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An Effective CFO Estimation Method Based on Unitary Transformation for Interleaved OFDMA Uplink Systems

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Abstract

This paper presents an effective carrier frequency offset (CFO) algorithm based on unitary transformation and MUSIC technique, for interleaved orthogonal frequency-division multiple-access (OFDMA) uplink systems. Compared with other recently proposed estimation approaches, the proposed method offers several advantages. Firstly, the proposed method reduces the computational complexity significantly by dealing with only real-valued computations. Secondly, the proposed method incorporates the data stacking technology and the unitary transformation, which adds structure to the data model for the implementation of the proposed method, and leads to an improved estimation performance. Simulation results demonstrate the efficacy of the proposed algorithm.

Keywords: Carrier Frequency Offset (CFO); orthogonal frequency-division multiple-access (OFDMA); unitary transformation.

1. Introduction

Effective carrier frequency offset estimation is one of the most essential technologies for Orthogonal frequency-division multiple access (OFDMA) uplink systems [1]-[7]. In [1], an effective iterative scheme is proposed. However, it requires an exhaustive grid searching to estimate each CFO, which results in unattractive complexity. In [2], a CFO estimation scheme is reported by making use of the repetitive structure of the users' training sequences. In [3], a simple iterative algorithm is derived via exploiting the

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fact that the tile structure in 802.16e. The algorithms based on MUSIC and ESPRIT are proposed in [4-5], respectively. A CFO matrix algorithm is reported in [6]. In [7], an optimum CFO estimator is proposed by truncating the series expansion of the correlation matrix in maximum likelihood function.

To reduce the computational load but gain an improved performance, an efficient CFO estimation algorithm is proposed for the interleaved OFDMA uplink systems in this paper. The proposed algorithm possesses several attractive features. Firstly, it is derived in terms of the real-valued computations throughout except the data preprocessing, which reduces the computational complexity considerably, since the dimension of the matrices is not increased. Secondly, it provides improved estimation accuracy by using the data stacking technology and the unitary transformation.

The rest of this paper is organized as follows. Section II introduces the OFDMA uplink systems model. Section III presents the proposed method. Simulation results are given in Section IV. Section V draws a conclusion.

2. Data model for OFDMA uplink

Consider an interleaved OFDMA uplink system with N sub-carriers shared by K users. The N sub-carriers are evenly divided into Q $(Q > K)$ sub-channels, each of which has $P = N/Q$ sub-carriers. Each user only occupies one sub-channel, and the q th sub-channel is assigned to the k th user, whose sub-carrier indices is defined as $\{q, Q+q, \cdots, (P-1)Q+q\}$ $(q = 0,1,\cdots,Q-1)$, After the cyclic prefix (CP) is removed, the signal sample of the n th sub-carrier of one OFDMA block at BS can be described as

$$y(n) = \sum_{k=1}^{K} y_k(n) + z(n) \quad (n = 0,1,\cdots,N-1) \tag{1}$$

where K denotes the total number of users. $z(n)$ is the AWGN with zero-mean and equal variance σ^2. $y_k(n) = \sum_{p=0}^{P-1} H_{k,p} X_{k,p} e^{j\frac{2\pi}{N}n(pQ+q+\xi_k)}$ is the signal of the n th sub-carrier of the k th user at the receiver, where $H_{k,p}$ and $X_{k,p}$ stand for the channel frequency response and the data symbol on the p th sub-carrier of the kth user, respectively. ξ_k is the normalized CFO of the k th user.

From (1), we can construct a $Q \times P$ matrix $\boldsymbol{Y}$ by a data stacking technology:

$$\boldsymbol{Y} = \begin{bmatrix} y(0) & y(1) & \cdots & y(P-1) \\ y(P) & y(P+1) & \cdots & y(2P-1) \\ \vdots & \vdots & \ddots & \vdots \\ y(N-P) & y(N-P+1) & \cdots & y(N-1) \end{bmatrix} = \boldsymbol{VS} + \boldsymbol{Z} = \boldsymbol{V}[\boldsymbol{C} \odot (\boldsymbol{BW})] + \boldsymbol{Z} \tag{2}$$

where $\odot$ indicates an element-by-element product. The kth column of $\boldsymbol{V}$ is $\boldsymbol{v}_k = [1, e^{j2\pi\theta_k}, e^{j4\pi\theta_k}, \cdots, e^{j2\pi(Q-1)\theta_k}]^{\mathrm{T}}$, where $\theta_k = (q+\xi_k)/Q$ is the effective CFO of the kth user. $\boldsymbol{b}_k$ and $\boldsymbol{c}_k$ are the kth row of $\boldsymbol{B}$ and $\boldsymbol{C}$, respectively, with $\boldsymbol{c}_k = [1, e^{j2\pi\theta_k/P}, \cdots, e^{j2\pi(P-1)\theta_k/P}]$, and $\boldsymbol{b}_k = [H_{k,1}X_{k,1}, \cdots, H_{k,P}X_{k,P}]$. $\boldsymbol{W}$ is an IFFT matrix and $\boldsymbol{Z}$ is the white Gaussian noise matrix.

3. Algorithm formulation

Let $\boldsymbol{y}_l, \boldsymbol{s}_l, \boldsymbol{z}_l$ $(l = 1,2,\cdots,P)$ denote the l th column of $\boldsymbol{Y}, \boldsymbol{S}, \boldsymbol{Z}$, respectively. Thus, we have

$$\boldsymbol{y}_l = \boldsymbol{V}\boldsymbol{s}_l + \boldsymbol{z}_l \tag{3}$$

where the Vandermonde matrix $\boldsymbol{V}$ is defined in (2) with $\boldsymbol{v}_k = [1, e^{j2\pi\theta_k}, \cdots, e^{j2\pi(Q-1)\theta_k}]^{\mathrm{T}}$.

Define a phase factor (CFO matrix) matrix $\boldsymbol{\Phi}_Q$ as $\boldsymbol{\Phi}_Q = \mathrm{diag}[e^{j\pi(1-Q)\theta_1}, e^{j\pi(1-Q)\theta_2}, \cdots, e^{j\pi(1-Q)\theta_K}]$, then (3) can be factored as

$$\boldsymbol{y}_l = \boldsymbol{V}\boldsymbol{\Phi}_Q\boldsymbol{\Phi}_Q^{-1}\boldsymbol{s}_l + \boldsymbol{z}_l = \boldsymbol{A}\boldsymbol{\Phi}_Q^{-1}\boldsymbol{s}_l + \boldsymbol{z}_l \tag{4}$$

where $\boldsymbol{A} = \boldsymbol{V}\boldsymbol{\Phi}_Q = [\boldsymbol{a}_1, \cdots, \boldsymbol{a}_K]$. Since Q which represents the sub-channel number is an even number in a practical OFDMA systems, we know that $\boldsymbol{a}_k$ is a centro-Hermitian vector and has the following form

$$\boldsymbol{a}_k = [e^{j\pi(1-Q)\theta_k}, \cdots, e^{-j\pi\theta_k}, e^{j\pi\theta_k}, \cdots, e^{j\pi(Q-1)\theta_k}]^{\mathrm{T}} \tag{5}$$

The following developments rely on the fact that a complex centro-Hermitian vector can be converted into a real vector by a unitary transformation [8]. Notice that $\boldsymbol{a}_k$ contains the information of the CFO of k th user. Throughout the sequel, $\boldsymbol{a}_k$ is referred to as the CFO steering vector and the matrix $\boldsymbol{A}$ is referred to as the CFO steering matrix.

Firstly, we define the $Q \times Q$ unitary matrix as $\boldsymbol{Q}_Q = \frac{1}{\sqrt{2}}\begin{bmatrix} \boldsymbol{I}_m & j\boldsymbol{I}_m \\ \boldsymbol{\Pi}_m & -j\boldsymbol{\Pi}_m \end{bmatrix}$, where $\boldsymbol{I}_m$ denotes the $m \times m$ identity matrix. $\boldsymbol{Q}_Q$ is a sparse unitary matrix that transforms the centro-Hermitian vector $\boldsymbol{a}_k$ into the real CFO steering vector $\boldsymbol{d}_k = \boldsymbol{Q}_Q^{\mathrm{H}}\boldsymbol{a}_k$, where $\boldsymbol{d}_k$ has the following form

$$\boldsymbol{d}_k = \sqrt{2} \times \left[\cos\left((Q-1)\pi\theta_k\right), \cdots, \cos\left(\pi\theta_k\right), -\sin\left((Q-1)\pi\theta_k\right), \cdots, -\sin\left(\pi\theta_k\right)\right]^{\mathrm{T}} \tag{6}$$

It should be notice that $\boldsymbol{A} = [\boldsymbol{a}_1, \cdots, \boldsymbol{a}_K]$, we have $\boldsymbol{Q}_Q^{\mathrm{H}}\boldsymbol{A} = [\boldsymbol{d}_1, \cdots, \boldsymbol{d}_K]$. In fact, premultiplying both sides in (5) by the unitary matrix $\boldsymbol{Q}_Q^{\mathrm{H}}$, we have the equivalent expression as follows

$$\boldsymbol{Q}_Q^H\boldsymbol{y}_l = \boldsymbol{Q}_Q^H\boldsymbol{A}\boldsymbol{\Phi}_Q^{-1}\boldsymbol{s}_l + \boldsymbol{Q}_Q^H\boldsymbol{z}_l \,. \tag{7}$$

Let $\tilde{\boldsymbol{y}}_l = \boldsymbol{Q}_Q^H\boldsymbol{y}_l$, $\boldsymbol{D} = \boldsymbol{Q}_Q^H\boldsymbol{A}$, $\tilde{\boldsymbol{s}}_l = \boldsymbol{\Phi}_Q^{-1}\boldsymbol{s}_l$, $\tilde{\boldsymbol{z}}_l = \boldsymbol{Q}_Q^H\boldsymbol{z}_l$, (8) can be rewritten into

$$\tilde{\boldsymbol{y}}_l = \boldsymbol{D}\tilde{\boldsymbol{s}}_l + \tilde{\boldsymbol{z}}_l \,. \tag{8}$$

Define a real matrix as $\boldsymbol{x}_l = [\mathrm{Re}\{\tilde{\boldsymbol{y}}_l\}, \mathrm{Im}\{\tilde{\boldsymbol{y}}_l\}]$, and let $\boldsymbol{R}$ denote the covariance matrix of $\boldsymbol{x}_l$. We know $\boldsymbol{R} = \mathrm{E}\{\boldsymbol{x}_l\boldsymbol{x}_l^{\mathrm{T}}\} = \boldsymbol{D}\mathrm{E}\{\tilde{\boldsymbol{s}}_l\tilde{\boldsymbol{s}}_l^{\mathrm{H}}\}\boldsymbol{D}^T + \boldsymbol{Q}_Q^H\mathrm{E}\{\boldsymbol{z}_l\tilde{\boldsymbol{z}}_l^{\mathrm{H}}\}\boldsymbol{Q}_Q = \boldsymbol{D}\mathrm{E}\{\tilde{\boldsymbol{s}}_l\tilde{\boldsymbol{s}}_l^{\mathrm{H}}\}\boldsymbol{D}^T + \mathrm{E}\{\boldsymbol{z}_l\tilde{\boldsymbol{z}}_l^{\mathrm{H}}\}$ (9)

According to (8) and (9), it can be readily shown that the covariance matrix $\boldsymbol{R}$ of the real matrix $\boldsymbol{x}_l$ is same with that of the complex matrix $\boldsymbol{y}_l$. The eigenvalue decomposition (EVD) of $\boldsymbol{R}$ is

$$\boldsymbol{R} = \sum_{m=1}^{Q}\lambda_m\boldsymbol{u}_m\boldsymbol{u}_m^{\mathrm{H}} = \boldsymbol{U}_s\boldsymbol{\Lambda}_s\boldsymbol{U}_s^{\mathrm{H}} + \boldsymbol{U}_n\boldsymbol{\Lambda}_n\boldsymbol{U}_n^{\mathrm{H}} \tag{10}$$

where $\lambda_1 \geq \lambda_2 \geq \cdots \geq \lambda_K > \lambda_{K+1} = \cdots = \lambda_Q$, $\boldsymbol{U}_s = [\boldsymbol{u}_1, \cdots, \boldsymbol{u}_K]$, and $\boldsymbol{U}_n = [\boldsymbol{u}_{K+1}, \cdots, \boldsymbol{u}_n]$. $\boldsymbol{U}_s$ and $\boldsymbol{U}_n$ are composed of the eigenvectors corresponding to the K "biggest" and the $Q-K$ "smallest" eigenvalues.

Notice that θ_k has one important property, i.e., different users have distinct effective CFOs. So, if the k th user occupies sub-channel $\{q\}$, then $\theta_k \in \left((q-0.5)/Q, (q+0.5)/Q\right)$, since $|\xi_k| < 0.5$. Because different users occupy different sub-channels, their effective CFOs fall in non-overlapping ranges.

Having the above analysis, it is readily to know that, θ_k $(k = 1, 2, \cdots, K)$ correspond to the largest K local maximum of

$$P(\theta) = \frac{1}{\boldsymbol{d}^{\mathrm{T}}(\theta)\boldsymbol{U}_n\boldsymbol{U}_n^{\mathrm{T}}\boldsymbol{d}(\theta)} \tag{11}$$

where $\boldsymbol{d}(\theta)$ is a real vector, and has the same structure with (6).

So, the carrier frequency offsets of K users can be obtained via searching the peaks of (11). It is necessary to note that $\boldsymbol{d}(\theta)$ is a real vector and $\boldsymbol{U}_n$ is a real matrix. Thus, the process of peak searching is based on real value totally.

Summary of Unitary-MUSIC

(1) Form the data matrix $\boldsymbol{Y}$ according to (2) and compute the covariance matrix $\boldsymbol{R} = \frac{1}{2P}\sum_{l=1}^{2P}\boldsymbol{x}_l\boldsymbol{x}_l^{\mathrm{T}}$, where $\boldsymbol{x}_l = [\mathrm{Re}\{\boldsymbol{Q}_Q^H\boldsymbol{y}_l\},\mathrm{Im}\{\boldsymbol{Q}_Q^H\boldsymbol{y}_l\}]$.

(2) Compute the EVD of $\boldsymbol{R} = \sum_{m=1}^{Q}\lambda_m\boldsymbol{u}_m\boldsymbol{u}_m^{\mathrm{H}}$, where $\lambda_1 \ge \lambda_2 \ge \cdots \ge \lambda_K \ge \cdots \ge \lambda_Q$.

(3) Obtain the noise subspace estimate $\boldsymbol{U}_n$ via the $Q-K$ "smallest" eigenvectors of $\boldsymbol{R}$.

(4) Find the largest K peaks of (11) to estimate the effective CFOs θ_k $(k=1,2,\cdots,K)$.

(5) Compute the normalized CFOs ξ_k via $\xi_k = Q\theta_k - q(k=1,2,\cdots,K)$.

4. Simulation

In this section, the performance of the proposed algorithm has been assessed by several simulations. Assume that the simulated interleaved OFDMA system has $N = 512$, and $Q = 8$. The length of CP is 64. Each user transmits his data stream using QPSK signals. The multipath fading channel has 6 paths and an exponentially decaying power delay profiles is selected. The normalized CFOs are uniformly distributed in $(-0.5, 0.5)$ with 500 random trial runs. The normalized root mean square error (RMSE) is used to evaluate the performance of the proposed method.

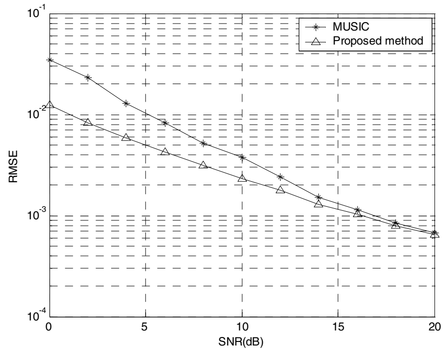

Figure 1. RMSE curves versus SNR

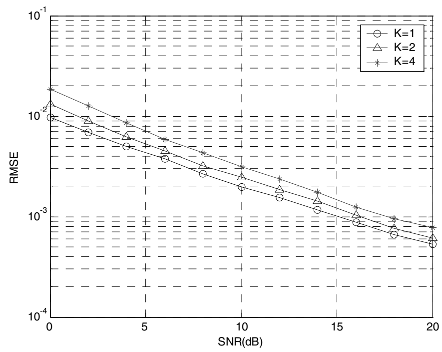

Figure 2. RMSE curves versus SNR for various K

Case 1. Fig.1 shows the normalized RMSE performance curves of the CFO estimation obtained via the proposed method and the MUSIC method. In this simulation, assume $K = 3$. From Fig.1, we know that, the RMSE of each method improves as SNR increases. But the estimation accuracy of the proposed method is obviously outperforms that of the MUSIC method.

Case 2. In the second test example, we examine the performance of the proposed method for different active user number. Fig.2 shows the RMSE curves of the proposed algorithm for $K = 1, 2, 4$, respectively. As shown in Fig.2, the estimation accuracy of CFOs increases as the number of the active user reduces.

5. Conclusions

In this paper, an efficient algorithm is proposed for CFO estimation in interleaved OFDMA uplink systems. The proposed algorithm can be enforced mainly based on real-valued computations by using the unitary transformation, which leads to a reduced significantly computational complexity. The improved CFO estimation accuracy is obtained by using the data stacking technique and the unitary transformation. Simulation results demonstrate the effectiveness of the proposed method.

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