2}\right)}{\Gamma\left(1+\frac{\alpha\beta}{2}-\beta\right)}t^{\frac{\alpha\beta}{2}-\beta} + \frac{a_0^\alpha}{v^2}\frac{(s_0)^{\alpha+1}\Gamma\left(1+\frac{\alpha\beta}{2}\right)}{\Gamma\left(1+\frac{\alpha\beta}{2}-2\beta\right)\Gamma(\alpha+3)}t^{\frac{\alpha\beta}{2}-2\beta} + \frac{a_0^\alpha}{v^3}\frac{(s_0)^{2\alpha+2}\Gamma\left(1+\frac{\alpha\beta}{2}\right)}{\Gamma\left(1+\frac{\alpha\beta}{2}-3\beta\right)\Gamma(2\alpha+4)}t^{\frac{\alpha\beta}{2}-3\beta} + \cdots\right)\right]$$

and so on. Therefore, approximate analytical solution of $s(t)$ is given by

$$s(t) = s_0 + s_1 + s_2 + \cdots \quad (19)$$

5. Numerical comparison and discussion

In this section, numerical results for height of sediment $\eta(x,t)$ and shoreline positions $s(t)$ are calculated using MATHEMATICA software and depicted through figures. The solution of the problem is discussed in detail by considering three particular cases:

Case1. When $\alpha = 1$, $\beta = 1$, the Eqs. (6)–(10) reduce to the Eqs. (1)–(5) which is standard moving boundary problem. In order to show the accuracy of the proposed approximate solution, we compare it with the existing exact solution for integer order given by Voller et al. [2]. Figs. 2 and 3 represent the dependence of height of sediment $\eta(x,t)$ on space x for standard moving boundary problem ($\alpha = 1$, $\beta = 1$) at the fixed value of diffusion coefficient ($v = 2.0$), sediment line flux ($q = 0.5$) and time $t = 3.0$ for $\gamma = 10$ and $\gamma = 15$, respectively. Figs. 4 and 5 depict the dependence of the shoreline position on time at the fixed value of

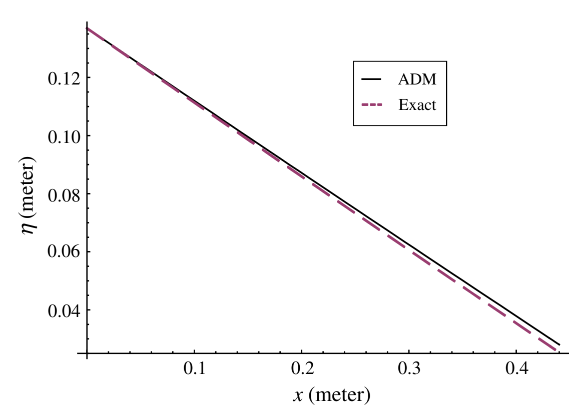

Figure 2 Plot of $\eta(x,t)$ vs. x for $q = 0.5$, $v = 2.0$ and $\gamma = 10$.

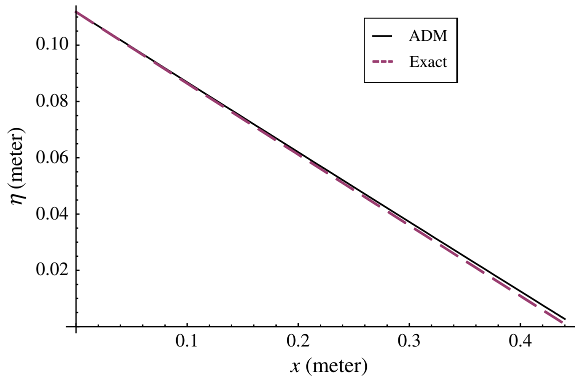

Figure 3 Plot of $\eta(x,t)$ vs. x for $q = 0.5$, $v = 2.0$ and $\gamma = 15$.

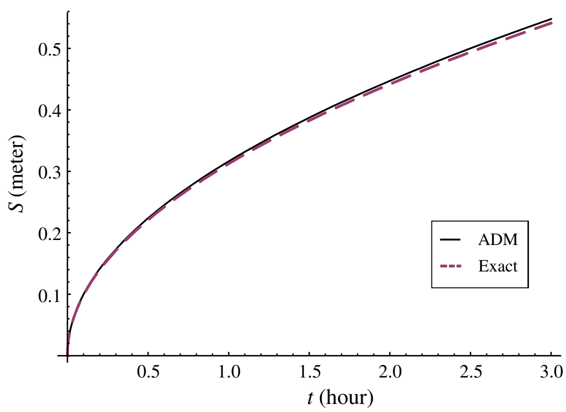

Figure 4 Plot of $s(t)$ vs. t for $q = 0.5$, $v = 2.0$ and $\gamma = 10$.

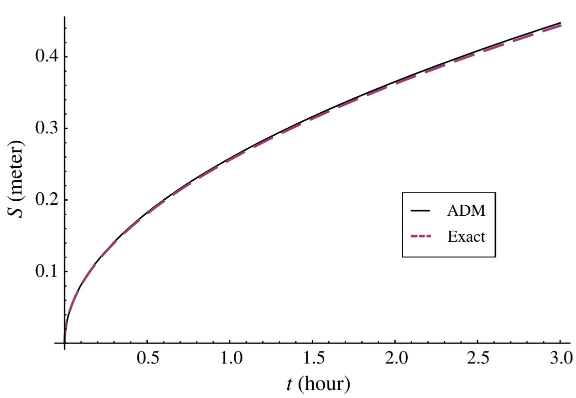

Figure 5 Plot of $s(t)$ vs. t for $q = 0.5$, $v = 2.0$ and $\gamma = 15$.

diffusion coefficient ($v = 2.0$) and sediment line flux ($q = 0.5$) for $\gamma = 10$ and $\gamma = 15$, respectively. It can be seen from Figs. 2–5 that the proposed approximate solution is close to the exact solution. Moreover, it is clear from Figs. 4, 5 that the movement of shoreline position decreases as the value of γ increases. In this case the sedimentation process becomes slow and the sediments will be deposited towards the land side which causes the increase of the thickness of earlier sediments. As a consequence of this there will be least shifting of the contact point towards the land side and sedimentation process will be slower.

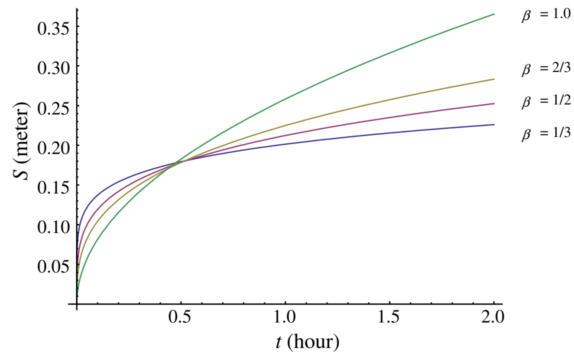

Figure 6 Plot of $s(t)$ vs. t for $q = 0.5$, $v = 2$ and $\gamma = 15$.

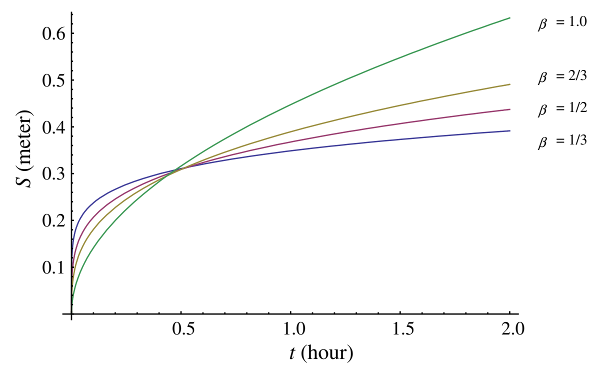

Figure 7 Plot of $s(t)$ vs. t for $q = 1.5$, $v = 2$ and $\gamma = 15$.

Case 2. When $\alpha = 1, 0 < \beta < 1$, the Eqs. (6)–(10) degenerate into a moving boundary problem governed with time-fractional derivative. Figs. 6 and 7 explain the dependence of shoreline position on time for different Brownian motion $\beta = \frac{1}{3}, \frac{1}{2}, \frac{2}{3}$, and also for the standard motion $\beta = 1.0$ at the fixed value of $v = 2.0$, $\gamma = 15$ and $\alpha = 1$. It is observed from Figs. 6, 7 that the rate of increase of $s(t)$ decreases with the increase of β which confirms the exponential decay of regular Brownian motion. This result is in good agreement with the result of Das and Rajeev [28].

Case 3. When $0 < \alpha < 1, \beta = 1$, the proposed problem becomes a moving boundary problem with space-fractional derivatives. Figs. 8 and 9 show the plot of shoreline position

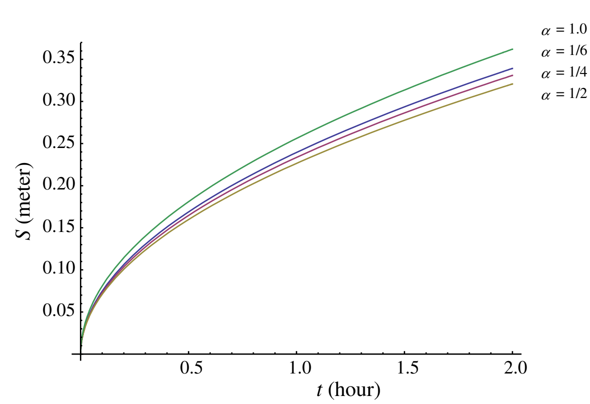

Figure 8 Plot of $s(t)$ vs. t for $q = 0.5$, $v = 2$ and $\gamma = 15$.

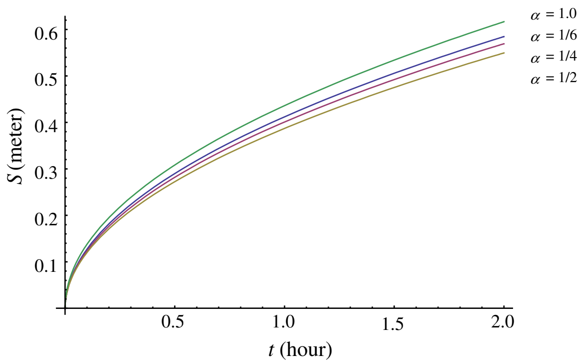

Figure 9 Plot of $s(t)$ vs. t for $q = 1.5$, $v = 2$ and $\gamma = 15$.

$s(t)$ on time for different values of$\alpha = \frac{1}{6}, \frac{1}{4}, \frac{1}{2}$ and 1 at the fixed value of $v = 2.0$ and $\gamma = 15$ for $\beta = 1$.

It can be seen from the Figs. 6–9 that if the sediment line flux q increases($q = 0.5$, 1.5), the movement of the contact point (shoreline position) increases towards sea side with formation of inclined strata along the off-shore sediment wedge. This conclusion show the fact that the models are well consistent with truth. Figs. 6–9 also show that trajectory of the movement of contact point deviates more from standard motion for the case of time fractional than space fractional case during sedimentation process.

6. Conclusion

In this work, we discussed a mathematical model governed by space–time fractional derivative in Caputo sense for a moving boundary problem which occurs in fluvio-deltaic sedimentation process on earth surface. The solution of the proposed problem is obtained by Adomian decomposition method (ADM). It is found that sedimentation process becomes slow as the value of γ increases and sedimentation process becomes fast as the sediment line flux increases for standard as well as fractional Brownian motion. It is observed that time fractional is more pronounced than space fractional during sedimentation process. Moreover, it is seen that ADM is a powerful and accurate method for finding the solution of moving boundary problem. It is straight forward and avoids the hectic work of calculations. The author believes that the procedure as described in the present study will considerably benefit to engineers and scientists working in this field.

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