Hierarchical majorana neutrinos from democratic mass matrices

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ABSTRACT

In this paper, we obtain the light neutrino masses and mixings consistent with the experiments, in the democratic texture approach. The essential ansatz is that ν_{Ri} are assumed to transform as "right-handed fields" $\mathbf{2_R} + \mathbf{1_R}$ under the $S_{3L} \times S_{3R}$ symmetry. The symmetry breaking terms are assumed to be diagonal and hierarchical. This setup only allows the normal hierarchy of the neutrino mass, and excludes both of inverted hierarchical and degenerated neutrinos.

Although the neutrino sector has nine free parameters, several predictions are obtained at the leading order. When we neglect the smallest parameters ζ_ν and ζ_R, all components of the mixing matrix U_{PMNS} are expressed by the masses of light neutrinos and charged leptons. From the consistency between predicted and observed U_{PMNS}, we obtain the lightest neutrino masses $m_1 = (1.1 \to 1.4)$ meV, and the effective mass for the double beta decay $\langle m_{ee} \rangle \simeq 4.5$ meV.

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1. Introduction

The observation of the neutrino oscillation [1,2] clarified finite masses of the neutrinos and lepton flavor nonconservation. Furthermore, the Daya Bay and RENO experiments [3,4] discovered that U_{e3} is nonzero and relatively large. However, these experiments shed us a further mysteries, *e.g.*, dozen of unexplained parameters, and the origin of the flavor. In particular, the lepton mixing matrix U_{PMNS} [5,6] is remarkably different from the quark mixing matrix U_{CKM} [7,8].

Innumerable models have been proposed so far, to explain the mysterious flavor structures of the standard model. As representative approaches, researchers explore the continuous or discrete flavor symmetries [9–11], and specific flavor textures [12,13]. In the texture approach, the democratic texture [14–23], realized by the $S_{3L} \times S_{3R}$ symmetry is widely studied. It assumes that the Yukawa interactions of the fermions $f = u, d, e$ have the "democratic matrix" in Eq. (1). In particular, Fujii, Hamaguchi and Yanagida [24] has derived the large mixing angles of light neutrinos by the seesaw mechanism [25], assuming almost degenerated neutrino Yukawa matrix $Y_\nu \sim c_\nu \text{diag}(1, 1, 1)$. This degenerated Y_ν is aesthetically unsatisfactory, because it is realized by assuming that the right-handed neutrinos ν_{Ri} transform as "left-handed fields" $\mathbf{2_L} + \mathbf{1_L}$ under the $S_{3L} \times S_{3R}$ symmetry. Furthermore, the degenerated Y_ν is undesirable in viewpoints of grand unified theory (GUT). A part of previous authors also have considered the democratic

matrices in SU(5) GUT [26]. However, degenerated Y_ν can not be unified to other Yukawa matrices.

Then, in this paper, ν_{Ri} are assumed to transform as "right-handed fields" $\mathbf{2_R} + \mathbf{1_R}$ under the $S_{3L} \times S_{3R}$ symmetry. The symmetry breaking terms are assumed to be diagonal and hierarchical, which is basically same as the previous studies. These assumptions realize hierarchical Y_ν and forbid degenerated Y_ν. It enables us to treat quarks and leptons uniformly under a simple framework. By the seesaw mechanism, we obtain the light neutrino masses and mixings consistent with the experiments. This setup only allows the normal hierarchy of the neutrino masses, and excludes both of inverted hierarchical and degenerated neutrinos.

Although the neutrino sector has nine free parameters, several predictions are obtained at the leading order. When we neglect the smallest parameters ζ_ν and ζ_R, the resulting neutrino matrix m_ν has only three parameters and then determined from the neutrino masses m_i. Therefore, all components of the mixing matrix U_{PMNS} are expressed by the masses of light neutrinos and charged leptons. From the consistency between predicted and observed U_{PMNS}, we obtain the lightest neutrino masses $m_1 = (1.1 \to 1.4)$ meV, and the effective mass for the double beta decay $\langle m_{ee} \rangle \simeq 4.5$ meV.

In the second-order perturbation, the predictability becomes little lower. However, the hierarchical Y_ν can be unified to other Yukawa interactions in SO(10) GUT or Pati–Salam models. Relating Y_ν and Y_u in some manner, several free parameters in the neutrino sector are expected to be removed. Meanwhile, the derivation in this paper remains only at tree level. The radiative corrections [35–37] and threshold correction [38] will modify the results. We leave it for our future work.

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This paper is organized as follows. In the next section, we review the Yukawa matrices with democratic texture. In Sect. 3 and 4, we present the parameter analysis of the light neutrino mass. Section 5 is devoted to conclusions and discussions.

2. Yukawa matrices with democratic texture

In the democratic mass matrix approach [14–23], the Yukawa matrices are assumed to be the following texture:

$$Y_f = \frac{K_f}{3} \begin{pmatrix} 1 & 1 & 1 \\ 1 & 1 & 1 \\ 1 & 1 & 1 \end{pmatrix} + \begin{pmatrix} \zeta_f & 0 & 0 \\ 0 & \epsilon_f & 0 \\ 0 & 0 & \delta_f \end{pmatrix}, \qquad (1)$$

where f is the SM fermions $f = u, d, e$. The first term (often called "democratic" mass matrix [17,18]) is realized by assigning fermions $f_{L,R}$ as $\mathbf{1_{L,R}} + \mathbf{2_{L,R}}$ under the $S_{3L} \times S_{3R}$ symmetry;

$$f'_{(L,R)i} = S^{(abc)}_{(L,R)ij} f_{(L,R)j}. \qquad (2)$$

For example, right-handed fields are explicitly written as

$$u_{Ri} = \begin{pmatrix} u_R \\ c_R \\ t_R \end{pmatrix}, \quad d_{Ri} = \begin{pmatrix} d_R \\ s_R \\ b_R \end{pmatrix}, \quad e_{Ri} = \begin{pmatrix} e_R \\ \mu_R \\ \tau_R \end{pmatrix}, \qquad (3)$$

and the left-handed fermions are written as similar way. The representation of $S^{(abc)}_{ij}$ is

$$S^{(123)}_{(L,R)} = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}, S^{(213)}_{(L,R)} = \begin{pmatrix} 0 & 1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 1 \end{pmatrix},$$

$$S^{(132)}_{(L,R)} = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & 1 & 0 \end{pmatrix}, \qquad (4)$$

$$S^{(321)}_{(L,R)} = \begin{pmatrix} 0 & 0 & 1 \\ 0 & 1 & 0 \\ 1 & 0 & 0 \end{pmatrix}, S^{(312)}_{(L,R)} = \begin{pmatrix} 0 & 0 & 1 \\ 1 & 0 & 0 \\ 0 & 1 & 0 \end{pmatrix},$$

$$S^{(231)}_{(L,R)} = \begin{pmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 1 & 0 & 0 \end{pmatrix}. \qquad (5)$$

The second term in Eq. (1) breaks the permutation symmetry slightly [15,16]. Here, the hierarchical relation

$$K_f \gg \delta_f \gg \epsilon_f \gg \zeta_f, \qquad (6)$$

is assumed. For the sake of simplicity of the discussion, we assume all breaking parameters are real. The discussion on the CP violation is given later.

The previous study by Fujii, Hamaguchi, and Yanagida [24] has derived the large mixing angles of light neutrinos by the seesaw mechanism, assuming almost degenerated neutrino Yukawa matrix $Y_\nu \sim c_\nu \mathrm{diag}(1, 1, 1)$. This degenerated Y_ν is aesthetically unsatisfactory, because it is realized by assuming that the right-handed neutrinos ν_{Ri} transform as "left-handed fields" $\mathbf{2_L} + \mathbf{1_L}$ under the $S_{3L} \times S_{3R}$ symmetry. Furthermore, the degenerated Y_ν is undesirable in viewpoints of grand unified theory (GUT).

Then, in this paper, ν_{Ri} are assumed to transform as "right-handed fields" $\mathbf{2_R} + \mathbf{1_R}$ under the $S_{3L} \times S_{3R}$ symmetry. The charge assignments of the leptons are shown in the Table 1. The symmetry breaking terms are assumed to be diagonal and hierarchical, which is basically same as the previous studies. These assumptions realize hierarchical Y_ν and forbid degenerated Y_ν. The texture of Yukawa matrices are determined as Eq. (1) for all SM leptons $f = \nu, e$.

Table 1
The charge assignments of the leptons under the discrete symmetries.

	S_{3L}	S_{3R}
l_{Li}	$\mathbf{1_L + 2_L}$	$\mathbf{1_R}$
ν_{Ri}, e_{Ri}	$\mathbf{1_L}$	$\mathbf{1_R + 2_R}$

Due to the charge assignment, the majorana mass term of ν_{Ri} invariant under the S_{3R} symmetry is found to be

$$M_R = m_R \left[\frac{K_R}{3} \begin{pmatrix} 1 & 1 & 1 \\ 1 & 1 & 1 \\ 1 & 1 & 1 \end{pmatrix} + c_R \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix} \right.$$
$$\left. + \begin{pmatrix} \zeta_R & 0 & 0 \\ 0 & \epsilon_R & 0 \\ 0 & 0 & \delta_R \end{pmatrix} \right]. \qquad (7)$$

Here, we assume the symmetry breaking term to M_R is also the diagonal. The term proportional to c_R is forbidden for the Y_ν by the assignment. In order to cancel out the hierarchy of Y_ν in the seesaw mechanism, the mass matrix (7) should be strongly hierarchical. Then, $K_R \gg c_R$ is required. Since the term $c_R \mathrm{diag}(1, 1, 1)$ is symmetric under $S_{3L} \times S_{3R}$, the parameter c_R need not necessarily be small parameter. Then, we assume

$$K_R \gg c_R \gg \delta_R \gg \epsilon_R \gg \zeta_R. \qquad (8)$$

When we analyze the matrices (1), at first the democratic matrix is diagonalized by the following unitary matrix:

$$U_{DC} = \begin{pmatrix} \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{6}} & \frac{1}{\sqrt{3}} \\ -\frac{1}{\sqrt{2}} & \frac{1}{\sqrt{6}} & \frac{1}{\sqrt{3}} \\ 0 & -\frac{2}{\sqrt{3}} & \frac{1}{\sqrt{3}} \end{pmatrix}. \qquad (9)$$

It is explicitly written as,

$$U^\dagger_{DC} Y_f U_{DC} \qquad (10)$$

$$= \begin{pmatrix} \frac{1}{\sqrt{2}} & -\frac{1}{\sqrt{2}} & 0 \\ \frac{1}{\sqrt{6}} & \frac{1}{\sqrt{6}} & -\frac{2}{\sqrt{3}} \\ \frac{1}{\sqrt{3}} & \frac{1}{\sqrt{3}} & \frac{1}{\sqrt{3}} \end{pmatrix} \left[\frac{K_f}{3} \begin{pmatrix} 1 & 1 & 1 \\ 1 & 1 & 1 \\ 1 & 1 & 1 \end{pmatrix} + \begin{pmatrix} \zeta_f & 0 & 0 \\ 0 & \epsilon_f & 0 \\ 0 & 0 & \delta_f \end{pmatrix} \right]$$

$$\times \begin{pmatrix} \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{6}} & \frac{1}{\sqrt{3}} \\ -\frac{1}{\sqrt{2}} & \frac{1}{\sqrt{6}} & \frac{1}{\sqrt{3}} \\ 0 & -\frac{2}{\sqrt{3}} & \frac{1}{\sqrt{3}} \end{pmatrix} \qquad (11)$$

$$= K_f \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

$$+ \begin{pmatrix} \frac{1}{2}(\zeta_f + \epsilon_f) & \frac{1}{2\sqrt{3}}(\zeta_f - \epsilon_f) & \frac{1}{\sqrt{6}}(\zeta_f - \epsilon_f) \\ \frac{1}{2\sqrt{3}}(\zeta_f - \epsilon_f) & \frac{1}{6}(\zeta_f + \epsilon_f + 4\delta_f) & \frac{1}{3\sqrt{2}}(\zeta_f + \epsilon_f - 2\delta_f) \\ \frac{1}{\sqrt{6}}(\zeta_f - \epsilon_f) & \frac{1}{3\sqrt{2}}(\zeta_f + \epsilon_f - 2\delta_f) & \frac{1}{3}(\zeta_f + \epsilon_f + \delta_f) \end{pmatrix}. \qquad (12)$$

From the hierarchical relation (6), approximate form of this matrix found to be the "cascade texture" [27]

$$U^\dagger_{DC} Y_f U_{DC} \cong \frac{1}{6} \begin{pmatrix} 3\epsilon_f & -\sqrt{3}\epsilon_f & -\sqrt{6}\epsilon_f \\ -\sqrt{3}\epsilon_f & 4\delta_f & -2\sqrt{2}\delta_f \\ -\sqrt{6}\epsilon_f & -2\sqrt{2}\delta_f & 6K_f \end{pmatrix}. \qquad (13)$$

If we assign $\zeta_f = -\epsilon_f$, it leads to the zero texture $(U^\dagger_{DC} Y_f U_{DC})_{11} = 0$ [15,16,19,28], that corresponds the "hybrid texture" in Ref. [27].

Eq. (12) is perturbatively diagonalized as

$$U_f^\dagger Y_f U_f = \text{diag}(y_{1f}, y_{2f}, y_{3f}), \tag{14}$$

where

$$y_{1f} = (\zeta_f + \epsilon_f + \delta_f)/3 - \xi_f/6, \tag{15}$$

$$y_{2f} = (\zeta_f + \epsilon_f + \delta_f)/3 + \xi_f/6, \tag{16}$$

$$y_{3f} = K_f + (\zeta_f + \epsilon_f + \delta_f)/3, \tag{17}$$

with

$$\xi_f = \sqrt{(2\delta_f - \epsilon_f - \zeta_f)^2 + 3(\epsilon_f - \zeta_f)^2}. \tag{18}$$

Here, the second order perturbations $O(\zeta_f^2/\epsilon_f, \epsilon_f^2/\delta_f, \delta_f^2/K_f)$ are all ignored. If we use Eq. (6), the eigenvalues (15)–(17) are approximated as the diagonal elements of Eq. (12),

$$y_{1f} \simeq \frac{1}{2}\epsilon_f, \quad y_{2f} \simeq \frac{2}{3}\delta_f, \quad y_{3f} \simeq K_f. \tag{19}$$

The unitary matrices $U_f = U_{\text{DC}} B_f$ are found to be [15,16,20]

$$B_f = \begin{pmatrix} \cos\theta_f & \sin\theta_f & \lambda_f \sin 2\theta_f \\ -\sin\theta_f & \cos\theta_f & -\lambda_f \cos 2\theta_f \\ -\lambda_f \sin 3\theta_f & \lambda_f \cos 3\theta_f & 1 \end{pmatrix}, \tag{20}$$

where

$$\tan 2\theta_f \simeq -\sqrt{3}\frac{\epsilon_f - \zeta_f}{2\delta_f - \epsilon_f - \zeta_f}, \quad \lambda_f = \frac{\xi_f}{3\sqrt{2}K_f}. \tag{21}$$

Note that this system can be interpreted as a toy model of the mixing between neutral mesons π^0, η^0, η'^0. Indeed, in Eq. (1), the first democratic term corresponds the gluonic anomaly that provides η'^0 mass and the second term does the small quark masses $m_{u,d,s}$ [29]. The mixing angle θ_f (21) is the same form to the π^0–η^0 mixing in the chiral perturbation theory [30].

The similarity between the Yukawa interactions and the neutral meson mixing is indicated since long years ago [31,32], and it suggests that fermion mass matrices might be ruled by some mass gap phenomena or unknown underlying principle.

3. Simplified case: $\zeta_\nu = \zeta_R = c_R = 0$

From the Yukawa matrices Eq. (1) and the mass matrix Eq. (7), the small neutrino mass is obtained by the seesaw mechanism [25]

$$m_\nu = \frac{v^2}{2} Y_\nu^T M_R^{-1} Y_\nu, \tag{22}$$

where $v/\sqrt{2} = \langle H \rangle$ is the vacuum expectation value of the Higgs boson. As the simple and important example, let us consider a simplified parameter set, $\zeta_\nu = \zeta_R = c_R = 0$. In this case, the resulting small neutrino mass is also democratic type with the diagonal breaking term:

$$m_\nu^{(0)} = \frac{v^2}{2}\frac{1}{m_R}\left[\frac{K_\nu^2}{3K_R}\begin{pmatrix} 1 & 1 & 1 \\ 1 & 1 & 1 \\ 1 & 1 & 1 \end{pmatrix} + \begin{pmatrix} 0 & 0 & 0 \\ 0 & \epsilon_\nu^2/\epsilon_R & 0 \\ 0 & 0 & \delta_\nu^2/\delta_R \end{pmatrix}\right]. \tag{23}$$

Note that this is the exact results and no approximation is used.

In order to obtain observed large mixing angles of U_{PMNS}, the diagonalization of m_ν should have only small mixing angles. Otherwise, the diagonalization of m_ν cancels out that of the charged lepton mass, or Y_e, which is almost diagonalized by U_{DC} (9). Then, the following hierarchical relation is required phenomenologically:

$$\frac{K_\nu^2}{3K_R} \ll \frac{\epsilon_\nu^2}{\epsilon_R} \ll \frac{\delta_\nu^2}{\delta_R}. \tag{24}$$

Accordingly, the normal hierarchy (NH) $m_1 \ll m_2 \ll m_3$ is forced for these parameter sets, and both inverted hierarchical and degenerated masses are excluded.

If we treat m_1 as a small perturbation, the mass matrix (23) is diagonalized at the leading order as

$$m_\nu^{(0)} = \begin{pmatrix} m_1 & m_1 & m_1 \\ m_1 & m_2 & m_1 \\ m_1 & m_1 & m_3 \end{pmatrix} \equiv V_\nu m_\nu^{\text{diag}} V_\nu^\dagger, \tag{25}$$

$$\simeq \begin{pmatrix} 1 & \frac{m_1}{m_2-m_1} & \frac{m_1}{m_3-m_1} \\ -\frac{m_1}{m_2-m_1} & 1 & \frac{m_1}{m_3-m_1} \\ -\frac{m_1}{m_3-m_1} & -\frac{m_1}{m_3-m_1} & 1 \end{pmatrix}\begin{pmatrix} m_1 & 0 & 0 \\ 0 & m_2 & 0 \\ 0 & 0 & m_3 \end{pmatrix}$$

$$\times \begin{pmatrix} 1 & -\frac{m_1}{m_2-m_1} & -\frac{m_1}{m_3-m_1} \\ \frac{m_1}{m_2-m_1} & 1 & -\frac{m_1}{m_3-m_1} \\ \frac{m_1}{m_3-m_1} & \frac{m_1}{m_3-m_1} & 1 \end{pmatrix}, \tag{26}$$

where

$$m_1 = \frac{v^2}{2m_R}\frac{K_\nu^2}{3K_R}, \quad m_2 = \frac{v^2}{2m_R}\left(\frac{\epsilon_\nu^2}{\epsilon_R} + \frac{K_\nu^2}{3K_R}\right),$$

$$m_3 = \frac{v^2}{2m_R}\left(\frac{\delta_\nu^2}{\delta_R} + \frac{K_\nu^2}{3K_R}\right). \tag{27}$$

As a result, the neutrino mixing matrix is calculated as

$$U_{\text{PMNS}}^0 = U_e^\dagger V_\nu = B_e^\dagger U_{\text{DC}}^\dagger V_\nu \tag{28}$$

$$\simeq \begin{pmatrix} 1 & \frac{m_e}{\sqrt{3}m_\mu} & \frac{3m_e}{\sqrt{6}m_\tau} \\ -\frac{m_e}{\sqrt{3}m_\mu} & 1 & \frac{m_\mu}{\sqrt{2}m_\tau} \\ -\frac{2m_e}{\sqrt{6}m_\tau} & -\frac{m_\mu}{\sqrt{2}m_\tau} & 1 \end{pmatrix}\begin{pmatrix} \frac{1}{\sqrt{2}} & -\frac{1}{\sqrt{2}} & 0 \\ \frac{1}{\sqrt{6}} & \frac{1}{\sqrt{6}} & -\frac{\sqrt{2}}{\sqrt{3}} \\ \frac{1}{\sqrt{3}} & \frac{1}{\sqrt{3}} & \frac{1}{\sqrt{3}} \end{pmatrix}$$

$$\times \begin{pmatrix} 1 & \frac{m_1}{m_2} & \frac{m_1}{m_3} \\ \frac{-m_1}{m_2} & 1 & \frac{m_1}{m_3} \\ \frac{-m_1}{m_3} & \frac{-m_1}{m_3} & 1 \end{pmatrix} \tag{29}$$

$$\simeq \begin{pmatrix} \frac{1}{\sqrt{2}} + \frac{m_1}{\sqrt{2}m_2} & -\frac{1}{\sqrt{2}} + \frac{m_1}{\sqrt{2}m_2} & 0 \\ \frac{1}{\sqrt{6}} + \frac{m_\mu}{\sqrt{6}m_\tau} - \frac{m_1}{\sqrt{6}m_2} & \frac{1}{\sqrt{6}} + \frac{m_\mu}{\sqrt{6}m_\tau} + \frac{m_1}{\sqrt{6}m_2} & -\sqrt{\frac{2}{3}} + \frac{m_\mu}{\sqrt{6}m_\tau} \\ \frac{1}{\sqrt{3}} - \frac{m_\mu}{2\sqrt{3}m_\tau} - \frac{m_1}{\sqrt{3}m_2} & \frac{1}{\sqrt{3}} - \frac{m_\mu}{2\sqrt{3}m_\tau} + \frac{m_1}{\sqrt{3}m_2} & \frac{1}{\sqrt{3}} + \frac{m_\mu}{\sqrt{3}m_\tau} \end{pmatrix}. \tag{30}$$

In the final expression, we neglect the ratios m_e/m_μ, m_e/m_τ and m_1/m_3.

In this simplified case, we have six free parameters. However, at leading order, free parameters are only three neutrino masses m_i in the mixing matrix (30). In particular, $U_{\mu 3}$ and $U_{\tau 3}$ are expressed by masses of heavy leptons:

$$U_{\mu 3} \simeq -\sqrt{\frac{2}{3}} + \frac{m_\mu}{\sqrt{6}m_\tau} \simeq -0.766, \tag{31}$$

$$U_{\tau 3} \simeq \frac{1}{\sqrt{3}} + \frac{m_\mu}{\sqrt{3}m_\tau} \simeq 0.645. \tag{32}$$

Here, we used the pole masses $m_\mu = 105.6$ MeV and $m_\tau = 1776$ MeV. These components are in the 3σ range of the latest global analysis [33]:

$$|U| = \begin{pmatrix} 0.801 \to 0.845 & 0.514 \to 0.580 & 0.137 \to 0.158 \\ 0.225 \to 0.517 & 0.441 \to 0.699 & 0.614 \to 0.793 \\ 0.246 \to 0.529 & 0.464 \to 0.713 & 0.590 \to 0.776 \end{pmatrix}. \tag{33}$$

Note that the difference between the realistic pattern $|U|$ (33) and the leading order mixing U_{DC} (9) is of $O(0.1)$ for all matrix elements. It suggests that $|U|$ (33) might result from properly perturbed U_{DC} [34]. Therefore, in the next section, we will explore the proper parameter sets of this neutrino mass system, including parameters ζ_ν, ζ_R and c_R.

4. General case: $\zeta_\nu \neq \zeta_R \neq c_R \neq 0$

The system of neutrinos analyzed here has nine free parameters, $K_{\nu,R}$, c_R, $\delta_{\nu,R}$, $\epsilon_{\nu,R}$, and $\zeta_{\nu,R}$ (m_R is essentially not free parameter because its magnitude can be absorbed into other parameters). Hereafter, we will treat the following quantities as free parameters:

$$m_{1,2,3}, \quad c_R, \quad \zeta_\nu, \quad \zeta_R, \quad \frac{K_\nu}{K_R} \equiv r_K, \quad \frac{\delta_\nu}{\delta_R} \equiv r_\delta, \quad \frac{\epsilon_\nu}{\epsilon_R} \equiv r_\epsilon. \tag{34}$$

4.1. Case 1: $\zeta_\nu \neq \zeta_R \neq 0$, $c_R = 0$

For the finite (but small) ζ_ν, ζ_R and $c_R = 0$, the mass matrix m_ν calculated from the seesaw formula (22) will be perturbed expression from the mass matrix for $\zeta_\nu = \zeta_R = c_R = 0$ (23). When we expand the mass matrix in the first order of ζ_ν, ζ_R, the perturbation is found to be

$$m_\nu^{(1)} = m_\nu^{(0)} + \delta m_\nu, \tag{35}$$

where

$$\delta m_\nu = \frac{v^2}{2 m_R} \begin{pmatrix} 0 & -r_\epsilon & -r_\delta \\ -r_\epsilon & 0 & 0 \\ -r_\delta & 0 & 0 \end{pmatrix} \zeta_\nu + \frac{v^2}{2 m_R} \begin{pmatrix} 0 & 0 & 0 \\ 0 & -r_\epsilon^2 & -r_\epsilon r_\delta \\ 0 & -r_\epsilon r_\delta & -r_\delta^2 \end{pmatrix} \zeta_R. \tag{36}$$

Here, we used the relation $r_\delta, r_\epsilon \gg r_K$, obtained from the hierarchical relations (6), (8), and the phenomenologically required relation (24). At first the mass matrix is diagonalized by $V_\nu^{(0)} = V_\nu$ in Eq. (26), and further diagonalized by the proper perturbation $V_\nu^{(1)}$:

$$m_\nu^{(1)} = V_\nu^{(1)\dagger} V_\nu^{(0)\dagger} (m_\nu^{(0)} + \delta m_\nu) V_\nu^{(0)} V_\nu^{(1)}. \tag{37}$$

As a result, the mixing matrix is modified from Eq. (30)

$$U_{PMNS}^{(1)} = U_e^\dagger V_\nu^{(0)} V_\nu^{(1)} = U_{PMNS}^{(0)} V_\nu^{(1)}. \tag{38}$$

Although the explicit form of $V_\nu^{(1)}$ is troublesome, U_{e3} found to be

$$U_{e3} \simeq \frac{\epsilon_\nu \zeta_R}{\sqrt{2} \delta_\nu \epsilon_R} - \frac{\zeta_\nu}{\sqrt{2} \delta_\nu}. \tag{39}$$

It suggests rather large parameters ζ_n, $\zeta_R \simeq 0.2 \delta_\nu$, and contradicts to the hierarchical assumption Eqs. (6), (8). If we consider the unification between quarks and leptons such as SO(10) GUT, this possibility is undesirable because $Y_\nu = Y_u$ and the resulting θ_{13} is too suppressed by $\zeta_\nu/\delta_\nu \lesssim m_u/m_c$, $\epsilon_\nu/\delta_\nu \simeq m_u/m_c$. However, with the assumption ζ_n, $\zeta_R \simeq 0.2 \delta_\nu$, we found some parameter regions where all elements are in 3σ range of Eq. (33).

4.2. Case 2: $\zeta_\nu = \zeta_R = 0$, $c_R \neq 0$

Since the term c_R diag $(1, 1, 1)$ is symmetric under $S_{3L} \times S_{3R}$ symmetry, we assume $c_R \gg \delta_R \gg \epsilon_R$, as in Eq. (8). In this case, parameters δ_R, ϵ_R do not appear at the leading order.

The procedure is rather similar to the simplified case. The mass matrix is found to be

$$m_\nu \simeq \frac{v^2}{2} \frac{1}{m_R} \left[\frac{K_\nu^2}{3K_R} \begin{pmatrix} 1 & 1 & 1 \\ 1 & 1 & 1 \\ 1 & 1 & 1 \end{pmatrix} + \frac{1}{3c_R} \begin{pmatrix} 0 & 0 & 0 \\ 0 & 2\epsilon_\nu^2 & -\delta_\nu \epsilon_\nu \\ 0 & -\delta_\nu \epsilon_\nu & 2\delta_\nu^2 \end{pmatrix} \right]. \tag{40}$$

It is not exact result, because we used hierarchical relation (8).

Eq. (40) is diagonalized at the leading order as

$$m_\nu = \begin{pmatrix} m_1 & m_1 & m_1 \\ m_1 & m_2 & m_{23} \\ m_1 & m_{23} & m_3 \end{pmatrix} \equiv V_\nu m_\nu^{diag} V_\nu^\dagger, \tag{41}$$

$$V_\nu \simeq \begin{pmatrix} 1 & \frac{m_1}{m_2 - m_1} & \frac{m_1}{m_3 - m_1} \\ -\frac{m_1}{m_2 - m_1} & 1 & \frac{m_{23}}{m_3 - m_2} \\ -\frac{m_1}{m_3 - m_1} & -\frac{m_{23}}{m_3 - m_2} & 1 \end{pmatrix}. \tag{42}$$

Here,

$$m_1 = \frac{v^2}{2 m_R} \frac{K_\nu^2}{3K_R}, \tag{43}$$

$$m_2 = \frac{v^2}{2 m_R} \left(\frac{2\epsilon_\nu^2}{3c_R} + \frac{K_\nu^2}{3K_R} \right) \equiv m_1 + \delta m_{21}, \tag{44}$$

$$m_3 = \frac{v^2}{2 m_R} \left(\frac{2\delta_\nu^2}{3c_R} + \frac{K_\nu^2}{3K_R} \right) \equiv m_1 + \delta m_{31}, \tag{45}$$

and

$$m_{23} = \frac{v^2}{2 m_R} \left(\frac{K_\nu^2}{3K_R} - \frac{\delta_\nu \epsilon_\nu}{3c_R} \right). \tag{46}$$

Indeed, this parameter m_{23} is not independent from the mass m_1 and the mass differences $\delta m_{i1} \equiv m_i - m_1$ ($i = 2, 3$):

$$m_{23} = m_1 - \frac{1}{2} \sqrt{\delta m_{21} \delta m_{31}}. \tag{47}$$

Then, the mass matrix (41) is determined by the three neutrino masses m_i.

In this case, the neutrino mixing matrix U_{PMNS} is approximately given by product of Eq. (30) and a mixing matrix

$$U_{PMNS} = B_e^\dagger U_{DC}^\dagger V_\nu \simeq U_{PMNS}^{(0)} \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & \frac{-m_{23}}{m_3 - m_2} \\ 0 & \frac{m_{23}}{m_3 - m_2} & 1 \end{pmatrix}. \tag{48}$$

Accordingly, U_{e3} is found to be

$$U_{e3} \simeq \frac{1}{\sqrt{2}} \frac{m_{23}}{m_3 - m_2} \left(1 - \frac{m_1}{m_2} \right). \tag{49}$$

Then, if we treat U_{e3} as a perturbative input parameter, U_{PMNS} is expressed by known parameters except m_i:

U_{PMNS}

$$\simeq \begin{pmatrix} \frac{1}{\sqrt{2}} + \frac{m_1}{\sqrt{2}m_2} & -\frac{1}{\sqrt{2}} + \frac{m_1}{\sqrt{2}m_2} & 0 \\ \frac{1}{\sqrt{6}} + \frac{m_\mu}{\sqrt{6}m_\tau} - \frac{m_1}{\sqrt{6}m_2} & \frac{1}{\sqrt{6}} + \frac{m_\mu}{\sqrt{6}m_\tau} + \frac{m_1}{\sqrt{6}m_2} & -\sqrt{\frac{2}{3}} + \frac{m_\mu}{\sqrt{6}m_\tau} \\ \frac{1}{\sqrt{3}} - \frac{m_\mu}{2\sqrt{3}m_\tau} - \frac{m_1}{\sqrt{3}m_2} & \frac{1}{\sqrt{3}} - \frac{m_\mu}{2\sqrt{3}m_\tau} + \frac{m_1}{\sqrt{3}m_2} & \frac{1}{\sqrt{3}} + \frac{m_\mu}{\sqrt{3}m_\tau} \end{pmatrix}$$

$$\times \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & -\sqrt{2}\,U_{e3}\left(\frac{m_1}{m_2}-1\right)^{-1} \\ 0 & \sqrt{2}\,U_{e3}\left(\frac{m_1}{m_2}-1\right)^{-1} & 1 \end{pmatrix}.$$

$$(50)$$

At the leading order, $U_{\mu3}$ and $U_{\tau3}$ are written by all known parameters:

$$U_{\mu3} \simeq -\frac{1}{\sqrt{3}}U_{e3} - \sqrt{\frac{2}{3}} + \frac{m_\mu}{\sqrt{6}m_\tau} = -0.701 \to -0.713, \qquad (51)$$

$$U_{\tau3} \simeq -\sqrt{\frac{2}{3}}U_{e3} + \frac{1}{\sqrt{3}} + \frac{m_\mu}{\sqrt{3}m_\tau} = 0.723 \to 0.740. \qquad (52)$$

Here, we used $m_\mu = 105.6$ MeV, $m_\tau = 1776$ MeV, and $U_{e3} = -0.137 \to -0.158$ (negative value is more preferred). These two elements are in 3σ range of Eq. (33). However, these formulas can have roughly $5 \sim 10\%$ error which come from the second order perturbations of m_1/m_2, m_2/m_3, and U_{e3}.[1] They are inevitable predictions of this model (assuming CP conservation).

If we set $m_1 = 0$, it determines other masses $\delta m_{21} = m_2 \simeq 0.008$ eV and $\delta m_{31} = m_3 \simeq 0.05$ eV. U_{e3} is also determined from Eq. (49),

$$U_{e3} = -\frac{\sqrt{m_2 m_3}}{2\sqrt{2}(m_3 - m_2)} \simeq -0.168. \qquad (53)$$

This value is close to the global fit $|U_{e3}| = 0.137 \to 0.158$. Then, treating m_1 as a perturbative parameter, we can predict m_1 from the current error of U_{e3}:

$$m_1 = (0.2 \to 0.6) \text{ meV}. \qquad (54)$$

On the other hand, consistency between Eq. (50) and the latest global analysis Eq. (33), the mass ratio of lighter neutrinos is predicted as

$$0.138 \lesssim \frac{m_1}{m_2} \lesssim 0.150. \qquad (55)$$

Then, the mass eigenvalues are found to be

$$m_1 \simeq (1.1 \to 1.4) \text{ meV}. \qquad (56)$$

There is a tension between the predictions from U_{e3} and $U_{e(1,2,3)}$, $U_{\mu(1,2,3)}$. However, if we adopt $\zeta_e = -\epsilon_e$ same as the previous study [24], U_{e3} becomes finite at the zeroth order, $U_{e3} = -\sqrt{\frac{2m_e}{3m_\mu}} \simeq -0.056$. In this case the tension will be successfully reconciled. Then we tentatively discard the prediction (54). The relation between U_{e3} and m_i will be reevaluated in the next study.

The neutrino masses predicted from Eq. (56) are found to be

$$m_1 \simeq (1.1 \to 1.4) \text{ meV}, \quad m_2 \simeq (8.5 \to 9.1) \text{ meV},$$

$$m_3 \simeq (48 \to 51) \text{ meV}. \qquad (57)$$

To show an example, when we set the parameters as follows,

$$\frac{m_1}{m_2} = 0.14, \quad \Rightarrow \quad U_{e3} = -0.127, \qquad (58)$$

and the numerical value of the U_{PMNS} will be

$$U_{\text{PMNS}} = \begin{pmatrix} 0.822 & -0.575 & -0.127 \\ 0.380 & 0.692 & -0.625 \\ 0.455 & 0.466 & 0.772 \end{pmatrix}. \qquad (59)$$

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[1] The terms like $U_{e3}\frac{m_1}{m_2}$ are also regarded as the second order perturbations.

In this matrix, all elements are in 3σ range of Eq. (33), except U_{e3}. It shows that the large mixing angles consistent with the experiments are possible from the democratic mass matrices. In the second-order perturbation, the predictability becomes little lower. However, we can consider SO(10) GUT or Pati–Salam models for the hierarchical Y_ν. Relating Y_ν and Y_u in some manner, several free parameters in the neutrino sector are expected to be removed. Furthermore, the derivation in this paper remains only at the tree level. The radiative corrections [35–37] and threshold correction [38] will modify the results. We leave it for our future work.

4.3. Relating observables and CP violation

Effective mass in double beta decay experiment $\langle m_{ee} \rangle$

$$\langle m_{ee} \rangle = \sum_{i=1}^{3} m_i U_{ei}^2, \qquad (60)$$

is calculated from Eqs. (50) and (57) as

$$\langle m_{ee} \rangle \simeq m_1 \left(\frac{1}{\sqrt{2}} + \frac{m_1}{\sqrt{2}m_2}\right)^2 + m_2 \left(-\frac{1}{\sqrt{2}} + \frac{m_1}{\sqrt{2}m_2}\right)^2 + m_3 U_{e3}^2$$

$$(61)$$

$$\simeq \frac{m_2 - m_1}{2} + m_3 U_{e3}^2 \simeq 4.5 \text{ meV}. \qquad (62)$$

In this study, we assumed all parameters are real. Meanwhile, several studies survey CP violation in democratic matrices [39,40]. Since the S_3 symmetry prohibits the relative phase between matrix elements, nontrivial phases are associate with the breaking parameters. When the breaking term is diagonal, CP violation is introduced by the following replacement

$$\begin{pmatrix} \zeta_f & 0 & 0 \\ 0 & \epsilon_f & 0 \\ 0 & 0 & \delta_f \end{pmatrix} \rightarrow \begin{pmatrix} \zeta_f e^{i\phi_1} & 0 & 0 \\ 0 & \epsilon_f e^{i\phi_2} & 0 \\ 0 & 0 & \delta_f e^{i\phi_3} \end{pmatrix}. \qquad (63)$$

These CP phases can produce baryon asymmetry of universe by the leptogenesis [41], as in the previous study [24]. Furthermore, the leptonic CP phases might be relate the hadronic ones in the viewpoint of GUT. In this case, the leptogenesis might also be restricted in some extent by the CKM phase.

5. Conclusions and discussions

In this paper, we obtain the light neutrino masses and mixings consistent with the experiments, in the democratic texture approach. The ansatz is that ν_{Ri} are assumed to transform as "right-handed fields" $\mathbf{2_R} + \mathbf{1_R}$ under the $S_{3L} \times S_{3R}$ symmetry. The symmetry breaking terms are assumed to be diagonal and hierarchical, which is basically same as the previous studies. This setup only allows the normal hierarchy of the neutrino masses, and excludes both of inverted hierarchical and degenerated neutrinos.

Although the neutrino sector has nine free parameters, several predictions are obtained at the leading order. When we neglect the smallest parameters ζ_ν and ζ_R, the resulting neutrino matrix m_ν has only three parameters and then determined from the neutrino masses m_i. Therefore, all components of the mixing matrix U_{PMNS} are expressed by the masses of light neutrinos and charged leptons. From the consistency between predicted and observed U_{PMNS}, we obtain the lightest neutrino masses $m_1 = (1.1 \to 1.4)$ meV, and the effective mass for the double beta decay $\langle m_{ee} \rangle \simeq 4.5$ meV.

In the second-order perturbation, the predictability becomes little lower. However, the hierarchical Y_ν can be unified to other Yukawa interactions in SO(10) GUT or Pati–Salam models. Relating

Y_ν and Y_u in some manner, several free parameters in the neutrino sector are expected to be removed. Meanwhile, the derivation in this paper remains only at tree level. The radiative corrections and threshold corrections will modify the results. We leave it for our future work.

Acknowledgements

References

[1] Y. Fukuda, et al., Super-Kamiokande Collaboration, Phys. Rev. Lett. 81 (1998) 1562, arXiv:hep-ex/9807003.
[2] Q.R. Ahmad, et al., SNO Collaboration, Phys. Rev. Lett. 89 (2002) 011301, arXiv:nucl-ex/0204008.
[3] F.P. An, et al., Daya Bay Collaboration, Phys. Rev. Lett. 108 (2012) 171803, arXiv:1203.1669 [hep-ex].
[4] J.K. Ahn, et al., RENO Collaboration, Phys. Rev. Lett. 108 (2012) 191802, arXiv:1204.0626 [hep-ex].
[5] Z. Maki, M. Nakagawa, S. Sakata, Prog. Theor. Phys. 28 (1962) 870.
[6] B. Pontecorvo, Sov. Phys. JETP 26 (1968) 984, Zh. Eksp. Teor. Fiz. 53 (1967) 1717.
[7] N. Cabibbo, Phys. Rev. Lett. 10 (1963) 531.
[8] M. Kobayashi, T. Maskawa, Prog. Theor. Phys. 49 (1973) 652.
[9] C.D. Froggatt, H.B. Nielsen, Nucl. Phys. B 147 (1979) 277.
[10] H. Ishimori, T. Kobayashi, H. Ohki, Y. Shimizu, H. Okada, M. Tanimoto, Prog. Theor. Phys. Suppl. 183 (2010) 1, arXiv:1003.3552 [hep-th].
[11] I. de Medeiros Varzielas, et al., arXiv:1210.6239 [hep-ph].
[12] H. Fritzsch, Phys. Lett. B 73 (1978) 317.
[13] H. Fritzsch, Z.z. Xing, Prog. Part. Nucl. Phys. 45 (2000) 1, arXiv:hep-ph/9912358.
[14] H. Harari, H. Haut, J. Weyers, Phys. Lett. B 78 (1978) 459.
[15] Y. Koide, Phys. Rev. D 28 (1983) 252.
[16] Y. Koide, Phys. Rev. D 39 (1989) 1391.
[17] M. Tanimoto, Phys. Rev. D 41 (1990) 1586.
[18] H. Fritzsch, J. Plankl, Phys. Lett. B 237 (1990) 451.
[19] H. Fritzsch, Z.z. Xing, Phys. Lett. B 372 (1996) 265, arXiv:hep-ph/9509389.
[20] M. Fukugita, M. Tanimoto, T. Yanagida, Phys. Rev. D 57 (1998) 4429, arXiv:hep-ph/9709388.
[21] H. Fritzsch, Z.-z. Xing, Phys. Lett. B 598 (2004) 237, arXiv:hep-ph/0406206.
[22] Z.-z. Xing, D. Yang, S. Zhou, Phys. Lett. B 690 (2010) 304, arXiv:1004.4234.
[23] S. Zhou, Phys. Lett. B 704 (2011) 291, arXiv:1106.4808 [hep-ph].
[24] M. Fujii, K. Hamaguchi, T. Yanagida, Phys. Rev. D 65 (2002) 115012, arXiv:hep-ph/0202210.
[25] P. Minkowski, Phys. Lett. B 67 (1977) 421;
     T. Yanagida, KEK Report No. KEK-79-18, p. 95;
     M. Gell-Mann, P. Ramond, R. Slansky, North-Holland, Amsterdam, 1979.
[26] M. Fukugita, M. Tanimoto, T. Yanagida, Phys. Rev. D 59 (1999) 113016, http://dx.doi.org/10.1103/PhysRevD.59.113016, arXiv:hep-ph/9809554.
[27] N. Haba, R. Takahashi, M. Tanimoto, K. Yoshioka, Phys. Rev. D 78 (2008) 113002, arXiv:0804.4055.
[28] H. Fritzsch, Z.-z. Xing, Phys. Lett. B 440 (1998) 313, arXiv:hep-ph/9808272.
[29] H. Fritzsch, P. Minkowski, Nuovo Cimento A 30 (1975) 393.
[30] A. Pich, Rep. Prog. Phys. 58 (1995) 563, arXiv:hep-ph/9502366.
[31] H. Fritzsch, MPI-PAE/PTh-60/88.
[32] H. Fritzsch, Print-88-0375 (MUNICH), MPI-PAE/PTh-22/88.
[33] M.C. Gonzalez-Garcia, M. Maltoni, T. Schwetz, arXiv:1512.06856, 2015.
[34] Z.-z. Xing, Chin. Phys. C 36 (2012) 281, arXiv:1203.1672.
[35] N. Haba, Y. Matsui, N. Okamura, T. Suzuki, Phys. Lett. B 489 (2000) 184, arXiv:hep-ph/0005064.
[36] J.-w. Mei, Z.-z. Xing, Phys. Lett. B 623 (2005) 227, arXiv:hep-ph/0506304.
[37] S. Ray, Int. J. Mod. Phys. A 25 (2010) 4339, arXiv:1005.1938.
[38] S. Gupta, C.S. Kim, P. Sharma, Phys. Lett. B 740 (2015) 353, arXiv:1408.0172.
[39] M. Fukugita, M. Tanimoto, T. Yanagida, arXiv:hep-ph/9903484.
[40] K. Fukuura, T. Miura, E. Takasugi, M. Yoshimura, Phys. Rev. D 61 (2000) 073002, http://dx.doi.org/10.1103/PhysRevD.61.073002, arXiv:hep-ph/9909415.
[41] M. Fukugita, T. Yanagida, Phys. Lett. B 174 (1986) 45, http://dx.doi.org/10.1016/0370-2693(86)91126-3.