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Drag coefficient for rigid vegetation in subcritical open channel

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Abstract

Vegetation in fluvial systems plays an important role in environmental and ecological aspects. Vegetation can cause energy dissipation via drag force due to the interaction between vegetation and flow, and the dimensionless drag coefficient (Cd) is of great importance for understanding and predicting the drag force. Currently Cd was determined through model experiments and mass literature about subcritical open channel flow with rigid vegetation can be obtained. A data processing method was introduced to seek an empirical prediction for Cd. Firstly, the influencing factors of Cd was determined by dimensional analysis method. Then based on the collected data, we used multi-parameter regression analysis to obtain an empirical expression of Cd with those dimensionless influencing factors. It is interesting to find that drag coefficients for subcritical and supercritical flows have opposite trends with varied vegetation density, and Cd remains almost constant with high Froude number for subcritical flow, or the Froude number has little impact on Cd; while for subcritical flow with low Froude number, the effect of Froude number cannot be ignored.

Keywords: drag coefficient; subcritical flow; rigid vegetation;

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1. Introduction

Aquatic plant is common in natural rivers, floodplains, and irrigation channels systems, and can affect the ecological environment in many respects. For instance, it improves water quality by producing oxygen and consuming excess nutrients (Tang et al., 2008; Nepf et al., 2007), and provides habitat for aquatic animals by creating overwater and low-flow regions (Kemp et al., 2000). Vegetation increases resistance to flow and thus affects the speed of water, enlarge local water level, reduce flood discharge and controls the fate of sediments. Generally vegetation-induced resistance can be considered as a superposition of viscous friction at the water-plant surface interface and form (pressure) drag often associated with flow separation (Albayrak et al., 2012), and the flow resistance caused by a single plant (F_p) can be described with the drag coefficient C_d as,

$$F_p = \frac{1}{2} C_d A_0 \rho u^2 \tag{1}$$

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where A_0 is the vertical projection area of a single plant facing the current, u is the velocity approaching the stem, and ρ is the fluid density. All the parameters in Eq.(1) except for C_d can be obtained directly by model test, and therefore C_d is crucial for a sound prediction of vegetation drag.

Studies on flow resistance due to cylindrical roughness can provide a better understanding for the resistance in vegetated open channels, for example, the relationship between the vegetation drag coefficient and some other related parameters (vegetation density, arrangement, average flow velocity, etc.) had been theoretically analyzed (Li and Shen, 1973). Many prior studies attempted to account for the effect of vegetation with those empirically determined resistance coefficients, and believed that C_d is varied with different flow condition . Schlichting et al.(1982) suggested the following relationship between C_d and stem Reynolds number R_d,

$$\begin{cases} C_d = 3.07 R_d^{-0.168} & (R_d < 800) \\ C_d = 1 & (800 \le R_d < 8000) \\ C_d = 1.2 & (8000 < R_d < 10^5) \end{cases} \tag{2}$$

where $R_d = \frac{\bar{u}d}{\upsilon}$, υ is the kinematic fluid viscosity, d is the stem diameter, $\bar{u}$ is the depth-averaged streamwise velocity, and Cheng and Nguyen (2010) conducted a series of experiments and presented an empirical relationship between C_d and R_v as,

$$C_d = \frac{50}{R_v^{0.43}} + 0.7(1 - e^{\frac{-Rv}{15000}}) \qquad (R_v = 5.2 \sim 5.6 \times 10^5) \text{ ,} \tag{3}$$

where $\mathrm{R}_v = \frac{\bar{u}r_v}{\upsilon}$, r_v is a vegetation-related hydraulic radius.

Ishikawa et al. (2000) conducted laboratory tests on the drag force of rigid vegetation placed in an open channel flow with varied Froude number ($\mathrm{F}r = \bar{u}/\sqrt{gH}$),for Fr <1, the flow is subcritical; while for Fr >1, the flow is supercritical), and indicated that the plant drag coefficient may decrease with the increase of Fr, and an empirical expression has been presented,

$$C_d = -0.32\mathrm{Fr} + 1.24 \tag{4}$$

Wang and Tang (2013) conducted a series of experiments and obtain a best-fit function describing the relationship between C_d and R_{ev} in subcritical flow.

$$C_d = \frac{90}{R_{ev}^{0.5}} + 4.5\frac{d}{\mathrm{H}} - 0.303\ln\lambda - 0.9 \tag{5}$$

Where λ is the area faction of cylinders (stems) and defined as the cross-sectional area per unit bed area in the array of cylinders, or $\lambda = \frac{\pi N(d/2)^2}{BL}$, R_{ev} is a parameter similar to the Reynolds number, $R_{ev} = \frac{(1-\lambda)\bar{u}H}{\upsilon}$, H is the flow depth.

For supercritical flow, Kothyari and Hayashi (2000) suggested a mathematical expression of C_d with Fr, vegetation density λ and R_d based on their own and Ishikawa's experimental data,

$$C_d = 1.8 R_d^{-3/50}[1 + 0.45\ln(1 + 100\lambda)](0.8 + 0.2\mathrm{Fr} - 1.5\mathrm{Fr}^2) \tag{6}$$

since flows in natural river channels are commonly subcritical, we collected experimental data about drag coefficient for rigid vegetation in subcritical open channel flow from literature, and an empirical formula about C_d was obtained by multi-parameter regression analysis method.

2. Data processing

2.1. Data collection

Most of the data we have collected used different parameters and experimental methods, and had different descriptions about flow resistance, so we need to reprocess the data to obtain the parameters with the same forms of expression to facilitate the processing. Here the effects of the side wall and the bottom of the channel are reasonably negligible, since vegetable resistance accounts for the vast majority of the total resistance of flow, or the main flow energy consumption is mainly caused by the vegetative resistance. The drag force on vegetation was selected to draw the effect of vegetation on the flow. For a two-dimension flow along the vegetation section (Fig.1), the forces include the water gravity, the friction force of the bed, dynamic pressure acted at the upstream and downstream section, and the resistance by vegetation.

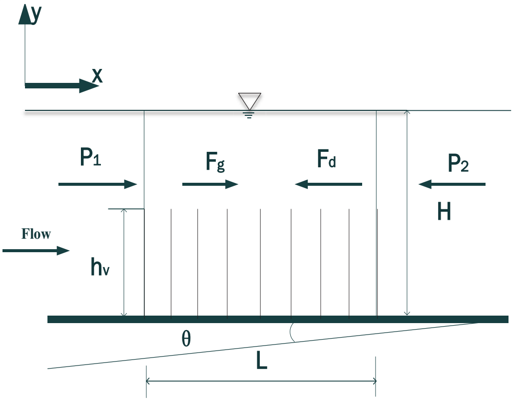

Fig. 1 Sketch of open channel flow with submerged rigid vegetation

The balance equation along the *X* direction is:

$$P_1 + F_g = P_2 + F_d \ ,$$

where F_g is the gravity of free body along the *X* direction, and $F_g = \rho V g \sin\theta$, $V = HBL$, *B* is the channel width, *L* is the length of the vegetation zone, h_v is stem height, P_1 is dynamic pressure at the upstream section and P_2 is that at the downstream section. Approximately, $P_1 = P_2$, θ is a very small value for natural rivers, and $\sin\theta$ approximatively equals $\tan\theta$; F_d is the resistance caused by all the stems and can be written as,

$$F_d = \sum F_p = \frac{1}{2} C_d A_p \rho u^2 \tag{7}$$

where F_d is the flow resistance of all stems, A_p is the vertical projection area of the total stems facing the current, and $A_p = \sum A_0 = N d h_v$, *N* is the total plant number. According to Eq.(1) and Eq. (7)， we can deduce an expression of C_d,

$$C_d = \frac{2gVS}{A_p u^2} = \frac{2gBLHS}{A_p u^2} \tag{8}$$

For most open channel flow with submerged vegetation, the depth-averaged velocity is not that approaching the stem. The ratio between the apparent vegetation layer velocity and the channel average velocity is (Stone and shen, 2002; Baptist, 2005),

$$\frac{\bar{u}}{u} = \sqrt{h^*}\left[\frac{1 - d\sqrt{N_0}}{1 - dh^*\sqrt{N_0}}\right] \tag{9}$$

where h^* is relative submergence depth, $h^* = h_v/h$, and for emergent cases, $h^*=1$; N_0 is the number of stems per unit bed area. Since the square bracket term of Eq.(9) is close to 1.0 for most practical cases, which means $\frac{\bar{u}}{u} = \sqrt{h^*}$, and then Eq.(9) can be written as,

$$C_d = \frac{2gBLHS}{A_p u^2} = \frac{2gBLHSh^*}{A_p \bar{u}^2} \tag{10}$$

Eq.(10) is used for calculating the value of C_d in this paper .

2.2. Influencing factors of C_d

Previously, many related experimental research have been undertaken to explore the vegetation-related resistance under different conditions, such as plant morphology and structure, flow velocity and water depth, hydraulic characteristics, plant species and distribution density. We determined some basic physical quantities by analyzing the physical process of drag force,

$$F(u, h, h_{\mathrm{v}}, \rho, s, d, \upsilon) = 0\ ,$$

where s is center to center distance between stems. By dimension analysis method, we determined that C_{d} was affected by several dimensionless parameters,

$$F(R_{\mathrm{d}}, \mathrm{F}r, h^{*}, \lambda) = 0\ ,$$

we collected about 500 sets of experimental data in subcritical flow, and the drag force acted on submerged or emergent rigid vegetation were used to calculated the C_{d}. Details were listed in Table 1.

Table 1. Summary of Experiments Conducted by Previous Investigators

Data sources	Vegetation Zone		Vegetation Configuration					
	Width L(m)	Length d(m)	Stem Diameter hv(m)	Stem Height density λ (%)	vegetation S(%)	bed gradient emergent	submerged or	B(m)
Schoneboom et al. [2011]	0.6	18.5	0.01	0.245	0.183	0.06-0.92	sub-	
Cheng and Nguyen[2011]	0.3	9.6	0.0032-0.0083	0.1	0.0045-0.12	0.4	sub-	
Dunn et al. [1996]	0.91	2.44	0.00635	0.097-0.161	0.14-1.32	0.36	sub-	
Kubrak et al.[2008]	0.58	3	0.007;0.0095	0.131-0.164	0.13-0.54	0.87;1.74	sub-	
Liu et al. [2008]	0.3	3	0.00635	0.076	0.31-1.57	0.3	sub-	
Meijer and Velzen [1998]	3	20.5	0.008	0.45-1.5	0.32-1.29	0.055-0.205	sub-	
Okamoto and Nezu [2010]	0.4	10	0.008	0.03-0.1	4.78	0.019-0.241	eme-,sub-	
Shimizu et al. [1991]	0.5	6	0.001	0.041	0.44-0.79	0.066-0.7	sub-	
Ston and shen [2002]	0.45	11	0.0032-0.0127	0.124	0.55-6.1	0.01-4.4	sub-	
Tang et al.[2014]	0.42	8	0.0006	0.06	1.13-2.83	0.0024-0.352	sub-	
Yan [2008]	0.42	8	0.006	0.06	1.414-5.655	0.065-1.28	sub-	
Zhao et al. [2014]	0.3	6	0.0083	0.11	1.3-12	0.04-1.02	eme-	

3. Analysis and discussion of data

3.1. Variation of drag coefficient with Reynolds number

Fig.2 shows the variation of C_{d} with R_{d}, and the change trend indicates that C_{d} exponentially decreased with R_{d} when the values of λ and h^{*} were fixed. To empirically describe the above relationship between C_{d} and R_{d}, we proposed a best-fit function,

$$C_{\mathrm{d}} = 4.6 R_{\mathrm{d}}^{-0.176} f_1(Fr, h^{*}, \lambda) \tag{11}$$

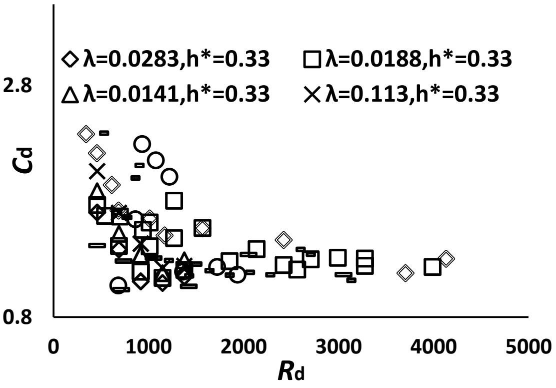

Fig. 2 Variation of C_d with R_d under fixed λ and h^*

One can find that the Eq.(11) is similar to Eq.(2) when $R_d<800$, and proved that the expression is reasonable.

3.2. Variation of Drag coefficient with vegetation density λ

To obtain the relationship between C_d and λ, a C_{d1} which is the ratio of C_d to R_d was used for further analysis,

$$\mathrm{C}_{d1} = \frac{\mathrm{C}_d}{4.6R_d^{-0.176}} = f_1(Fr, h^*, \lambda) \tag{12}$$

In Eq.(12), f_1 is a multivariate function including variables Fr, h^* and λ. For subcritical flow conditions (i.e., $Fr<1.0$), Kouwen and Fathi-Moghadam (2000) demonstrated that the flow resistance is independent on Fr, and here we temporarily ignore the influence of Fr in further analysis.

The variation of C_{d1} with λ is shown in Fig.3, and we can conclude a logarithmic decrease of C_{d1} with λ for a fixed value of h^*, then we obtain an optimum calculating formula by non-linear fitting,

$$C_{d1} = (-0.2\ln\lambda + 0.04) f_2(\mathrm{F}r, h^*) \tag{13}$$

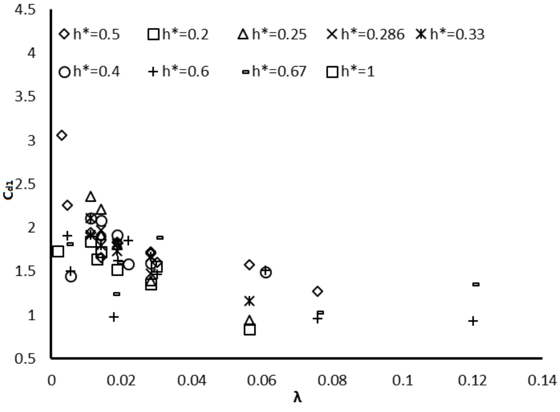

Fig.3 Variation of C_{d1} with λ under different h^*

3.3. Effects of Fr and h* on Cd

Similarly, a ratio C_{d2} was selected for further analysis，

$$C_{d2} = \frac{C_{d1}}{-0.2\ln\lambda + 0.04} = f_2(\mathrm{F}r, h^*) \tag{14}$$

where f_2 is a multivariate function including variables h^* and Fr.

The variation of C_{d2} with Fr was shown in Fig.4, and C_{d2} remains almost constant under fixed λ when Fr>0.1, or Fr has little impact on C_d; whereas C_d exponentially decreases with Fr when Fr<0.1. Then a subsection model can be obtained based on data fitting,

$$\begin{cases} f_2(\mathrm{Fr}, h^*) = 0.23h^* + 0.8 & (0.1 < \mathrm{Fr} < 1.0) \\ f_2(\mathrm{Fr}, h^*) = (0.19h^* + 0.67)\mathrm{Fr}^{-0.48} & (0 < \mathrm{Fr} < 0.1) \end{cases} \tag{15}$$

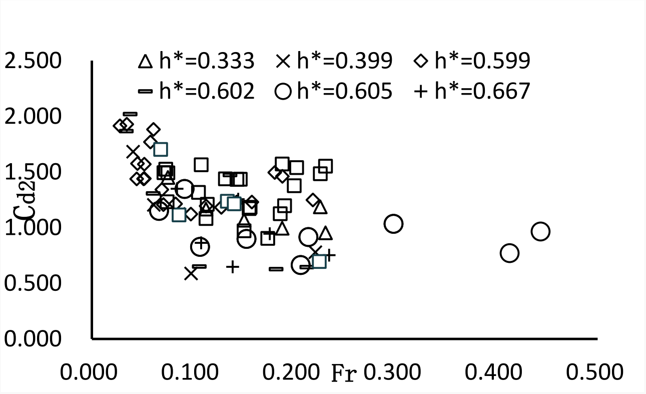

Fig.4 Variation of C_{d2} with Fr under fixed λ

3.4. Complete empirical expression of C_d with R_d, λ, h^* and Fr

According to Eq.(11-15), An optimum empirical formula was obtained by non-linear fitting,

$$\begin{cases} C_\mathrm{d} = 5.6R_\mathrm{d}^{-0.176}(-0.2\ln\lambda + 0.04)(0.23h^* + 0.8) & (0.1 < \mathrm{Fr} < 1.0) \\ C_\mathrm{d} = \dfrac{4.8R_\mathrm{d}^{-0.176}(-0.2\ln\lambda + 0.04)(0.19h^* + 0.67)}{\mathrm{Fr}^{0.48}} & (0 < \mathrm{Fr} < 0.1) \end{cases} \tag{16}$$

The C_d values calculated by Eq.(11) and Eq.(16) have been compared in Figs.(5-6), and the comparison shows that the drag coefficient C_d predicted using Eq.(16) agree well with that calculated by Eq.(11) for subcritical flow.

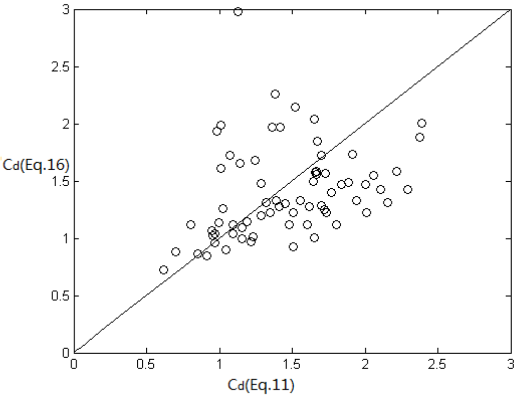

Fig.5 Comparison between C_d computed by Eq.(11) and those by Eq.(16) when Fr<0.1

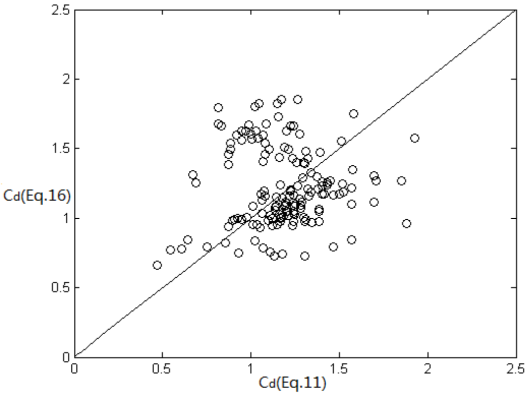

Fig.6 Comparison between C_d computed by Eq.(11) and those by Eq.(16) with 0.1<Fr<1.0

When vegetation is emergent in the flow, or h^*=1, is a constant, Eq.(16) can be simplified as,

$$\begin{cases} C_d = 5.77R_d^{-0.176}(-0.2\ln\lambda + 0.04) & (0.1 < Fr < 1.0) \\ C_d = 4.13R_d^{-0.176}(-0.2\ln\lambda + 0.04)Fr^{-0.48} & (0 < Fr < 0.1) \end{cases} \tag{17}$$

For supercritical flow, Eq.(6) shows a logarithmically increases of C_d with increasing λ, and as λ becomes large enough, C_d tends to a constant value (Kothyari and Hayashi,2009). While for subcritical flow, C_d logarithmically decreases with λ (Eq.(17) and Eq.(5)), we can preliminary concluded that C_d and λ have the opposite functional relationships in subcritical flow and supercritical flow and this has also been presented by Wu (1999) based on limited experimental results, whom concluded that the roughness coefficients for sub- and supercritical flows have opposite trends of variation.

4. Conclusions

The flow resistance caused by rigid vegetation stems in subcritical open channel flow is considered and drag coefficient C_d is selected for quantitatively accounting for the vegetation drag force, and different forms of empirical equation have been reviewed. Based on dimensional analysis, we found that the influence factors of C_d include hydraulic parameters such as Reynolds number, Froude number; and vegetation characteristics such as relative submergence of vegetation, and vegetation density. In this paper the four affecting factors were analyzed based on collected experimental data, and different forms of expression of C_d with varied parameters have been presented (Eqs.(11-18)), and the readers can select corresponding form for predicting C_d. For subcritical flow with Fr >0.1, the effect of Fr can be ignored; but for subcritical flow with Fr <0.1, the effect of Fr should be taken into account. We also concluded that the variation of C_d with vegetation density is different, but further experimental investigation should be conducted to reveal the variation.

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