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Relativistic generalizations of gravity-induced localization models

Sergio De Filippo [a,b], Filippo Maimone [a,b]

[a] Dipartimento di Fisica "E.R. Caianiello", Università di Salerno, Via Allende, I-84081 Baronissi (SA), Italy
[b] Unità INFM, INFN, Salerno, Italy

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Abstract

Nonunitary versions of Newtonian gravity leading to wavefunction localization admit natural special-relativistic generalizations. They include the first consistent relativistic localization models. At variance with the unified model of localization and gravity, the purely localizing version requires negative energy fields, which however are less harmful than usual and can be used to build ultraviolet-finite theories.

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Despite many claims according to which environment induced decoherence has solved the measurement problem in quantum mechanics, the issue is not yet settled [1]. One of the proposals to cope with the measurement problem is the modification of the evolution law in such a way to get the emergence of classicality even in closed systems [2–7]. Of course the modified dynamics must comply with strict constraints, imposed by the huge amount of experimental data consistent with the ordinary unitary, linear and deterministic evolution law generated by the Hamiltonian operator. Although it was shown that such constraints can be met by adding nonlinear stochastic terms to the ordinary Schroedinger equation, that was achieved at some expense. First, the proposed models require the introduction of phenomenological constants, which should be fitted by future experiments. Secondly, as observed by John Bell, who considered the main idea behind these models as a viable one, the special role assigned to position requires a smearing on space, which makes it quite problematic to find relativistic generalizations [6–8].

On the other hand the analysis of the possibility that the localization of macroscopic bodies is an unavoidable effect of gravity has a long history [9,10]. That idea led to the introduction of localization models inspired by gravity, with the unattained aim of getting rid of the mentioned free parameters [11,12]. It should also be mentioned that a strong support to the idea that gravity may imply a nonunitary generalization of quantum dynamics came from the emergence of the information loss paradox within black hole physics [13,14].

E-mail addresses: defilippo@sa.infn.it (S. De Filippo), maimone@sa.infn.it (F. Maimone).

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In some recent papers it was shown that suitable nonunitary modifications of Newtonian gravity lead to localization models without any free parameter [15–17]. While for previous nonunitary models inspired by black hole dynamics the basic idea is to have the given system interacting with a "hidden system" with "no energy of its own and therefore... not... available as either a net source or a sink of energy" [18], in the present models energy conservation is granted by the "hidden system" being a copy of the physical system, coupled to it only by gravity, and constrained in its same state and then with its same energy. The unitary dynamics and the states referred to the doubled operator algebra are what we call meta-dynamics and meta-states, while, by tracing out the hidden degrees of freedom, we get the nonunitary dynamics of the physical states. Pure physical states correspond then to meta-states without entanglement between physical and hidden degrees of freedom.

The hint that gravity may induce a nonunitary evolution came long ago even from the perturbative analysis of Einstein gravity leading to the emergence of higher order theories, which however are either nonunitary or plagued by ghosts [19,20]. An optimistic conclusion is that "the S matrix will be nearly unitary" [19]. In Ref. [21] a remedy for the ghost problem, leading to a nonunitary theory, was suggested by a redefinition of the Euclidean path integral. A different approach in real space–time was proposed in Refs. [22,23], thus avoiding analytical continuation, which amounts to a tricky operation outside the realm of a fixed flat geometry. As in Ref. [21], classical instability is cured at the expense of unitarity and the ensuing theory singles out one of the mentioned modifications of Newtonian gravity as its Newtonian limit. Of course the fully relativistic model may present the usual problems ensuing from the consideration of a general covariant theory of gravity within a quantum context.

In this Letter we want to prove that there are no fundamental obstructions to the building of relativistic localization models, by showing that the mentioned nonrelativistic models have natural special-relativistic generalizations, leading to the first well-defined localization models, both relativistic and without free parameters.

In particular the analysis of the possible relativistic extensions sheds some light on the ubiquitous presence of a divergent injection of energy in the previous attempts [6,7]. Within the field theoretic setting of the relativistic models presented here an uncontrollable energy injection may occur only in the presence of negative energy fields. On the other hand such fields are unavoidable within our approach only if one requires that the localizing interaction averages out to zero. On the contrary, if one accepts that the localizing interaction includes an average effect, which in the nonrelativistic limit corresponds to the ordinary Newtonian interaction, negative energy fields can be avoided.

To be specific, let $H_0[\psi^\dagger, \psi]$ be the second quantized form of an ordinary matter Hamiltonian in the absence of gravity. To define the nonunitary Newtonian limit of the general covariant model [23], we introduce a (meta-)matter algebra that is the product of two equivalent copies of the observable matter algebra, respectively generated by the $\psi^\dagger, \psi$ and $\tilde{\psi}^\dagger, \tilde{\psi}$ operators and a meta-Hamiltonian

$$\begin{aligned} H_G = {} & H_0[\psi^\dagger, \psi] + H_0[\tilde{\psi}^\dagger, \tilde{\psi}] - \frac{G}{2}\sum_{j,k} m_j m_k \int dx\, dy\, \frac{\psi_j^\dagger(x)\psi_j(x)\tilde{\psi}_k^\dagger(y)\tilde{\psi}_k(y)}{|x-y|} \\ & - \frac{G}{4}\sum_{j,k} m_j m_k \int dx\, dy\, \frac{\psi_j^\dagger(x)\psi_j(x)\psi_k^\dagger(y)\psi_k(y)}{|x-y|} \\ & - \frac{G}{4}\sum_{j,k} m_j m_k \int dx\, dy\, \frac{\tilde{\psi}_j^\dagger(x)\tilde{\psi}_j(x)\tilde{\psi}_k^\dagger(y)\tilde{\psi}_k(y)}{|x-y|}, \end{aligned} \tag{1}$$

acting on the product $F_\psi \otimes F_{\tilde{\psi}}$ of the Fock spaces of the ψ and $\tilde{\psi}$ operators. Here two couples of meta-matter operators $\psi_j^\dagger, \psi_j$ and $\tilde{\psi}_j^\dagger, \tilde{\psi}_j$ appear for every particle species and spin component, while m_j is the mass of the jth particle species. The $\tilde{\psi}_j$ operators obey the same statistics as the corresponding operators ψ_j, while $[\psi, \tilde{\psi}]_- = [\psi, \tilde{\psi}^\dagger]_- = 0$.

The meta-state space S is defined by a symmetry constraint as the subspace of $F_\psi \otimes F_{\tilde{\psi}}$ including the meta-states obtained from the vacuum $\|0\rangle\rangle = |0\rangle_\psi \otimes |0\rangle_{\tilde{\psi}}$ by applying operators built in terms of the products $\psi_j^\dagger(x)\tilde{\psi}_j^\dagger(y)$

and symmetrical with respect to the interchange $\psi^\dagger \leftrightarrow \tilde{\psi}^\dagger$, which, then, have the same number of ψ and $\tilde{\psi}$ meta-particles of each species. As the observable algebra is identified with the ψ operator algebra, expectation values can be evaluated by preliminarily tracing out the $\tilde{\psi}$ operators.

It was shown that the ensuing nonunitary dynamics, while embodying the ordinary Newton interaction, gives rise to a dynamical localization that is compatible both with the wavelike behavior of microscopic particles and the emergence of classicality for macroscopic bodies [15–17,23,24].

In an interaction representation, where the free meta-Hamiltonian is $H_0[\psi^\dagger,\psi] + H_0[\tilde{\psi}^\dagger,\tilde{\psi}]$, if for simplicity we refer to one particle species only and $\psi^\dagger\psi$ denotes a quadratic scalar expression, the time evolution of a generic meta-state $\|\tilde{\Phi}(0)\rangle\rangle$ is represented by

$$\begin{aligned}\big\|\tilde{\Phi}(t)\big\rangle\big\rangle &= T\exp\frac{i}{\hbar}Gm^2\int dt\int dx\,dy\\ &\quad\times\left[\frac{\psi^\dagger(x)\psi(x)\psi^\dagger(y)\psi(y)}{4|x-y|}+\frac{\tilde{\psi}^\dagger(x)\tilde{\psi}(x)\tilde{\psi}^\dagger(y)\tilde{\psi}(y)}{4|x-y|}+\frac{\psi^\dagger(x,t)\psi(x,t)\tilde{\psi}^\dagger(y,t)\tilde{\psi}(y,t)}{2|x-y|}\right]\\ &\quad\times\big\|\tilde{\Phi}(0)\big\rangle\big\rangle\\ &\equiv U(t)\big\|\tilde{\Phi}(0)\big\rangle\big\rangle.\end{aligned}\tag{2}$$

Then, by a Stratonovich–Hubbard transformation [25], we can rewrite $U(t)$ as a functional integral over an auxiliary real scalar field φ:

$$\begin{aligned}U(t) &= \int D[\varphi]\exp\frac{ic^2}{2\hbar}\int dt\,dx\,\big[\varphi\nabla^2\varphi\big]\\ &\quad\times T\exp\left[-i\frac{mc}{\hbar}\sqrt{2\pi G}\int dt\,dx\,\varphi(x,t)\big(\psi^\dagger(x,t)\psi(x,t)+\tilde{\psi}^\dagger(x,t)\tilde{\psi}(x,t)\big)\right].\end{aligned}\tag{3}$$

This allows, in particular, to obtain, by tracing out the hidden degrees of freedom, an expression for the physical state ρ_{Ph} evolving from the generic pure state $|\Phi(0)\rangle_\psi$, which can be taken as an alternative definition of the model, independent of any reference to the hidden degrees of freedom [17,23]:

$$\begin{aligned}\rho_{\mathrm{Ph}}(t) &= \int D[\varphi,\varphi']\exp\frac{ic^2}{2\hbar}\int dt\,dx\,\big[\varphi\nabla^2\varphi-\varphi'\nabla^2\varphi'\big]\\ &\quad\times{}_\psi\big\langle\Phi(0)\big|T^{-1}\exp\left[i\frac{mc}{\hbar}\sqrt{2\pi G}\int dt\,dx\,\varphi'\psi^\dagger\psi\right]T\exp\left[-i\frac{mc}{\hbar}\sqrt{2\pi G}\int dt\,dx\,\varphi\psi^\dagger\psi\right]\big|\Phi(0)\big\rangle_\psi\\ &\quad\times T\exp\left[-i\frac{mc}{\hbar}\sqrt{2\pi G}\int dt\,dx\,\varphi\psi^\dagger\psi\right]\big|\Phi(0)\big\rangle_\psi\,{}_\psi\big\langle\Phi(0)\big|T^{-1}\exp\left[i\frac{mc}{\hbar}\sqrt{2\pi G}\int dt\,dx\,\varphi'\psi^\dagger\psi\right].\end{aligned}\tag{4}$$

The announced relativistic model is obtained by the immediate generalization of the equation above corresponding to the replacement of the matter fields with their relativistic generalization and of the Laplacian with the d'Alambertian operator. The same replacement transforms Eq. (3) into a mixed operator and path integral expression for a theory with meta-matter interacting with a quantum neutral scalar field by a Yukawa interaction. The ensuing theory is of course a well-defined renormalizable field theory without any instability leading to an uncontrollable increase of the matter energy.

If one assumes that the ensuing relativistic model is a real improvement on its Newtonian limit, one has to see if using the latter is consistent at all. In order to do that, consider that the Newtonian model gives a localization length $\Lambda \sim (\hbar^2 R^3/GM^3)^{1/4}$ for a body whose linear dimension is R and whose mass M is above the threshold, which for ordinary densities is $\sim 10^{11}$ proton masses (m_p) [15,23]. The localization process implies a localization energy $E_\Lambda \sim \hbar^2/(M\Lambda^2) \sim \hbar G^{1/2}\rho^{1/2}$, depending only on the body density ρ, which, for ordinary densities is

$E_\Lambda \sim 10^{-20}$ eV. This process takes a time $T_G \sim 10^{20}(M/m_p)^{-5/3}$ s [17,23] and consists of the transformation of potential into kinetic meta-energy, corresponding to twice the physical kinetic energy.

To estimate the radiated energy in the relativistic model, consider that the bound metastates, corresponding to localized states, are small oscillations in a potential $U(r) \sim (GM^2/R^3)r^2$, namely with a frequency $\omega \sim \sqrt{GM/R^3}$. The corresponding classical radiating power for the nth harmonic frequency is easily seen to be $w_n = (n\omega)^2 G M_n^2/c$, where $\rho(x,t) \equiv \sum_n \rho_n(x)\exp(in\omega t)$, $M_n \equiv \int dx\, \rho_n(x)$. For ordinary densities and $M = 10^{12} m_p$, just above the threshold, one gets a total radiating power $w \lesssim 10^{-37}$ eV/s, which in the localization time $T_G \sim 1$ s amounts to an irrelevant fraction ($\sim 10^{-17}$) of the localization energy E_Λ. This means that, in order to estimate relativistic corrections, it makes sense just to replace in Eq. (1) the instantaneous interactions with the ones mediated by the retarded propagator.

Also in Pearle's proposal [7,26] matter is coupled to a scalar field by a Yukawa interaction. The main differences consist in the field being massive and in the fact that here it is coupled to a hidden copy of matter, whereas in Pearle's model it is coupled to a classical stochastic field, whose c-number character leads to an infinite growth rate of the energy of the scalar field [7]. Moreover, while our aim was to build a unified model of localization and gravity, the interaction introduced in Pearle's model is meant to produce localization only.

If one wanted to introduce a scalar field leading only to dynamical localization, without an average ensuing force, then one would be forced to take a negative energy field, namely an evolution operator

$$U_0(t) = \int D[\varphi]\exp\frac{i}{2\hbar}\int dt\,dx\left[-c^2\varphi\nabla^2\varphi + \varphi\partial_t^2\varphi\right] \times T\exp\left[-i\frac{mc}{\hbar}\sqrt{2\pi G}\int dt\,dx\,\varphi(x,t)\left(\psi^\dagger(x,t)\psi(x,t) - \tilde\psi^\dagger(x,t)\tilde\psi(x,t)\right)\right]. \tag{5}$$

In fact, if one integrates out the scalar field, in analogy with the Feynman's elimination of the electromagnetic potential [27] and one takes the $c \to \infty$ limit, one gets a Hamiltonian like the one in Eq. (1), but for the replacement $G \to -G$ in the last two terms. By paraphrasing the analysis performed in Refs. [15,23], one sees that, once the symmetry constraint is considered, no net force survives and that the localization properties are exactly the same as for the original model, since they depend only on the interaction between physical and hidden degrees of freedom. It should be remarked that if in Eq. (5) one replaces the scalar field with a positive energy one, by changing the sign in the exponent of the first exponential, one gets a model still with a vanishing net force, but without localization properties, as the interaction between physical and hidden degrees of freedom is repulsive.

Although our aim was the introduction of a well-defined relativistic theory, on a heuristic level one can introduce another relativistic model where both positive and negative energy fields are present, with the further bonus of Pauli–Villars-like cancellations, like in the general covariant theory [20,22,23]. In fact, if we consider a relativistic action

$$\mathcal{A} = \mathcal{A}_0[\psi^\dagger,\psi] + \mathcal{A}_0[\tilde\psi^\dagger,\tilde\psi] + \frac{1}{2}\int dt\,dx\left[c^2\varphi_1\nabla^2\varphi_1 - \varphi_1\partial_t^2\varphi_1 - c^2\varphi_2\nabla^2\varphi_2 + \varphi_2\partial_t^2\varphi_2 - 2mc\sqrt{2\pi G}\left(\psi^\dagger\psi + \tilde\psi^\dagger\tilde\psi\right)\varphi_1 - 2mc\sqrt{2\pi G}\left(\psi^\dagger\psi - \tilde\psi^\dagger\tilde\psi\right)\varphi_2\right], \tag{6}$$

where $\mathcal{A}_0[\psi^\dagger,\psi]$ is the ordinary relativistic matter action, its Newtonian limit is obtained by the elimination of the second and the third term and the replacement of G with $2G$ in Eq. (1). This nonrelativistic model is qualitatively equivalent to the Newtonian limit of the general covariant theory, apart from little quantitative changes in the localization properties due to the doubling of the localizing interaction. As to the relativistic model (6), it is remarkable that it contains no new ultraviolet divergences with respect to the ones already present in the traditional theory with action $\mathcal{A}_0[\psi^\dagger,\psi]$, as there is a complete cancellation of all self-energy and vertex graphs corresponding to the interaction of meta-matter with the scalar fields, due to the difference in sign of their propagators.

In conclusion some remarks are in order.

First it should be added that in our models (the avoidable) negative energy fields are less harmful than expected, since their average values are constrained to vanish, which makes such models stable, at least within a naive classical analysis, like it happens in the general covariant theory [22,23]. This follows from the fact that, in order for the evolution to be compatible with the symmetry constraint, one has to generalize the latter by replacing the symmetry transformation exchanging physical and hidden degrees of freedom with

$$\psi \to \tilde{\psi}, \qquad \tilde{\psi} \to \psi, \qquad \varphi \to -\varphi, \tag{7}$$

where φ is the negative energy field, in analogy to Eq. (12) in Ref. [23].

Secondly we should stress that, even though the obstruction to the formulation of consistent relativistic localization models is removed within a unified theory of localization and gravity, this does not mean that our special relativistic extensions may include a relativistic theory of gravity. In fact the Newtonian interaction is obtained starting from a Yukawa interaction, while a relativistic theory of gravity should involve the matter energy–momentum tensor. However, the present results, together with the observation that renouncing unitarity may tame the instabilities inherent in higher order gravity [22,23], appear to us to be a rather compelling indication that a unified relativistic theory of spontaneous localization and gravity may be easier to construct than a unitary theory of gravity.

Finally one can look in principle for spontaneous localization models in terms of a stochastic dynamics for pure states, which, when averaged, leads to our nonunitary evolution of the density operator [2–7]. Apart from the nonuniqueness of stochastic realizations [6], stochastic models can certainly be useful as computational tools. However the view advocated here considers density operators as the fundamental characterization of the system state and not just as a bookkeeping tool for statistical uncertainties. This point of view, apart from possibly being relevant to the quantum foundations of thermodynamics [23], avoids the ambiguities of the stochastic viewpoint, where the expectation of a local observable depends on the choice of a particular space-like surface in its entirety (Ref. [7], Section 14.2). The fact that, in measurement processes, the apparent uniqueness of the result seems to imply a real collapse is perhaps more an ontological than a physical problem, and presumably, if one likes it, that can be addressed by a variant of the Everett interpretation [28].

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