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Note

Characterization of graphs with equal domination and connected domination numbers☆

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Abstract

Arumugam and Paulraj Joseph (Discrete Math 206 (1999) 45) have characterized trees, unicyclic graphs and cubic graphs with equal domination and connected domination numbers. In this paper, we extend their results and characterize the class of block graphs and cactus graphs for which the domination number is equal to the connected domination number.
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1. Introduction

By a graph we mean a finite, undirected graph without loops or multiple edges. Terms not defined here are used in the sense of Arumugam [1].

Let $G=(V,E)$ be a simple graph of order n. The degree, neighborhood and closed neighborhood of a vertex v in the graph G are denoted by $d(v)$, $N(v)$ and $N[v]=N(v)\cup\{v\}$, respectively. For a subset S of V, $N(S)$ denotes the set of all vertices adjacent to some vertex in S and $N[S]=N(S)\cup S$. The graph induced by $S\subseteq V$ is denoted by $\langle S\rangle$. The minimum

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degree and maximum degree of the graph G are denoted by $\delta(G)$ and $\Delta(G)$, respectively. Let P_n denote the path with n vertices.

A subset S of V is called a *dominating set* if every vertex in $V - S$ is adjacent to some vertex in S. The *domination number* $\gamma(G)$ of G is the minimum cardinality taken over all dominating set of G. A dominating set is called a *connected dominating set* if the induced subgraph $\langle S\rangle$ is connected. The *connected domination number* $\gamma_{\mathrm{c}}(G)$ of G is the minimum cardinality taken over all minimal connected dominating sets of G. A connected dominating set S with cardinality $\gamma_{\mathrm{c}}(G)$ is called a γ_{c}-set. Let $S \subset V(G)$ and $x \in S$, we say that x has a private neighbour (with respect to S) if there is a vertex in $V(G) - S$ whose only neighbour in S is x. Let $\mathrm{PN}(x, S)$ denote the private neighbours set of x with respect to S.

A vertex v of G is called a *support* if it is adjacent to a pendant vertex. Any vertex of degree greater than one is called an *internal vertex*.

For any connected graph G a vertex $v \in V$ is called a *cutvertex* of G, if $G - v$ is no longer connected. A connected subgraph B of G is called a *block*, if B has no cutvertex and every subgraph $B' \subseteq G$ with $B \subseteq B'$ and $B \neq B'$ has at least one cutvertex. A block B of G is called an *end block*, if B contains at most one cutvertex of G. Such a cutvertex is called *an end block cutvertex*. A graph G is called a *block graph*, if every block in G is a complete graph. A cycle in G containing only one cutvertex is called an end cycle.

A graph G is called *unicyclic graph*, if G contains exactly one cycle. A graph G is called *cactus graph*, if every edge is in at most one cycle of G. Arumugam and Paulraj Joseph [1] have characterized trees, unicyclic graphs and cubic graphs with equal domination and connected domination numbers.

Lemma 1 (*Arumugam and Paulraj Joseph [1]*). *For a tree T of order $n \geqslant 3$, $\gamma_{\mathrm{c}}(T) = \gamma(T)$ if and only if every internal vertex of T is a support.*

Since a tree is a special case of block graphs, Lemma 1 is extended by Theorem 2 in Section 2.

Lemma 2 (*Arumugam and Paulraj Joseph [1]*). *Let G be a unicyclic graph with cycle C of length at least 5, and let X be the set of all vertices of degree 2 in C. Then $\gamma(G) = \gamma_{\mathrm{c}}(G)$ if and only if the following conditions hold*:

(a) *Every vertex of degree at least 2 in $V - N[X]$ is a support.*

(b) *$\langle X_i\rangle$ is connected and $|X_i| \leqslant 3$.*

(c) *If $\langle X\rangle = P_1$ or P_3, both vertices in $N(X)$ of degree greater than 2 are supports and if $\langle X\rangle = P_2$, at least one vertex in $N(X)$ of degree greater than 2 is support.*

Lemma 3 (*Arumugam and Paulraj Joseph [1]*). *Let G be a unicyclic graph of order $n \geqslant 4$ with cycle C of length 3, and let X be the set of all vertices of degree 2 in C. Then $\gamma(G) = \gamma_{\mathrm{c}}(G)$ if and only if the following conditions hold*:

(a) *Every vertex of degree at least 2 in $V - N[X]$ is a support.*

(b) *C contains exactly one vertex of degree at least 3 or every vertex of degree at least 3 in C is a support.*

Lemma 4 (*Arumugam and Paulraj [1]*). *Let G be a unicyclic graph of order $n \geqslant 5$ with cycle C of length 4, and let X be the set of all vertices of degree 2 in C. Then $\gamma(G) = \gamma_{\mathrm{c}}(G)$*

if and only if the following conditions hold:

(a) *Every vertex of degree at least* 2 *in* $V - N(X)$ *is a support*.

(b) *If* $|X| = 1$, *all the three remaining vertices of* C *are supports and if* $|X| \geqslant 2$, C *contains at least one support*.

Since unicyclic graph is a special case of cactus graph, Lemmas 2–4 are extended by Theorem 4 in Section 2.

In this paper, we characterize the class of block graphs and cactus graphs for which the domination number is equal to the connected domination number.

2. Main results

Obviously, complete graph and tree are special cases of block graph. For every connected block graph G, let F and l denote the set of cutvertices and the block number of G respectively. The connected domination number of the block graph G is given by the following theorem.

Theorem 1. *Let* G *be a connected block graph, then*

$$\gamma_c(G) = \begin{cases} 1 & \text{for } l = 1, \\ |F| & \text{for } l \geqslant 2. \end{cases}$$

Proof. If $l = 1$, then G is a complete graph. Obviously, $\gamma_c(G) = 1$. So, we only consider the case $l \geqslant 2$. If $l \geqslant 2$, then $F \neq \emptyset$. Since G is a connected block graph and every block contains at least one cutvertex, F is a connected dominating set of G. Hence, $\gamma_c(G) \leqslant |F|$. If $\gamma_c(G) < |F|$, then for arbitrary γ_c-set S of G, there exists a cutvertex v such that $v \notin S$. Let $G_1, G_2, \ldots, G_w$ denote the components of $G - \{v\}$. Let $S_i = S \cap V(G_i)$ for $i = 1, 2, \ldots, w$. Then $\bigcup_{1 \leqslant i \leqslant w} S_i = S$ and S is disconnected, which is a contradiction. Hence, $\gamma_c(G) = |F|$. □

If G is a complete graph, then $\gamma_c(G) = \gamma(G) = 1$. So we only consider the connected block graph with $l \geqslant 2$.

Theorem 2. *Let* G *be a connected block graph with* $l \geqslant 2$. *Then* $\gamma_c(G) = \gamma(G)$ *if and only if every cutvertex of* G *is an end block cutvertex*.

Proof. Since every cutvertex is in every connected dominating set and $\gamma_c(G) = |F|$, F is the unique minimum connected dominating set in G.

Now, let $\gamma_c(G) = \gamma(G)$. If there exists a cutvertex v such that v is not an end block cutvertex, then $F - \{v\}$ is a dominating set of G with cardinality $\gamma(G) - 1$, which is a contradiction.

Conversely, if every cutvertex v is an end block cutvertex, then $\gamma(G) \geqslant |F|$. Since $\gamma(G) \leqslant \gamma_c(G)$ and $\gamma_c(G) = |F|$, it follows that $\gamma_c(G) = \gamma(G)$.

Let G be a connected cactus graph, let $D = \{C_1, C_2, \ldots, C_t\}$ be the set of cycles in G. Let D_1 and D_2 denote the set of cycles in D with exactly one cutvertex and at least two cutvertices, respectively. Let $D_{21} \subseteq D_2$ satisfy that any cycle $C_i \in D_{21}$ contains at least one vertex of degree 2 but no two adjacent vertices of degree 2. Let $D_{22} \subseteq D_2$ satisfy that any cycle $C_i \in D_{22}$ has two cutvertices, say u and v, of C_i such that the length of the longest way between them in C_i not containing other cutvertices is at least three. Let L denote the set of pendant vertices in G. □

Theorem 3. *Let G be a connected cactus graph, then* $\gamma_c(G) = n - 2|D_1| - |D_{21}| - 2|D_{22}| - |L|$.

Proof. It is obvious that every γ_c-set of G does not contain pendant vertices and every cutvertex is in every connected dominating set of G. Hence, if a vertex is neither in any cycle nor a pendant vertex, then it must be a cutvertex and belongs to every γ_c-set. □

Case 1: For each $C_i \in D_1$, C_i has only one cutvertex. Hence, C_i is an end cycle and every γ_c-set must contain at least $|V(C_i)| - 2$ vertices of C_i. That is to say, for every γ_c-set S of G, C_i has at most two vertices such that they are not in S.

Case 2: If there exists a cycle C_i such that each vertex in C_i is of degree at least 3, then every vertex in C_i belongs to every γ_c-set S of G.

Case 3: For each $C_i \in D_{21}$, since C_i contains at least one vertex of degree 2 but no two adjacent vertices of degree 2, it follows that every γ_c-set must contain at least $|V(C_i)| - 1$ vertices of C_i. That is to say, for every γ_c-set S of G, C_i has at most one vertex that is not in S.

Case 4: For each $C_i \in D_{22}$, C_i has at least two cutvertices and there exist such two cutvertices such that the length of a longest way between them in C_i not containing other cutvertices is at least 3. Hence, every γ_c-set must contain at least $|V(C_i)| - 2$ vertices of C_i. That is to say, for every γ_c-set S of G, C_i has at most two vertices such that they are not in S.

By Cases (1)–(4),

$$\begin{aligned}\gamma_c(G) &\geqslant |V(G)| - \sum_{C_i \in D_1} 2 - \sum_{C_i \in D_{21}} 1 - \sum_{C_i \in D_{22}} 2 - |L| \\ &= n - 2|D_1| - |D_{21}| - 2|D_{22}| - |L|.\end{aligned}$$

Since it is obvious that there exists a connected dominating set with cardinality $n - 2|D_1| - |D_{21}| - 2|D_{22}| - |L|$, it follows that $\gamma_c(G) = n - 2|D_1| - |D_{21}| - 2|D_{22}| - |L|$.

Lemma 5. *If G is a cycle, then* $\gamma(G) = \gamma_c(G)$ *if and only if G is isomorphic to a cycle of length* 3 *or* 4.

For every cycle C_i, let X_i be the set of all vertices of degree 2 in C_i. Let $X = \bigcup_{1 \leqslant i \leqslant t} X_i$ and $A = V(G) - N[X]$. Let

$$B = \{v \in N(X) | d(v) \geqslant 3,\ N(v) \cap X_i \neq \emptyset \text{ for some } X_i \text{ with } |X_i| = 2,\ C_i \in D_2,\ 1 \leqslant i \leqslant t\},$$

$$Y_0 = \{v | v \in X_i \text{ and } C_i \text{ is an end 4-cycle for } 1 \leqslant i \leqslant t\},$$

$$Y_1 = \{v \in X_i | C_i \in D_2,\ |X_i| = 1, \text{ for } 1 \leqslant i \leqslant t\},$$

$$Y_2 = \{v \in X_i | C_i \in D_2, |X_i| = 2, \text{ for } 1 \leqslant i \leqslant t\},$$

$$Y_3 = \{v \in X_i | C_i \in D_2, |X_i| = 3, \text{ for } 1 \leqslant i \leqslant t\}.$$

If G is a connected cactus graph with no cutvertex, then G is a cycle or a path with at most two vertices. By Lemma 5, the characterization of cycles G with $\gamma(G) = \gamma_c(G)$ is well known. So, we only consider the connected cactus graph with at least one cutvertex. Let H be a family of cactus graphs such that for each graph of H the following two conditions hold: (1) every cycle is a 4-cycle and every vertex and edge is in at least one 4-cycle; (2) every cycle has at most two cutvertices, and if a cycle has two cutvertices, say u and v, then u and v are adjacent.

Theorem 4. *Let G be a connected cactus graph with at least one cutvertex. Then $\gamma(G) = \gamma_c(G)$ if and only if G is isomorphic to one graph of H, or the following conditions hold*:

(a) *Every end cycle is a* 3-*cycle or* 4-*cycle*.

(b) $\langle X_i \rangle$ *is connected and* $|X_i| \leqslant 3$ *for* $1 \leqslant i \leqslant t$.

(c) *For each* $v \in A$, v *is either a pendant vertex or a support*.

(d) *For each vertex v of degree at least* 3 *in* $N(X)$, *at least one of the following conditions holds*:

(d1) v *is either a support or a cutvertex of an end* 3-*cycle*.

(d2) $v \in B$ *and the component of* $\langle B \cup Y_2 \rangle - E(\langle B \rangle)$ *containing v has at least a vertex* $u \in B$ *such that u is a support or a cutvertex of an end* 3-*cycle in G*.

Proof. First, we prove the necessity. Let S be any minimum connected dominating set of G. Then $|S| = \gamma(G) = \gamma_c(G)$. If G is isomorphic to one graph of H, then we are done. So, we only consider the case that G is not isomorphic to any graph of H.

If (a) does not hold, then there exists an end cycle $C_i = v_1 v_2 \ldots v_t v_1$ with $t \geqslant 5$. It is obvious that exactly two adjacent vertices of C_i do not belong to S. Without loss of generality, assume v_1 is the cutvertex and S contains $v_1, v_2, \ldots, v_{t-2}$. Hence, $S - \{v_2\}$ is a dominating set of G with cardinality $\gamma(G) - 1$, which is a contradiction.

Suppose (b) does not hold. If $\langle X_i \rangle$ is disconnected, S contains all vertices of at least one component, say X_i', of $\langle X_i \rangle$. Say $v_i \in V(X_i')$. Then $S - \{v_i\}$ is a dominating set of G with cardinality $\gamma(G) - 1$, which is a contradiction. If $\langle X_i \rangle$ is connected and $|X_i| \geqslant 4$, we may assume without loss of generality that $X_i = \{v_1, v_2, \ldots, v_s\}$ where $s \geqslant 4$. Then exactly two adjacent vertices of X_i do not belong to S.

If $v_1, v_2 \notin S$, then $S - \{v_s\}$ is a dominating set of G with cardinality $\gamma(G) - 1$, which is a contradiction.

If $v_{s-1}, v_s \notin S$, then $S - \{v_1\}$ is a dominating set of G with cardinality $\gamma(G) - 1$, which is a contradiction.

If $v_i, v_{i+1} \notin S$, then $(S - \{v_{i-1}, v_{i+2}\}) \cup \{v_i\}$ is a dominating set of G with cardinality $\gamma(G) - 1$, which is a contradiction.

(c) For each $v \in A$, if v is neither a pendant vertex nor a support, then any vertex in $N[v]$ is a cutvertex. So, $N[v] \subseteq S$. Hence, $S - \{v\}$ is a dominating set of G with cardinality $\gamma(G) - 1$, which is a contradiction.

(d) Since every vertex v of degree at least 3 in $N(X)$ is a cutvertex, it follows that $v \in S$. Suppose $|S| = 1$. It is obvious that $\mathrm{PN}(v, S) \neq \emptyset$. Suppose $|S| \geqslant 2$. If $\mathrm{PN}(v, S) = \emptyset$, then

$S - \{v\}$ is a dominating set of G with cardinality less than $\gamma(G)$, which is a contradiction. So, $\mathrm{PN}(v, S) \neq \emptyset$. Let $\mathrm{PN}(v, S) = \{v_1, v_2, \ldots, v_m\}$. If v is neither a support nor a cutvertex of an end 3-cycle, then $\mathrm{PN}(v, S) \subseteq \bigcup_{j=0}^{3} Y_j$ and we have the following claims. □

Claim 1. $\mathrm{PN}(v, S) - Y_0 \neq \emptyset$.

Suppose every vertex $v_i \in \mathrm{PN}(v, S)$ is a vertex of an end 4-cycle, say $vv_iw_il_iv$, for $1 \leqslant i \leqslant m$. Since S is a γ_c-set, $l_i \in S$ for $1 \leqslant i \leqslant m$. If G has only one cutvertex, then G is isomorphic to one graph of H, which is a contradiction. Hence, G has at least two cutvertices. Since $\langle S \rangle$ is connected, it follows that v is dominated by other cutvertex in S. So, $S' = (S - \{v, l_1, l_2, \ldots, l_m\}) \cup \{w_1, w_2, \ldots, w_m\}$ is a dominating set of G with cardinality $\gamma(G) - 1$, which is a contradiction.

Claim 2. $\mathrm{PN}(v, S) - Y_3 \neq \emptyset$.

Suppose every vertex $v_i \in \mathrm{PN}(v, S)$ is a vertex of a cycle $C_i \in D_2$ with $|X_i| = 3$. Assume, $vv_iw_il_iu_i$ is a segment of C_i, where v and u_i are cutvertices of G and $\{v_i, w_i, l_i\} = X_i$ for $1 \leqslant i \leqslant m$. Since S is a γ_c-set of G, $V(C_i) \backslash \{v_i, w_i\} \subseteq S$ for $1 \leqslant i \leqslant m$. Hence, $S' = (S - \{v, l_1, l_2, \ldots, l_m\}) \cup \{w_1, w_2, \ldots, w_m\}$ is a dominating set of G with cardinality $\gamma(G) - 1$, which is a contradiction.

In a similar way, we can prove that

Claim 3. $\mathrm{PN}(v, S) - (Y_0 \cup Y_3) \neq \emptyset$.

Claim 4. $\mathrm{PN}(v, S) \cap Y_1 = \emptyset$.

By Claims 1–4, it follows that $\mathrm{PN}(v, S) \cap Y_2 \neq \emptyset$. Thus, $v \in B$.

By the proofs of Claims 1–4, we assume without loss of generality that every vertex v of degree at least 3 in $N(X)$ has all its private neighbours in Y_2. Otherwise, if $\mathrm{PN}(v, S) \cap (Y_0 \cup Y_3) \neq \emptyset$, then let $\mathrm{PN}(v, S) \cap Y_0 = \{v_1, v_2, \ldots, v_l\}$ and $\mathrm{PN}(v, S) \cap Y_3 = \{v_{l+1}, v_{l+2}, \ldots, v_s\}$. So $1 \leqslant s \leqslant m$. Suppose that every vertex v_i is a vertex of an end 4-cycle $C_i : vv_iw_il_iv$ for $i = 1, 2, \ldots, l$. Suppose that every vertex v_j is a vertex of a cycle C_j with $|X_j| = 3$ and $vv_jw_jl_ju_j$ is a segment of C_j, where v and u_j are cutvertices of G, for $j = l + 1, l + 2, \ldots, s$. Let $S' = (S - \{l_1, l_2, \ldots, l_s\}) \cup \{w_1, w_2, \ldots, w_s\}$. Then S' is a γ-set of G such that $\mathrm{PN}(v, S') \cap (Y_0 \cup Y_3) = \emptyset$. Furthermore every cutvertex is in S'.

It is obvious that each component of $\langle B \cup Y_2 \rangle - E(\langle B \rangle)$ is a tree. If the component B_i of $\langle B \cup Y_2 \rangle - E(\langle B \rangle)$ containing v satisfies that every vertex $u \in V(B_i) \cap B$ is neither a support nor a cutvertex of an end 3-cycle in G, then we have the following contradictions:

Case 1: If there exists a vertex $u \in V(B_i) \cap B$ such that u is adjacent to at least one other cutvertex of G not in B_i, then partition B_i into levels according to the distance from u, i.e., a vertex is in level i if it has distance i from u. Let $M = \{v \in V(B_i) | d(u, v) \equiv 0 \pmod 3\}$ and $W = \{v \in V(B_i) | d(u, v) \equiv 2 \pmod 3\}$. It is obvious that $u \in M$ and $M \subset S$. Since every vertex of W is adjacent to exactly one vertex of $M \backslash \{u\}$, it follows that $|W| = |M| - 1$. Let $S' = (S - M) \cup W$. Since each vertex of $(\bigcup_{v \in V(B_i) \cap B} \mathrm{PN}(v, S)) \cup (V(B_i) \backslash \{u\})$ is dominated by W and u is dominated by other cutvertex of G not in B_i, it follows that S' is a dominating set of G with cardinality $\gamma(G) - 1$, which is a contradiction.

Case 2: If every vertex $u \in V(B_i) \cap B$ is not adjacent to any cutvertex of G not in B_i, then each cycle C_i where X_i belongs to the component B_i is a 4-cycle. Furthermore, for every edge ut_i that does not belong to $E(\langle V(B_i)\rangle)$, since t_i is neither a cutvertex nor a pendent vertex of G, it follows that ut_i must be an edge of some cycle C_i. Suppose that $|V(C_i)| \geqslant 6$. Since $|X_i| \leqslant 3$, it follows that u is adjacent to at least one cutvertex of G not in B_i, which is a contradiction. Suppose that $|V(C_i)| = 5$. If $|X_i| = 3$, then u is adjacent to at least one cutvertex of G not in B_i, which is a contradiction. If $|X_i| \leqslant 2$, then u is still adjacent to at least one cutvertex of G not in B_i, which is a contradiction. Since u is not a cutvertex of an end 3-cycle, it follows that C_i is a 4-cycle. Suppose that C_i is not an end 4-cycle. If $|X_i| = 1$, then u is adjacent to at least one cutvertex of G not in B_i, which is a contradiction. If $|X_i| = 2$, then $ut_i \in E(\langle B_i\rangle)$, which is a contradiction. Hence, C_i is an end 4-cycle. So, G is isomorphic to H, which is a contradiction.

Now, we prove the sufficiency.

Case 1: G is isomorphic to H. It is clear that $\gamma(G) = \gamma_c(G)$.

Case 2: G is not isomorphic to H.

Let S be a γ-set of G with minimum pendant vertices. Then S does not contain every pendant vertex. Otherwise, let S' be obtained from S by replacing one of pendant vertices in S with its support, then S' is a γ-set of G with fewer pendant vertices than S, which is a contradiction. If v is a cutvertex of an end 3-cycle C_i and $v \notin S$, then there exists a vertex $u \in V(C_i)$ such that $u \in S$. Hence $(S - \{u\}) \cup \{v\}$ is a γ-set of G that contains v. Without loss of generality, assume that every cutvertex of end 3-cycle belongs to S. By (c), for each $v \in A$, if v is a pendant vertex, then v belongs to neither S nor any γ_c-set of G. If v is a support, then v belongs to S and any γ_c-set of G.

By (d), for each vertex v of degree at least 3 in $N(X)$, if v is either a support or a cutvertex of an end 3-cycle, then v belongs to S and any γ_c-set of G. We assume that $v \in B$. Since the component B_i of $\langle B \cup Y_2\rangle - E(\langle B\rangle)$ containing v has at least a vertex $u \in V(B_i) \cap B$ such that u is a support or a cut vertex of an end 3-cycle in G, then partition B_i into levels according to the distance from u, i.e., a vertex is in level i if it has distance i from u. It follows that S and any γ_c-set of G contain the same number of vertices in B_i. For each end 4-cycle $v_i t_i w_i l_i v_i$, where v_i is an end cutvertex, $|S \cap \{t_i, w_i, l_i\}| \geqslant 1$. For each cycle $C_i \in D_2$ with $|X_i| = 3$, let $v_i t_i w_i l_i u_i$, where v_i and u_i are cutvertices, $|S \cap \{t_i, w_i, l_i\}| \geqslant 1$. Hence, $\gamma(G) \geqslant \gamma_c(G)$. Since $\gamma(G) \leqslant \gamma_c(G)$, it follows that $\gamma(G) = \gamma_c(G)$.

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