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Global optimum spacecraft orbit control subject to bounded thrust in presence of nonlinear and random disturbances in a low earth orbit

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Abstract The primary objective of this work is to develop an effective spacecraft orbit control algorithm suitable for spacecraft orbital maneuver and/or rendezvous. The actual governing equation of a spacecraft orbiting the earth is merely nonlinear. Disturbance forces resulting from aerodynamic drag, oblateness of the earth till the fourth order (i.e. J_4), and random disturbances are modeled for the initial and target orbits. These disturbances increase the complexity of nonlinear governing equations. Global optimum solutions of the control algorithm parameters are determined throughout real coded genetic algorithms such that the steady state difference between the actual and desired trajectories is minimized. The resulting solutions are constrained to avoid spacecraft collision with the surface of the earth taking into account limited thrust budget.

1. Introduction

Spacecraft orbital maneuver is used frequently for many reasons, such as station keeping, orbital transfer, and rendezvous. It is classified, based on a control sense, as a tracking rather than a stabilization problem. In order to execute such maneuvers, a good modeling of some environmental effects is required. The most important environmental effects are aerodynamic drag, and oblateness of the earth. The most common methods existing in the literature depend on C–W equations which describe the relative motion for two vicinal spacecraft as shown in Yang et al. (2010) and Zhang et al. (2011). Unfortunately, these equations did not include the effect of aerodynamic drag, oblateness of the earth, or bounded thrust budget. Naasz (2002) describes the problem of spacecraft orbit control taking into account only the effect of earth oblateness, up to the term, J_2, only. Another important note regarding the problem at hand is that, it is a constrained optimization problem. The nature of the first constraint results from the fact that the spacecraft must avoid collision with the surface of the earth during the execution of its orbital maneuver. The second constraint results logically from the bounded thrust budget. Luo and Tang (2005) suffers from

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the same drawbacks presented. In addition, the control force magnitude is not considered at all, the number of controller parameters to be tuned are very huge (actually 18 while as the current research requires only two parameters), and the control algorithm requires high computational burden over the spacecraft onboard computer (which is usually fully loaded) due to mathematical processes required for the 18 controller parameters. Massary and Bernelli-Zazzera (2009), is concerned with satellite formation maneuver which is usually constrained with the relative distance between the satellites of the formation. This in turn, as mentioned in Massary and Bernelli-Zazzera (2009), requires much more computational resources.

The objective of the research at hand is to establish an effective control algorithm suitable for station keeping, orbital transfer, and rendezvous. System describing equations are based on Cowell's formulation (Vallado, 1997). This formulation is found to be very useful in many applications because any arbitrary disturbing acceleration could be simply added to the two-body equation so as to produce a more precise solution to the ordinary differential equation. Thus, disturbing acceleration resulting from aerodynamic drag, and oblateness of the earth to the nth order could be simply modeled (Tamer, 2009). Deriving the control law based on Cowell's formulation, as seen in subsequent sections, is a straight forward process to determine the initial guessing of the control gains. Furthermore, Cowell's formulation could be easily differentiated to obtain the state transition matrix (Tamer, 2009; Montenbruck and Gil, 2005). A very important constraint that must be satisfied is that the spacecraft must avoid collision with the surface of the earth during its orbital maneuvers. Transfer orbit which do not satisfy this condition must be eliminated. The effect of bounded thrust budget is taken into consideration during the control law design. Global optimum solutions for the control gains are found throughout real coded genetic algorithms (RCGA) given in Jamshidi et al. (2003). These solutions of course consider the effect of aerodynamic drag, J_4, and bounded thrust budget.

The main contribution of the research at hand is its simple and effective controller design method based on Cowell's formulation taking into account various sources of nonlinearity, such as aerodynamic disturbance, oblateness of the earth, J_4, and bounded thrust budget. Concurrently with these nonlinearities, the orbital maneuver is restricted such that the maneuvering satellite must not hit the surface of the earth. Moreover, the number of controller parameters to be selected is only two, so the computational burden over the spacecraft onboard computer (which is usually fully loaded) is drastically reduced compared to the algorithm presented in Luo and Tang (2005).

2. Spacecraft orbital dynamics

The translation motion model of a spacecraft is derived using Newton's law of gravitation between two particles i, and j as seen in Fig. 1. The force acting on a particle i due to the existence of a particle j is given by the relation (Tamer, 2009).

$$\vec{F}_{ij} = \frac{-Gm_i m_j}{\|r_{ij}\|^2}\frac{r_{ij}}{\|r_{ij}\|} \tag{1}$$

where G, is the gravitational constant ($G = 6.673 \times 10^{-11} \pm 0.001 \times 10^{-11} \frac{m^3}{kg.s^2}$); m_i, is the mass of

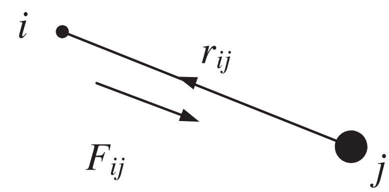

Figure 1 Gravitational force.

particle i; m_j, is the mass of particle j; r_{ij}, is the vector from the particle j to the particle i, as illustrated in Fig. 1.

Eq. (1) is known in the literature as the two-body problem. In the following treatment it is assumed that

- There are no aerodynamic forces.
- There are no thrust forces.
- The spacecraft is a point mass.
- The earth is a point mass.
- There is not any disturbing force acting on the spacecraft.

For a spacecraft, Eq. (1) describes the forces acting on the spacecraft due to the presence of the earth. This force, according to Newton's second law, is equal to the spacecraft mass times its acceleration. Therefore, in an inertial frame of reference we could write (Montenbruck and Gil, 2005)

$$\vec{F}_{ij} = \frac{-Gm_E m_{sc}}{\|R\|^3} R\bigg|_I = m_{sc}\frac{d^2R}{dt^2}\bigg|_I \tag{2}$$

Where m_E, is the earth mass; m_{sc}, is the spacecraft mass; R, is the spacecraft position vector defined in the inertial frame of reference (the subscript (I) denotes an inertial frame of reference. i.e. $R|_I = R_I = [X_I \quad Y_I \quad Z_I]^T$); $\|R\|$, is the distance between the spacecraft and the earth center. and consequently,

$$\overset{oo}{R}_I + \frac{\mu_E}{\|R_I\|^3}R_I = 0 \tag{3}$$

with μ_E, is the earth's gravitational constant ($\mu_E = 3.986 \times 10^{14}\ \mathrm{m^3/s^2}$).

Orbital disturbances such as those resulting from earth oblateness, third body effects, etc. could be included in Eq. (3) simply by adding an inertial perturbing acceleration, aI to Eq. (3). Thus, Eq. (3) results in the Cowell's formulation (Tamer, 2009).

$$\begin{bmatrix} \overset{o}{X}_I \\ \overset{o}{Y}_I \\ \overset{o}{Z}_I \\ \overset{oo}{X}_I \\ \overset{oo}{Y}_I \\ \overset{oo}{Z}_I \end{bmatrix} = \begin{bmatrix} 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \\ \frac{-\mu_E}{\|R_I\|^3} & 0 & 0 & 0 & 0 & 0 \\ 0 & \frac{-\mu_E}{\|R_I\|^3} & 0 & 0 & 0 & 0 \\ 0 & 0 & \frac{-\mu_E}{\|R_I\|^3} & 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} X_I \\ Y_I \\ Z_I \\ \overset{o}{X}_I \\ \overset{o}{Y}_I \\ \overset{o}{Z}_I \end{bmatrix} + \begin{bmatrix} 0 \\ 0 \\ 0 \\ a_{XI} \\ a_{YI} \\ a_{ZI} \end{bmatrix} \tag{4}$$

Or in a more compact form

$$\overset{oo}{R}_I = \frac{-\mu_E}{\|R_I\|^3}R_I + a_I = f + a_I = f_E(R_I)R_I + a_I \tag{5}$$

3. Perturbation models

3.1. The earth oblateness

The gravity field affecting the motion of a spacecraft in a two-body keplerian orbit assumes a perfectly spheroid shape of the earth. However, for higher accuracies of spacecraft position the ellipsoidal shape of the earth must be used instead. The gravitational potential function of the earth could be expanded in a spherical harmonic form as

$$U = \frac{\mu_E}{r_S}\sum_{l=0}^{l\max}\sum_{m=0}^{l}\left(\frac{R_\oplus}{r_S}\right)^l P_{lm}\{C_{lm}\cos(m\phi_S) + S_{lm} \times \sin(m\phi_S)\} \quad (6)$$

with $R_\oplus$, is the earth's mean equatorial radius ($R_\oplus = 6378.1363 \times 10^3$m); P_{lm}, are associated Legendre functions; m, is the model order; l, is the model degree; C_{lm} and S_{lm}, are the dimension-less coefficients used to describe the shape and mass distribution inside the earth; r_S, is the geocentric distance; ϕ_S, is the east longitude from Greenwich.

If the selected coordinate system in which the potential function is calculated coincides with the geocentric equatorial axes, the terms, $C_{1,0}$, $C_{1,1}$, and $S_{1,0}$ become zero. The term $C_{0,0}$ is equal to 1 and is corresponding to a spherical earth model. Thus, Eq. (6) turns out to be

$$U = \frac{\mu_E}{r_S}\left(1 + \sum_{l=2}^{l\max}\sum_{m=0}^{l}\left(\frac{R_\oplus}{r_S}\right)^l P_{lm}\{C_{lm}\cos(m\phi_S) \times + S_{lm}\sin(m\phi_S)\}\right) \quad (7)$$

the associated Legendre polynomial could be computed recursively by the relations:-

$$P_{0,0} = 1 \quad (8)$$

$$P_{l,l} = (2l-1)P_{l-1,l-1}\cos\lambda \quad (9)$$

$$P_{l,0} = \frac{(2l-1)P_{l-1,0}\sin\lambda - (l-1)P_{l-2,0}}{l} \quad (10)$$

$$P_{l,m} = P_{l-2,m} + (2l-1)P_{l-1,m-1}\cos\lambda \quad (11)$$

where λ: is the latitude.

The partial derivatives of the gravitational potential function are

$$\frac{\partial U}{\partial r_S} = \frac{-\mu}{r_S^2}\sum_{l=2}^{l\max}\sum_{m=0}^{l}\left(\frac{R_\oplus}{r_S}\right)^l (l+1)P_{l,m}\{C_{l,m}\cos(m\phi_S) + S_{l,m}\sin(m\phi_S)\} \quad (12)$$

$$\frac{\partial U}{\partial \lambda} = \frac{\mu}{r_S}\sum_{l=2}^{l\max}\sum_{m=0}^{l}\left(\frac{R_\oplus}{r_S}\right)^l \{P_{l,m+1} - m\tan(\lambda)P_{l,m}\}\{C_{l,m}\cos(m\phi_S) + S_{l,m}\sin(m\phi_S)\} \quad (13)$$

$$\frac{\partial U}{\partial \phi_S} = \frac{\mu}{r_S}\sum_{l=2}^{l\max}\sum_{m=0}^{l}\left(\frac{R_\oplus}{r_S}\right)^l mP_{l,m}\{S_{l,m}\cos(m\phi_S) - C_{l,m}\sin(m\phi_S)\} \quad (14)$$

and finally, the perturbation acceleration in the earth centered earth fixed (ECEF) coordinate system is:

$$a_{XE} = \left\{\frac{1}{r_S}\frac{\partial U}{\partial r_S} - \frac{Z_E}{r_S^2\sqrt{X_E^2+Y_E^2}}\frac{\partial U}{\partial \lambda}\right\}X_E - \frac{1}{X_E^2+Y_E^2}\frac{\partial U}{\partial \phi_S}Y_E \quad (15)$$

$$a_{YE} = \left\{\frac{1}{r_S}\frac{\partial U}{\partial r_S} - \frac{Z_E}{r_S^2\sqrt{X_E^2+Y_E^2}}\frac{\partial U}{\partial \lambda}\right\}Y_E - \frac{1}{X_E^2+Y_E^2}\frac{\partial U}{\partial \phi_S}X_E \quad (16)$$

$$a_{ZE} = \frac{1}{r_S}\frac{\partial U}{\partial r_S}Z_E + \frac{\sqrt{X_E^2+Y_E^2}}{r_S^2}\frac{\partial U}{\partial \lambda} \quad (17)$$

where X_E, Y_E, and Z_E: are the spacecraft position vector components in ECEF coordinate system.

The difference between ellipsoidal and spherical earth model results in a difference in the position vector of order 20 km (Vallado, 1997). This difference reaches a maximum value at the poles and vanishes at the equator. The accelerations resulting from oblateness of the earth could be transformed from the ECEF frame of reference to the inertial frame (ECI) through the relation

$$\begin{bmatrix} a_{XI} \\ a_{YI} \\ a_{ZI} \end{bmatrix} = \begin{bmatrix} \cos\alpha_g & -\sin\alpha_g & 0 \\ \sin g_g & \cos\alpha_g & 0 \\ 0 & 0 & 1 \end{bmatrix}\begin{bmatrix} a_{XE} \\ a_{YE} \\ a_{ZE} \end{bmatrix} \quad (18)$$

and α_g is the Greenwich right ascension determined from Bate and White (1971)

$$\alpha_g = \alpha_{g0} + 1.002737903 \times 2\pi \times D \quad (19)$$

with αg_0, 1.74933340 rad at 1/1/1970 0 h:0 m:0 s; D, time in day fraction elapsed since 1/1/1970 0 h:0 m:0 s.

3.2. Aerodynamic drag

The aerodynamic force, df_{Aero} on a satellite surface element dA, is given by Tamer, (2003)

$$df_{Aero} = -\frac{1}{2}C_D\rho V^2 dA \quad (20)$$

Where V, is the the translational velocity of the satellite t relative to the incident stream. ρ, is the atmospheric density; CD, is the drag coefficient. For practical applications, CD may be set in the range of 2.0.

The atmospheric density is modeled based on interpolation between the values given in Larson and Wertz (1999).

4. Spacecraft orbit control

In order to simplify the problem, a PD control strategy is employed. The initial estimates of the required control gains are determined using classical control techniques. Optimum estimates of these initial values are obtained throughout genetic algorithms. For a general state feedback approach, we could write system equation as (Franklin and Emami, 2010)

$$\overset{o}{X} = FX + Gu \tag{21}$$

where X is the state vector, F is a matrix of constants, G is the system input matrix, and u is the system input. Using state feedback approach

$$u = -K(X - X^{\text{commanded}}) \tag{22}$$

substitution from Eq. (22) into Eq. (21) results in

$$\overset{o}{X} = FX - GK(X - X^{\text{commanded}}) \tag{23}$$

rearranging Eq. (23) gives

$$\overset{o}{X} - FX + GKX = GKX^{\text{commanded}} \tag{24}$$

taking Laplace transform

$$\begin{aligned}\ell\{\overset{o}{X} - FX + GKX\} &= (sI - \{F - GK\})X(s) \\ &= GKX^{\text{commanded}}(S)\end{aligned} \tag{25}$$

so, the transfer function becomes

$$\frac{X(s)}{X^{\text{commanded}}} = \frac{GK}{sI - \{F - GK\}} \tag{26}$$

To apply Eq. (21) we could simplify Eq. (4) by considering only the X_I direction, so we could write

$$\overset{oo}{X}_I = \frac{-\mu_E}{\|R_I\|^3} X_I + a_{XI} = -\omega_0^2 X_I + a_{XI} \tag{27}$$

or, in a canonical state space form

$$\overset{o}{X} = \begin{bmatrix} \overset{o}{X}_I \\ \overset{o}{V}_{Ix} \end{bmatrix} = \begin{bmatrix} 0 & 1 \\ -\omega_0^2 & 0 \end{bmatrix} \begin{bmatrix} X_I \\ V_{Ix} \end{bmatrix} + \begin{bmatrix} 0 \\ 1 \end{bmatrix} a_{XI} = FX + Gu \tag{28}$$

based on Eq. (28), the characteristic equation is given as,

$$|sI - F| = \left| \begin{bmatrix} s & 0 \\ 0 & s \end{bmatrix} - \begin{bmatrix} 0 & 1 \\ -\omega_0^2 & 0 \end{bmatrix} \right| = \left| \begin{bmatrix} s & -1 \\ \omega_0^2 & s \end{bmatrix} \right| = s^2 + \omega_0^2 \tag{29}$$

the controlled system similarly has the characteristic equation given by Eq. (28)

$$\begin{aligned}|sI - \{F - GK\}| &= \left| \begin{bmatrix} s & 0 \\ 0 & s \end{bmatrix} - \left(\begin{bmatrix} 0 & 1 \\ -\omega_0^2 & 0 \end{bmatrix} - \begin{bmatrix} 0 \\ 1 \end{bmatrix} [k_x \ k_{xd}] \right) \right| \\ &= \left| \begin{bmatrix} s & 0 \\ 0 & s \end{bmatrix} - \left(\begin{bmatrix} 0 & 1 \\ -\omega_0^2 & 0 \end{bmatrix} - \begin{bmatrix} 0 & 0 \\ k_x & k_{xd} \end{bmatrix} \right) \right| = \left| \begin{bmatrix} s & 0 \\ 0 & s \end{bmatrix} - \begin{bmatrix} 0 & 1 \\ -\omega_0^2 - k_x & -k_{xd} \end{bmatrix} \right| \\ &= \left| \begin{bmatrix} s & 0 \\ 0 & s \end{bmatrix} - \begin{bmatrix} 0 & 1 \\ -\omega_0^2 - k_x & -k_{xd} \end{bmatrix} \right| = \left| \begin{bmatrix} s & -1 \\ \omega_0^2 + k_x & s + k_{xd} \end{bmatrix} \right| \\ &= s^2 + k_{xd}s + \omega_0^2 + k_x\end{aligned} \tag{30}$$

comparing Eq. (30) with the standard characteristic equation of a second order system given by

$$s^2 + 2\zeta\omega_n s + \omega_n^2 = 0 \tag{31}$$

we find that

$$k_x = \omega_n^2 - \omega_0^2, \quad 2\zeta\omega_n = k_{xd} \tag{32}$$

now, rearranging Eq. (27) gives

$$a_{XI} = \overset{oo}{X}_I + \omega_0^2 X_I \tag{33}$$

and from Eq. (22) we could write

$$\begin{aligned}a_{XI} = u = -K(X - X^{\text{commanded}}) &= \begin{bmatrix} k_x & k_{xd} \end{bmatrix} \begin{bmatrix} X_I^{\text{commanded}} - X_I \\ V_{xI}^{\text{commanded}} - V_{xI} \end{bmatrix} \\ &= k_x(X_I^{\text{commanded}} - X_I) + k_{xd}(V_{xI}^{\text{commanded}} - V_{xI})\end{aligned} \tag{34}$$

the overall state space form of the spacecraft is given by

$$\begin{aligned}\overset{o}{X}_T = \begin{bmatrix} \overset{o}{X}_I \\ \overset{o}{Y}_I \\ \overset{o}{Z}_I \\ \overset{o}{V}_{Ix} \\ \overset{o}{V}_{Iy} \\ \overset{o}{V}_{Iz} \end{bmatrix} &= \begin{bmatrix} 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \\ -\omega_0^2 & 0 & 0 & 0 & 0 & 0 \\ 0 & -\omega_0^2 & 0 & 0 & 0 & 0 \\ 0 & 0 & -\omega_0^2 & 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} X_I \\ Y_I \\ Z_I \\ V_{Ix} \\ V_{Iy} \\ V_{Iz} \end{bmatrix} \\ &+ \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \\ 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} a_{XI} \\ a_{YI} \\ a_{ZI} \end{bmatrix} = F_T X_T + G_T u_T\end{aligned} \tag{35}$$

and the gains to be determined are, k_x, k_{xd}, k_y, k_{yd}, k_z, and k_{zd}. These gains are required to satisfy certain performance parameters. Two meaningful parameters of the controlled system are selected, namely, the damping ratio, ζ_c, and the time constant, τ_c. The time constant is related to the damping ratio through the relation

$$\tau_c = \frac{1}{\zeta_c \omega_{nc}} \tag{36}$$

where ω_{nc} is the required natural frequency of the controlled system.

5. Global optimum solutions for the control gains

Although the algorithm presented in the previous section is based on linear control theory, the problem at hand is to some extent nonlinear. This nonlinearity results from the describing equation, nonlinear disturbances, and actuator saturation represented by the bounded thrust budget. Also, we should note that during the execution of the orbital maneuver the satellite must not hit the surface of the earth.

For nonlinear systems there exists several sets of solutions those could minimize (or maximize) a certain objective function locally without being related to the true solution. In another words, the simplex algorithm (which is a local minimum maximum search algorithm) could stick in a local minimum of the objective function and fails to find the global optimum solution (Cappelleri et al., 2006). The simplex algorithm is based on moving and resizing of a multidimensional polygon (or simplex) along a downhill direction to a local minimum (Sayeed, 2003). The downhill direction is obtained numerically without the need to evaluate gradient calculations. Genetic algorithms on the other hand could be used to search for the global optimum solutions (Abdelkhalik, 2005). Ref. Jamshidi et al. (2003) represents a distinguished text book that handles genetic algorithms for real numbers instead of binary coded numbers. The control gains are initially determined by the linear control theory without any constraint. Afterwards, the constraints are applied to the objective function to be minimized by the real coded genetic algorithms (RCGA). The objective function to be minimized is given by

$$J = \sum_{i=1}^{N} e_i^2 \tag{37}$$

where e_i is defined as the magnitude of the difference between the actual satellite position and the target position. An individual, c should now be formed from a number of genes equal to the controller parameters to be determined. First, let's discuss

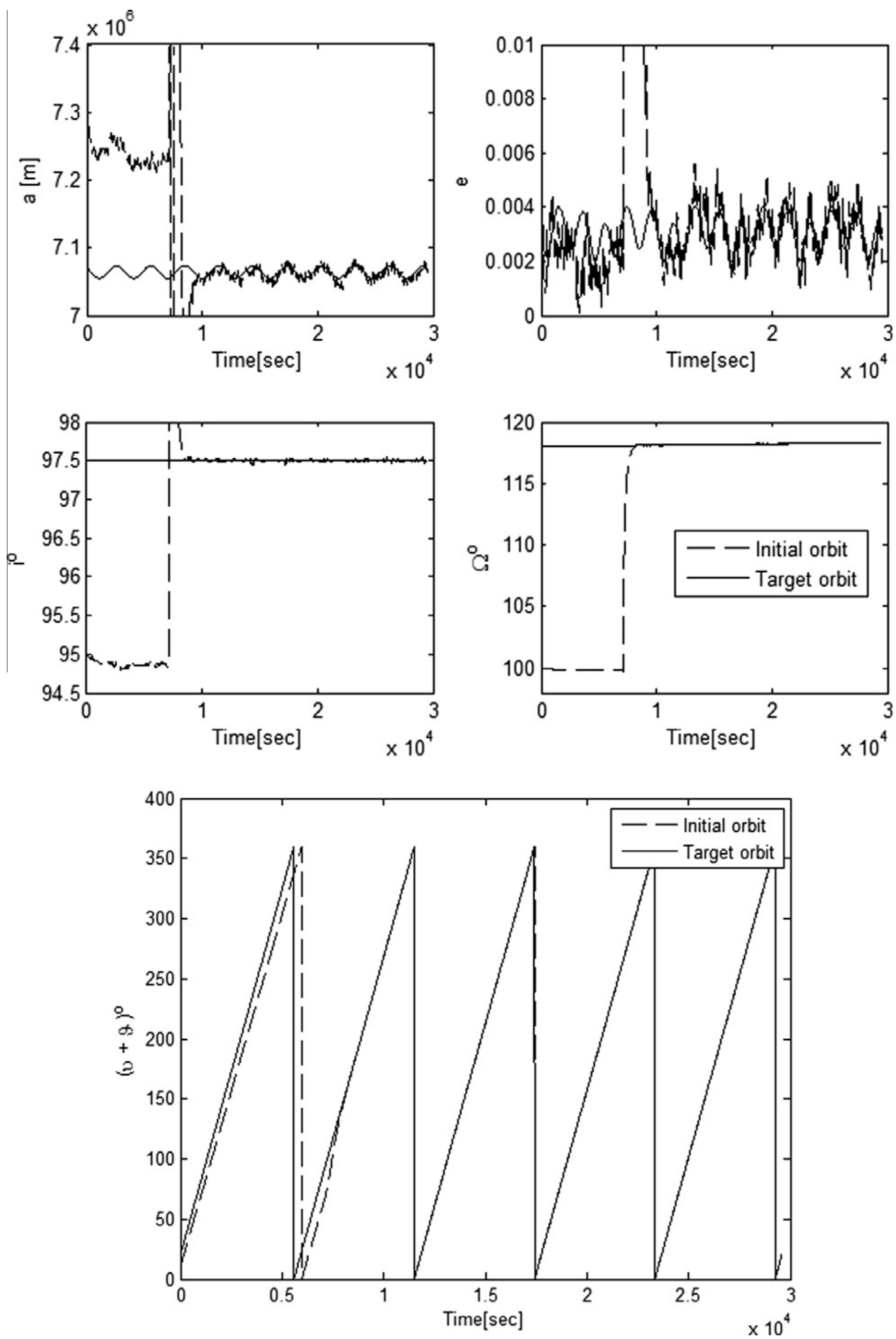

Figure 2 Orbital parameters for the initial and target orbits.

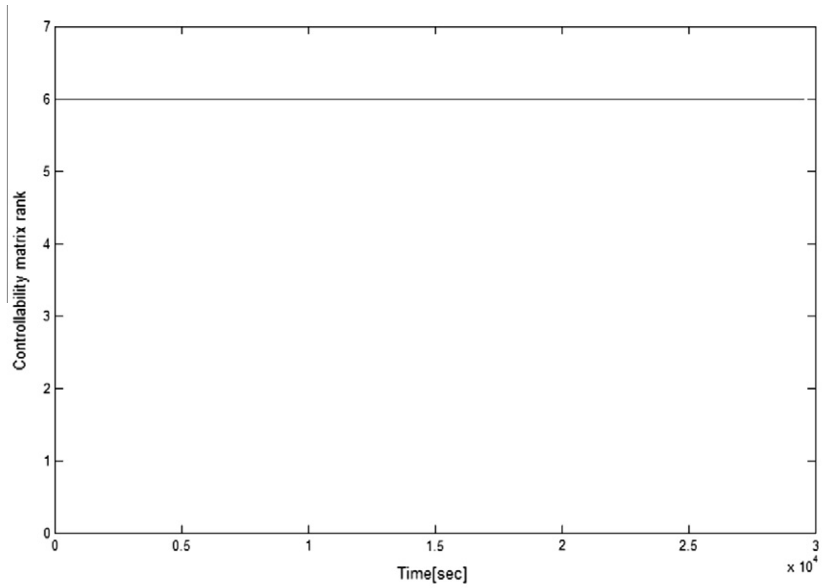

Figure 3 Controllability matrix rank during orbital maneuver execution.

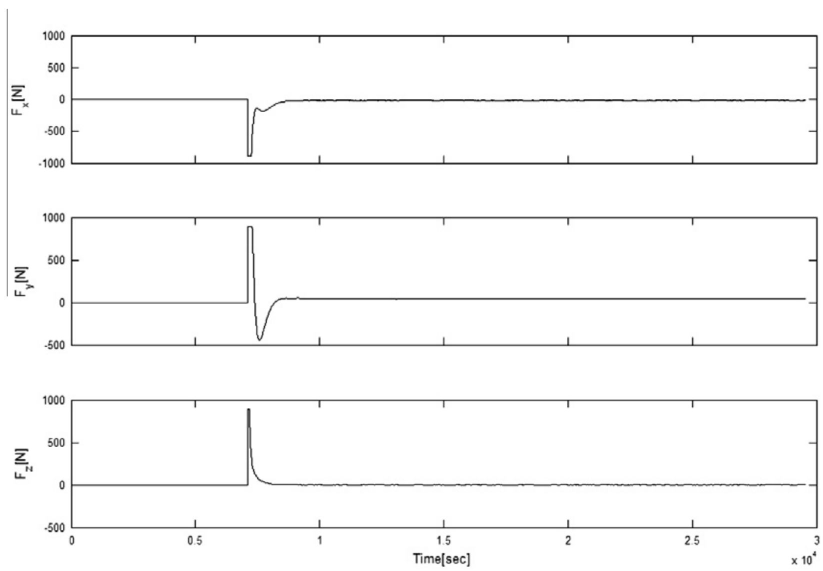

Figure 4 Thrust force time history.

the time constant parameter. If we chose a different time constant for each axis of the X_I, Y_I, and Z_I axes, there will exist an axis that reaches its steady state value fast and another one reaching its steady state value slow. The overall time constant of the system will be equal to the slowest one. Thus, there will be no gain at all if we select a certain axis to respond faster than the others, because the overall system will always be restricted to the slowest responding axis. Furthermore, faster responding axes require more control effort. Based on this discussion it is reasonable to select the time constant for all of the axes to be the same. In addition, the damping ratio is selected to be also the same for all of the axes. Thus, an individual c consists from,

$$c = [\tau_c \quad \zeta_c]^T \tag{38}$$

the time constant, τ_c and the damping ratio, ζ_c are directly related to the control gains through Eqs. (32) and (36). The choice of equal time constant, and damping ration for all of the axes implies that

$$k_x = k_y = k_z \tag{39}$$

and

$$k_{xd} = k_{yd} = k_{zd} \tag{40}$$

note that, during the execution of spacecraft orbital maneuver, the spacecraft must not hit the surface of the earth. This con-

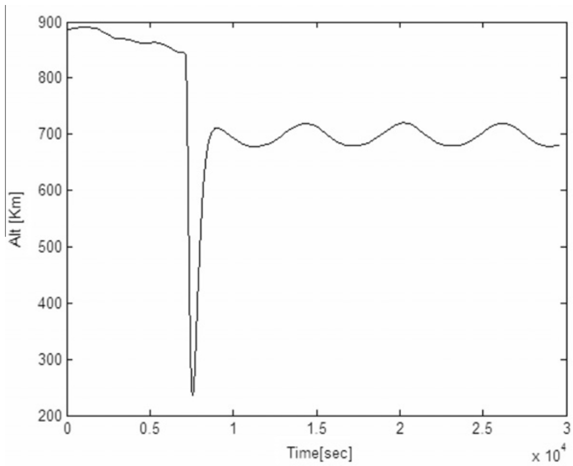

Figure 5 Satellite altitude above sea level.

dition (constraint) is expressed as a negative fitness function with a value of -10^{100} .

6. Controllability analysis

Controllability analysis is based mainly on the two matrices F_T and G_T. As seen from Eq. (27), $\omega_0 = -\mu_E/||R_I||^3$, which is not constant at all during maneuver execution. This behavior causes deviation of the performance of the control gains designed based on linear control theory. To alleviate such difficulty, RCGA are utilized to obtain the global optimum values of the gains. Thus, the matrix F_T in general, is time dependent. The controllability matrix is computed from the relation

$$C = [G_T \quad F_T G_T \quad F_T^2 G_T \quad F_T^3 G_T \quad F_T^4 G_T \quad F_T^5 G_T] \tag{41}$$

The controllability matrix given in Eq. (41) must have a full rank (i.e. 6).

7. Simulation parameters, results, and testing

The initial orbital parameters are a (semi major axis) = 72,71,200 m, e (orbit eccentricity) = 0.002, i (orbit inclination) = 95°, Ω (right ascension of ascending node) = 100°, ω (argument of perigee) = 0°, and υ (true anomaly) = 10°. The target orbital parameters of the case study satellite are a = 70,71,200 m, e = 0.003, i = 97.5°, Ω = 118°, ω = 0°, and υ = 20°. Epoch time (18/7/2010 0 h:0 m:0 s). Time step = 10 s. Thrust is applied after 7101 s measured since the epoch time. A maximum thrust value of 890 N is applied. The initial desired control loop natural frequency and damping ratio are 0.00357 rad/s and 0.91, respectively. After application of RCGA, the optimum values of the desired control loop natural frequency and damping ratio are found to be 0.004869 rad/s and 0.9621 respectively. Random disturbances are assumed to affect the satellite position and velocity with a standard deviation of 10 m, and 1 m/s, respectively. Fig. 2 shows the behavior of the satellite classical orbital elements over 30,000 s. And as clear in this figure, the satellite had a successful orbital maneuver after 10,000 s approximately.

Fig. 3 represents the rank of the controllability matrix defined earlier. As clarified in Fig. 3, the rank of the controllability matrix is six which indicate a full rank. Fig. 4 represents the thrust force required to execute the orbital maneuver. As shown in this figure, the maximum thrust bound is 890 N in all of the thrust directions.

A typical liquid rocket engine to provide such thrust is given in Larson and Wertz (1999). It is the R42 engine developed by Marquardt. Engine mass is 4.54 kg. The choice of any rocket engine in this case study was done based on those engines found in Larson and Wertz (1999). Some engines with less thrust force (ranging from 400 to 500 N) were utilized. But utilizing such low thrust engines had led to maneuver failure. Fig. 5 represents a check that the spacecraft did not hit the earth surface during its orbital maneuver. Note that, the sinusoidal behavior of the altitude and the semi-major axis is due to oblateness of the earth, as clarified in Section 3.1.

8. Conclusion

The proposed structure of the control algorithm had successfully brought the spacecraft from its initial orbit to its target orbit. Cowell's formulation enabled a simple design of the control algorithm based on linear control theories. Nonlinearities of the actual spacecraft behavior resulted from earth oblateness, aerodynamic drag, and bounded thrust budget are taken into consideration during the controller design process. RCGA are effectively used to compute the global optimum gains of the controller taking into account various sources of nonlinearity. Simulation results indicated that the spacecraft did not hit the earth surface during the execution of its orbital maneuver.

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